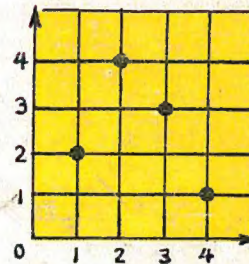
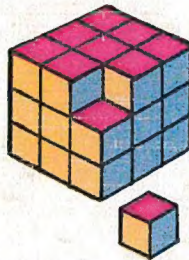


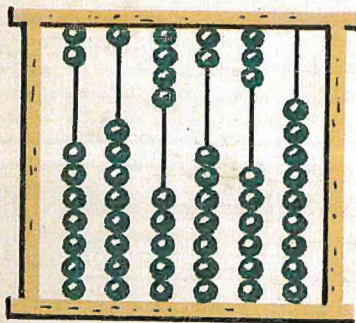
MATHEMATICS

4

3	5	4	
3	5	4	1
6	1	0	8
9	1	5	1
1	2	0	1
1	5	2	5
1	8	3	0
2	1	3	5
2	4	0	3
2	7	4	5



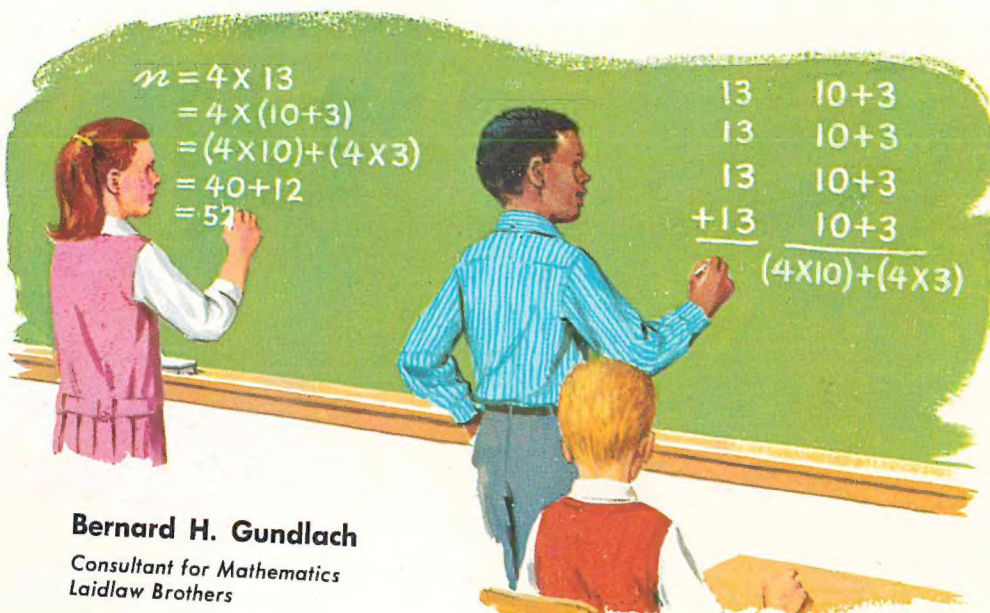
$$\square + 3 = 7$$
$$6 - 1 > 2$$



IV

4

MATHEMATICS 4



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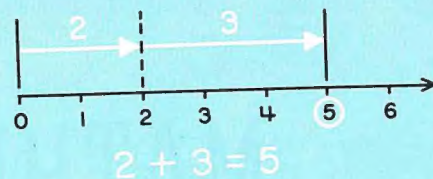
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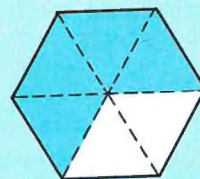
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$$4 \times 3 = 12$$



$$12 \div 3 = 4$$



$$\frac{4}{6}$$

$$\frac{2}{3}$$

1 inch

1 sq.in.



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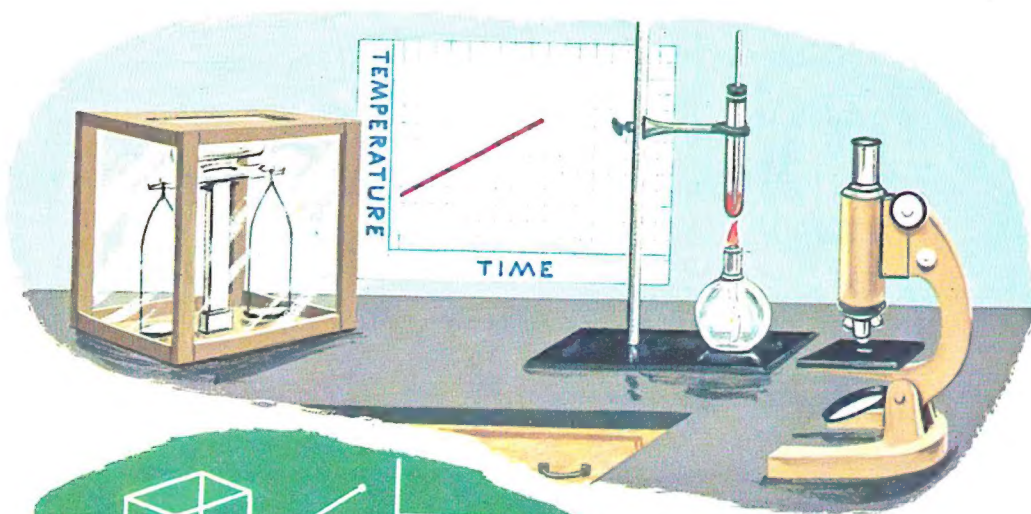
1 2 3 4 5 6 7 8 9 5 4 3 2 1 0 9 8 7

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$$3 \times 2 = \square$$



Chapter 1

SETS, SYMBOLS, AND SENTENCES

Sets and Subsets



Look at the picture above. What are some of the different kinds of objects shown? Think of all these objects as forming one collection. Then you are thinking of the objects as forming a set.

Each object in a set is called a **member** of that set. Is a roller skate a member of the set shown above? **Yes**

We can also think of sets *contained within* other sets. Is the set of stars a set contained within the set shown above? **Yes**

Name some other sets within the set shown above. A set contained within another set is called a **subset** of the set which contains it.

Oral Describe the sets which are indicated below. **Answers will vary.**

1. Three different subsets of the set shown above **set of stars; set of toys; set of cups**

2. A set of objects in your classroom **the set of pupils' desks**

3. A subset of the set described in Oral 2 **the set of pupils' desks in one row**

Written A set can be named by naming its members in braces, as

{Bob, Joe, Tom, Mary}

Use braces to name these sets.

Answers will vary.

1. A set of 3 boys in your class
{Bill, Bob, Jim}

2. A set of 4 girls in your class
{Mary, Joan, Ann, Sherry}

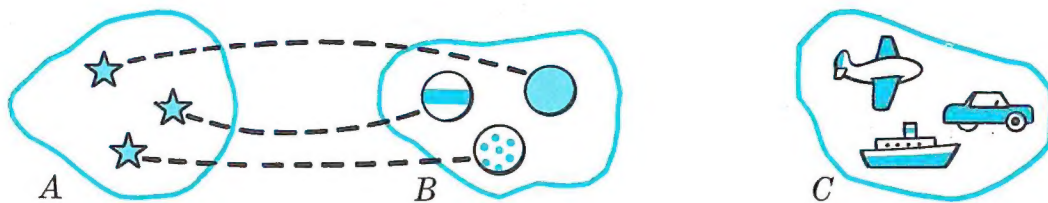
3. A set of 2 objects for writing
{pen, pencil}

*1 skate, ball, star, cup, block

*2 set of balls; set of cups; set of blocks

(NOTE: Throughout this text, symbols like *1, *2, etc., indicate that the answers are given as footnotes.)

Equivalent Sets



Look at the picture above. The stars are shown with a ring drawn around them. This is done to show that they are being thought of as a *set*. What other sets are shown above? **the set of 3 balls; the set of 3 toys**

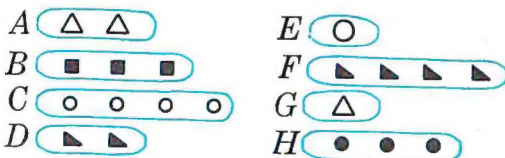
The letter *A* is printed below the picture of the set of stars to give this set a simpler name. Now, instead of saying "the set of stars shown above," we can say "set *A*." Which set has been named set *B*? Set *C*? **the set of 3 balls; the set of 3 toys**

Notice the lines that match or pair the members of set *A* with the members of set *B*. When each member of a set is paired with one and only one member of another set, the sets are said to be **matched one-to-one**. Can set *A* be matched one-to-one with set *C*? ^{*1} Sets which can be matched one-to-one are called **equivalent sets**. Is set *B* equivalent to set *C*? **Yes**

Oral For each set named below, describe two equivalent sets. **Answers will vary.**

- The set of wings on a bird
{your hands}; {your eyes}
- The set of legs on a horse
{wheels on a wagon}; {a, b, c, d}

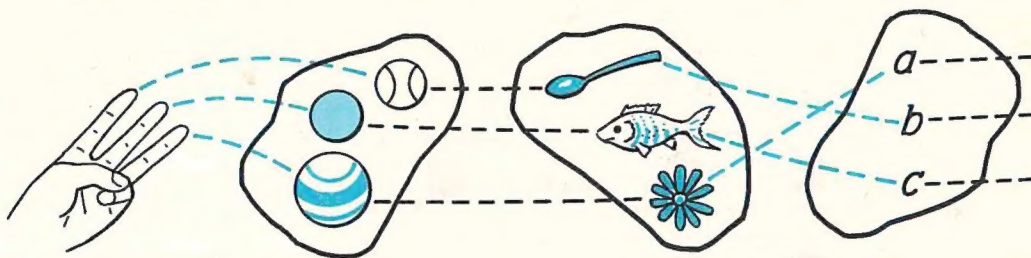
Written Copy and complete each sentence at the right. Refer to the set pictures shown below.



- Set *A* is equivalent to set D.
- Set *D* is equivalent to set A.
- Set B is equivalent to set *H*.
- Set H is equivalent to set *B*.
- Set G is equivalent to set *E*.
- Set *C* is equivalent to set F.
- Set *F* is equivalent to set C.
- Set E is equivalent to set *G*.

Number

All sets which can be matched one-to-one may be thought of as forming a *set of sets*. Study the set pictures below.



Can any set shown above be matched one-to-one with every other set shown? Can you think of other sets that could be matched to the sets above? **Yes; Yes**

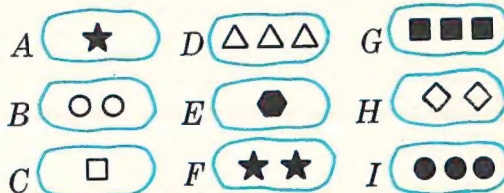
All sets containing this many members have a common property. This property is called a **number**. The *number* shown by the set of sets pictured above is named **three** or **3**.

All sets that can be matched one-to-one have the same number.

Oral For each set named below, name several other sets having the same number. **Answers will vary.**

1. The set of tusks on a walrus
{your legs}; {your arms}
2. The set of trunks on an elephant
{your head}; {a}
3. {a,b,c}
{wheels on a tricycle}; {1,2,3}
4. {1,2,3,4}
{four seasons}; {a,b,c,d}
5. The set of all the days of the week
{continents on earth};
{1,2,3,4,5,6,7}
6. {#, @, €, *}
{corners of a square}; {legs on a dog}

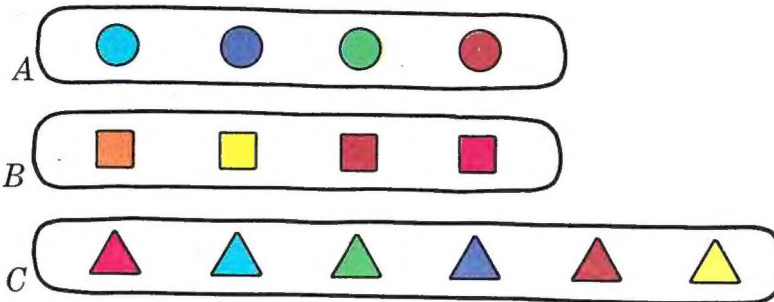
Written Use the set pictures below to complete each sentence.



1. Sets A, C, and E all have the same number.
2. Sets D, G, and I all have the same number.
3. Sets H, B, and F all have the same number.

Numerals and Number Words

The **numerals** 0,1,2,3,4,5,6,7,8, and 9 are names for numbers. The **number words** zero, one, two, three, four, five, six, seven, eight, and nine are also names for numbers. Which number word names the same number as the numeral 4? Which numeral names the same number as the number word six? **four; 6**



Let us use the symbol $n(A)$ to stand for the number of set A. In the same way, $n(B)$ will stand for the number of set B. What will $n(C)$ stand for? **the number of set C**

We use the **equality symbol** $=$ to show that two symbols name the same number. For example, $n(A)=4$ means that $n(A)$ and 4 name the same number. We read $n(A)=4$ as *the number of set A is equal to four*.^{*1} How would you read $n(B)=4$?^{*2} How would you read $n(C)=6$?^{*3} How would you read $n(A)=n(B)$?^{*3}

Oral Answer each of the following questions.

1. If set B can be described as the set of all the days of the week, which number word names $n(B)$?

seven

2. We may think of the set having no members. This set is called the **empty set**. Which number word names the number of the empty set?

zero

3. If set E can be described as the set of all of your classmates who are 70 years old, is it true that $n(E)=0$? **Yes**

4. How do you read $n(E)=0$? **The number of set E is equal to zero.**

5. If set A is named as $\{x,y\}$, then $n(A)=2$. If set B is named as $\{z\}$, then $n(B)=1$. If set C is named as $\{\}$, which numeral names $n(C)$? **0**








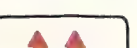


8

*1 The number of set B is equal to four.

*2 The number of set C is equal to six.

*3 The number of set A is equal to the number of set B.

Written Copy and complete the table below so that the set picture, numeral, and number word in each row show the same number.

	Set picture	Number word	Numeral
1.		<u>three</u>	<u>3</u>
2.		five	<u>5</u>
3.		<u>one</u>	<u>1</u>
4.		<u>four</u>	<u>4</u>
5.		<u>nine</u>	<u>9</u>
6.		zero	<u>0</u>
7.		<u>eight</u>	<u>8</u>
8.		<u>two</u>	<u>2</u>
9.		seven	<u>7</u>
10.		<u>six</u>	<u>6</u>

Tell how In your own words, tell how each of the following statements may be explained.

1. $n(A) = n(A)$ Every whole number is equal to itself.
2. If $n(A) = n(B)$, then $n(B) = n(A)$.
See below.
3. If $n(A) = n(B)$ and $n(B) = n(C)$, then $n(A) = n(C)$. See below.

- Tell how**
2. The symbols on either side of the $=$ may be interchanged.
 3. If two numbers, $n(A)$ and $n(C)$, are equal to the same number, $n(B)$, they are equal to each other.

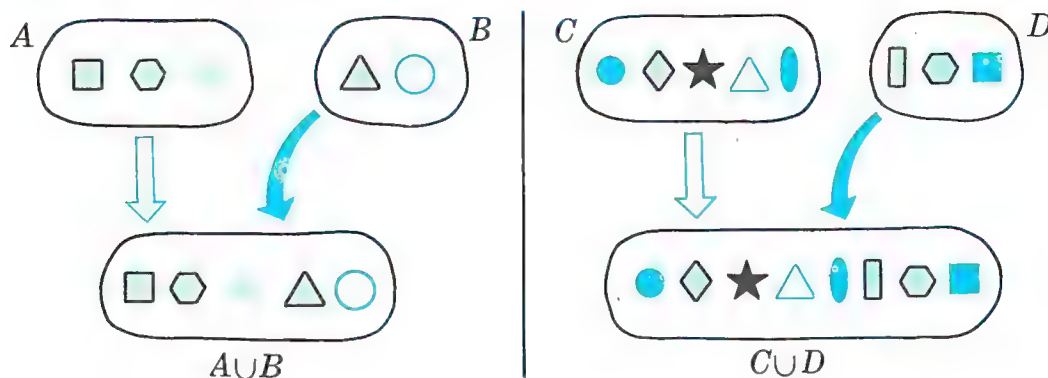
Can you do this? Besides being used to tell how many members are in a set, the numerals 1, 2, 3, 4, 5, 6, 7, 8, and 9 may be used to tell the *positions* or the *order* of the members of a set. For example, we can write "this is desk number 2 in row number 4." This statement tells us the position of the desk in relation to the other desks in the room.

When used in this way, the numerals refer to **ordinal numbers**. The ordinal words **first**, **second**, **third**, **fourth**, **fifth**, **sixth**, **seventh**, **eighth**, and **ninth** may also be used to tell position or order.

Copy. Complete the following sentences by filling each blank with an ordinal word.

1. Desk number 3 in row number 5 may also be called the ___ desk in the ___ row. **third; fifth**
2. If chair number 6 is in front of chair number 7, then the ___ chair is behind the ___ chair. **seventh; sixth**
3. John entered the room before anyone else. So John was the ___ person here. **first**
4. Al entered right after John. So Al was the ___ person here. **second**
5. Alice was the last child in a line of nine children. So she was the ___ child in line. **ninth**

Union of Sets and Addition of Numbers



Two sets may be joined to form a new set. For example, look at the picture at the left above. Set B may be joined to set A . The resulting set is called the **union** of set A and set B . The symbol $A \cup B$ is used to name the union of set A and set B . The symbol $A \cup B$ is read “ A union B .”

Look at the picture at the right above. How is $C \cup D$ read? How was the set named $C \cup D$ formed? **C union D ; by joining set D to set C**

The union of sets may be used to explain the addition of numbers. For sets A and B shown above, the addition of the numbers of the sets may be explained as follows. The number of set B **added** to the number of set A is equal to the number of set $A \cup B$. This last sentence may also be written as follows.

$$n(A) + n(B) = n(A \cup B)$$

By counting, we can find that $n(A)=3$, $n(B)=2$, and $n(A \cup B)=5$. Therefore, $n(A) + n(B) = n(A \cup B)$ becomes the following addition sentence.

$$3 + 2 = 5$$

For sets C and D shown above, does $n(D)=3$, $n(C)=5$, and $n(C \cup D)=8$? Then, what addition sentence may be used for $n(C) + n(D) = n(C \cup D)$? **Yes; Yes; Yes; $5 + 3 = 8$**

Oral Several sets are named below by listing the names of their members within braces. Use this list to name the members of each union of sets indicated below.

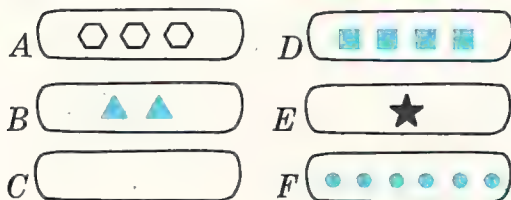
$A = \{\text{Bob, Tom}\}$ $B = \{\text{Phil}\}$
 $C = \{\text{Mary, Joan, Sue}\}$ $D = \{\}$
 $E = \{\text{Jim, Carol, Doris, John}\}$

Answers for column
 a b

1. $A \cup B$
 $\{\text{Bob, Tom, Phil}\}$
 2. $B \cup A$
 $\{\text{Phil, Bob, Tom}\}$
 3. $E \cup B$
 $\{\text{Jim, Carol, Doris, John, Phil}\}$
 4. $B \cup E$
 $\{\text{Phil, Jim, Carol, Doris, John}\}$
 5. $A \cup D$
 $\{\text{Bob, Tom}\}$
 6. $D \cup A$
 $\{\text{Bob, Tom}\}$
 7. $A \cup E$
 $\{\text{Bob, Tom, Jim, Carol, Doris, John}\}$

b shown below.
 $B \cup C$
 $C \cup B$
 $A \cup C$
 $C \cup A$
 $E \cup D$
 $D \cup E$
 $B \cup D$

Written Look at the set pictures below. Then write addition sentences by using numerals to replace $n(A)$, $n(B)$, $n(A \cup B)$, and so on, below and in the next column.



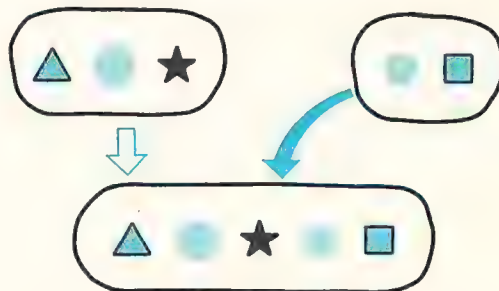
1. $n(A) + n(B) = n(A \cup B)$ $3+2=5$

2. $n(B) + n(A) = n(B \cup A)$ $2+3=5$

- Oral 1b $\{\text{Phil, Mary, Joan, Sue}\}$
 2b $\{\text{Mary, Joan, Sue, Phil}\}$
 3b $\{\text{Bob, Tom, Mary, John, Sue}\}$
 4b $\{\text{Mary, Joan, Sue, Bob, Tom}\}$
 5b $\{\text{Jim, Carol, Doris, John}\}$
 6b $\{\text{Jim, Carol, Doris, John}\}$ 7b $\{\text{Phil}\}$

3. $n(E) + n(A) = n(E \cup A)$
 $1+3=4$
 4. $n(C \cup A) = n(C) + n(A)$
 $3+0=3$
 5. $n(A \cup D) = n(A) + n(D)$
 $7=3+4$
 6. $n(B \cup F) = n(B) + n(F)$
 $8=2+6$
 7. $n(A) + n(C) = n(A \cup C)$
 $3+0=3$
 8. $n(A) + n(C) = n(A)$
 $3+0=3$
 9. $n(C) + n(D) = n(D)$
 $0+4=4$

Study the following set picture for the addition sentence $3+2=5$.



Draw a set picture, as shown above, for each addition sentence below.

See typical answer above.

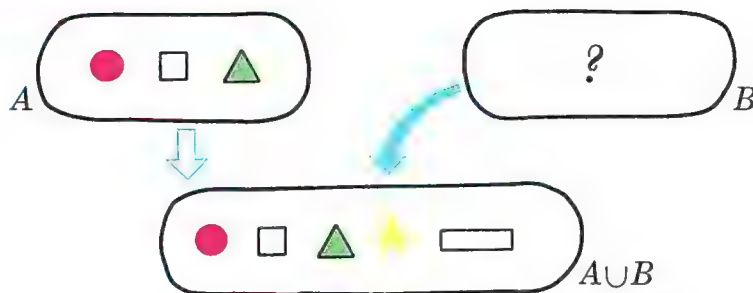
10. $2+3=5$ $4+1=5$ $1+4=5$

11. $4+2=6$ $3+3=6$ $2+4=6$

12. $4+3=7$ $5+2=7$ $6+1=7$

Tell why If you know set A and set B , then it is possible to name set $A \cup B$. However, if you know set $C \cup D$, it is *not* always possible to name set C and set D . Tell why. You cannot tell which members come from set C and which members come from set D .

Finding an Unnamed Addend



Suppose you know which objects are the members of the union of two sets and you also know the members of one of the sets. Can you find which objects are the members of the other set? Look at the picture above. Which objects are the members of $A \cup B$? Of set A ? Which objects must be the members of set B ? Why? \star and \square ; These objects are not in A , but are in $A \cup B$.

Since we know which objects are the members of set $A \cup B$ and set A , we also know $n(A \cup B)$ and $n(A)$. What are these two numbers? We can write the following addition sentence.

$$3 + n(B) = 5$$

The numbers being added are called **addends**. Does the sentence above show that one addend is not named by a numeral? We call this addend an **unnamed addend**. Which objects should be in set B ? What number is $n(B)$ or the unnamed addend? \star, \square ; 2

Oral In each of the following, you are given the union of two sets and one of the sets. Name the members of the other set. *Answers given for disjoint sets only.*

1. $A \cup B$ is {Jack, Al, Ray, Ed}; set A is {Jack, Ray}. *Al, Ed*

2. $C \cup D$ is {a, b, c, d, e, f}; set D is {b, d, f}. *a, c, and e*

3. $E \cup F$ is {x, y, z}; set E is { }. *x, y, and z*

12

*1 Yes

*2 $\triangle, \bigcirc, \square, \star$, and \square

*3 \triangle, \bigcirc , and \square

*4 3 and 5

*5 Yes

4. $G \cup H$ is {r, s, t, u}; set G is {r, s, t, u}. *Set H is { }.*

5. $I \cup J$ is { }; set I is { }. *Set J is { }.*

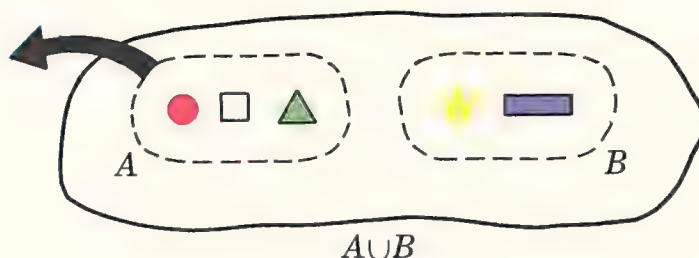
Written Write an addition like $3 + n(B) = 5$ for Oral 1–5. Find the unnamed addends. Name them as follows: $n(B) = 2$. *See below.*

Tell why If set A is {a, b, c}, then set $A \cup B$ cannot be {a, b}. *Set $A \cup B$ must contain every member of set A.*

Written

1. $2 + n(B) = 4$; $n(B) = 2$
2. $3 + n(C) = 6$; $n(C) = 3$
3. $0 + n(F) = 3$; $n(F) = 3$
4. $4 + n(H) = 4$; $n(H) = 0$
5. $0 + n(J) = 0$; $n(J) = 0$

Subtraction



The subtraction of numbers may be explained by using sets. For the sets shown above, if subset A is removed from set $A \cup B$, which subset remains? We write $n(A \cup B) - n(A) = n(B)$. What are the numbers $n(A \cup B)$ and $n(A)$? Then we can write the following subtraction sentence.

$$5 - 3 = n(B)$$

Since we know which objects are the members of set B , can we find $n(B)$? What number is $n(B)$? **Yes; 2**

Oral In each of the following, you are given the names of the members of a set and one of its subsets. Name the members of the other subset.

Answers for disjoint subsets only. 1. $A \cup B$ is {Jack, Al, Ray, Ed}; set A is {Jack, Ray}.

Ed, Al

2. $C \cup D$ is {a,b,c,d,e,f}; set D is {b,d,f}.

a, c, and e

3. $E \cup F$ is {x,y,z}; set E is { }.

x, y, and z

4. $G \cup H$ is {r,s,t,u}; set G is {r,s,t,u}.

Set H is { }.

5. $I \cup J$ is { }; set I is { }.

Set J is { }.

*1 subset B

*2 5, 3

Written For each exercise in *Oral*, write a subtraction sentence like $5 - 3 = n(B)$ and then find $n(B)$.

See below.

Another way Subtraction may also be explained as finding an unnamed addend. For example, both of the sentences below may be thought of as asking the same question: What number must be added to 3 to get 5?

$$5 - 3 = n(B)$$

$$3 + n(B) = 5$$

For each subtraction sentence you wrote for *Written* above, write an addition sentence. Then, find the unnamed addend.

Written

Another way

1. $4 - 2 = n(B)$; 2

1. $2 + n(B) = 4$; 2

2. $6 - 3 = n(C)$; 3

2. $3 + n(C) = 6$; 3

3. $3 - 0 = n(F)$; 3

3. $0 + n(F) = 3$; 3

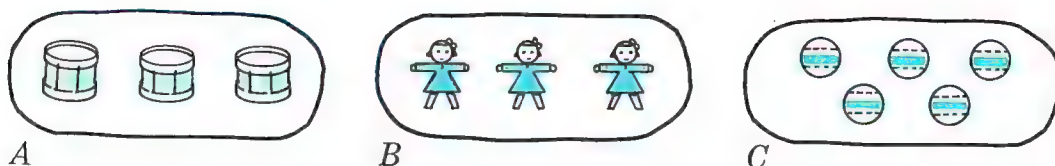
4. $4 - 4 = n(H)$; 0

4. $4 + n(H) = 4$; 0

5. $0 + 0 = n(J)$; 0

5. $0 + n(J) = 0$; 0

Non-Equivalent Sets



Can set A be matched one-to-one with set B ? Is set A equivalent to set B ? Does $n(A) = n(B)$? **Yes; Yes; Yes**

Can set B be matched one-to-one with set C ? Is set B equivalent to set C ? Does $n(B) = n(C)$? **No; No; No**

Two sets which cannot be matched one-to-one are called **non-equivalent sets**.

Non-equivalent sets do not have the same number of members.

We show that the numbers of two sets are not the same by using the **inequality symbol** \neq as follows.

$$n(B) \neq n(C)$$

Reading \neq as *is not equal to*, how would you read $n(B) \neq n(C)$?
The number of set B is not equal to the number of set C .

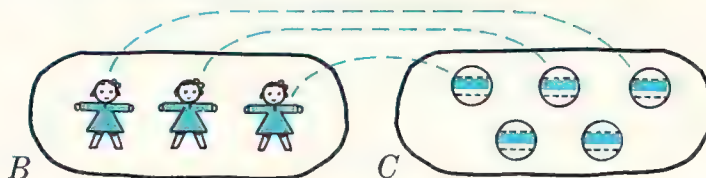
Oral Tell whether the sets in each pair shown below are equivalent or non-equivalent.

1. A B
non-equivalent
2. C D
equivalent
3. E F
non-equivalent
4. G H
non-equivalent

Written Copy each of the sentences below. Replace each \neq with the correct symbol ($=$ or \neq). Use the sets given in *Oral* for exercises 1 and 2.

1. $n(A) \neq n(B)$ $n(C) \neq n(D)$
2. $n(E) \neq n(F)$ $n(A) \neq n(C)$
3. $2+2 \neq 4$ $3+4 \neq 9$
4. $7-4 \neq 5$ $8 \neq 2+6$

Is Less Than; Is Greater Than



Are sets B and C above equivalent or non-equivalent? Why?
non-equivalent; Their members cannot be matched one-to-one.

Which set shown above has some unmatched members? Which set has all of its members matched? The set with some unmatched members is said to have the **greater** number. The set with all of its members matched is said to have the **lesser** number.

The inequality symbols $<$ and $>$ are used to show which of two numbers is the lesser or greater. The symbol $<$ is read *is less than*. The symbol $>$ is read *is greater than*. How would you read the following two sentences?

The number of set B $n(B) < n(C)$ is less than the number of set C . $n(C) > n(B)$ The number of set C is greater than the number of set B .

Oral Tell whether the first number of each pair named below is greater or lesser than the second number.

- | a | b |
|---------------------|------------------|
| 1. 3 ● 5
less | 5 ● 3
greater |
| 2. 2 ● 1
greater | 1 ● 2
less |
| 3. 8 ● 6
greater | 6 ● 8
less |
| 4. 4 ● 0
greater | 0 ● 4
less |
| 5. 0 ● 1
less | 1 ● 0
greater |

Written Copy the sentences in *Oral* above. Replace each ● with the correct symbol ($<$ or $>$).

- | | |
|----------|----------------------|
| *1 set C | <u>Written</u> |
| *2 set B | 1. $3 < 5$; $5 > 3$ |
| | 2. $2 > 1$; $1 < 2$ |
| | 3. $8 > 6$; $6 < 8$ |
| | 4. $4 > 0$; $0 < 4$ |
| | 5. $0 < 1$; $1 > 0$ |

Can you do this? The greater of two numbers can be named as the lesser number plus some other number greater than zero.

For example, $5 > 3$. Therefore, 5 can be named as 3 plus some number greater than zero as follows.

$$5 = 3 + 2$$

For each exercise in *Oral* name the greater number as the lesser number plus some other number.

See below.

Tell how The three symbols \neq , $<$, and $>$ may all be used to write a sentence about 5 and 3. How? $3 < 5$;

$$5 > 3; 5 \neq 3$$

Can you do this?

1. $3 + 2 = 5$; $5 = 3 + 2$
2. $2 = 1 + 1$; $1 + 1 = 2$
3. $8 = 6 + 2$; $6 + 2 = 8$
4. $4 + 0 = 4$; $0 + 4 = 4$
5. $0 + 1 = 1$; $1 = 0 + 1$

Equations and Inequalities

From your study of English, you probably know that a sentence usually contains a *verb*. Which of the following are sentences?

Jane walks to school. **sentence** He is playing ball. **sentence**
John and Mary. **not a sentence** She sews dresses. **sentence**

A number sentence must contain a verb. The verbs we shall use are expressed by $=$, \neq , $<$, and $>$. A number sentence which uses an equality symbol ($=$) is called an **equation**. A number sentence which uses an inequality symbol (\neq , $<$, or $>$) is called an **inequality**.

Number sentences may be *equations* or *inequalities*.

Oral Tell whether each of the following *is* or *is not* a sentence. If it is a sentence tell whether it is an equation or an inequality. _____ denotes
"Is not a sentence." a b c

1. $2+2=4$ **equation** $4+6$ _____ $9-4=5$ **equation**
2. $7-5\neq 4$ **inequality** $6-2<5$ **inequality** $3+2<7$ **inequality**
3. $5+(8-6)$ _____ $6-4$ _____ $8>5+3$ **inequality**

Copy the sentences below. Replace each \square with the correct symbol ($=$, $<$, or $>$).

- | | | |
|--------------------|-----------------|---------------|
| a | b | c |
| 4. $4 \square 5-1$ | $8-3 \square 7$ | $4 \square 6$ |
| 5. $5-1 \square 4$ | $9 \square 5+2$ | $6 \square 4$ |
| 6. $7 \square 8-3$ | $5+2 \square 9$ | $3 \square 5$ |

Written Copy the sentences below. Replace each \square with the correct symbol ($=$ or \neq).

- | | |
|--------------------|-----------------|
| a | b |
| 1. $3 \square 7-4$ | $8+1 \square 4$ |
| 2. $4+2 \square 6$ | $6 \square 4+2$ |
| 3. $8-5 \square 4$ | $4 \square 8-5$ |

For each true sentence, write T. Copy each false sentence, but change the symbol $=$, \neq , $<$, or $>$ to make it a true sentence.

- | | |
|----------------------------|---------------------------|
| a | b |
| 7. $2+0 \stackrel{T}{=} 2$ | $5-0 \stackrel{=}{=} 5$ |
| 8. $2+1 \neq 1+2$ | $2+1 \stackrel{T}{=} 1+2$ |
| 9. $0+3 \stackrel{=}{=} 3$ | $1 \stackrel{=}{=} 0+1$ |

Open and Closed Sentences

$3+5=9$

$7<6$

$3+5=8$

$7-3\neq 2$

$4-3>0$

$0>4-3$

*1

*2

Which of the sentences above are true? Which are false? Every sentence which is either true or false is called a **closed sentence**.

Are the following sentences true or false? **cannot tell**

He is a fourth grade pupil.

$6+\square=8$

Suppose you replace the word *He* by the name of a boy in your class. Would the sentence be true? ^{*3} Suppose you replace \square by 2. Would the sentence be true? ^{*4} Suppose you replace the word *He* by your father's name. Would the sentence be false? ^{*5} Suppose you replace \square by 3. Would the sentence be false? **Yes**

Before you made the replacements above, the two sentences were neither true nor false.

A sentence which is neither true nor false is called an **open sentence**. An *open sentence* in mathematics will contain some symbol, such as \square , \triangle , a , or n , which holds the place for a numeral.

Number sentences may be **open** or **closed**.

Oral Tell if each of the following is a sentence. Then tell if each sentence is open or closed.

- John Adams was President.
sentence; closed
- He is my father.
sentence; open
- ___ is my teacher.
sentence; open
- $5+4=7$
sentence; closed
- $5+\square=9$
sentence; open

Written For each open sentence, write both a true closed sentence and a false closed sentence. **Answers may vary. Correct answer given in \square .**

False sentence listed below.

- | | |
|-----------------------------|---------------------------------|
| 1. $5+2=\square$
$5+2=3$ | $\square=5+2$
$8=5+2$ |
| 2. $5+\square=7$
$5+9=7$ | $7=\square+2$
$7=1+2$ |
| 3. $4<\square$
$4<3$ | $4\neq\square$
$4\neq 4$ |
| 4. $\square>4$
$3>4$ | $4\neq\square+3$
$4\neq 1+3$ |

*1 $3+5=8$; $7-3\neq 2$; $4-3>0$

*2 $3+5=9$; $7<6$; $0>4-3$

*3 Yes

*4 Yes

*5 Yes

Replacement Sets

The closed sentences below set A are listed as *True* or *False*. They are formed by replacing \square in $3+\square=5$ by one of the numerals used to name the numbers in set A .

How are the closed sentences below set B formed? by replacing \square in $\square+2>3$ by one of the numerals used to name the numbers in set B

$$A = \{0, 1, 2, 3, 4\}$$

$$3 + \square = 5$$

$$B = \{1, 3, 5, 7\}$$

$$\square + 2 > 3$$

True
 $3+2=5$

False
 $3+0=5$
 $3+1=5$
 $3+3=5$
 $3+4=5$

True
 $3+2>3$
 $5+2>3$
 $7+2>3$

False
 $1+2>3$

The set of numbers which may be named in place of \square in an open sentence is called the **replacement set**.

Which set of numbers is the replacement set for $3+\square=5$?
Why? For $\square+2>3$? Why?

Each number in the replacement set for which the open sentence is true is called a **solution**.

The number 2 is a solution of $3+\square=5$. What are the solutions of $\square+2>3$? 3, 5, 7

Oral Use $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ as the replacement set. Tell the solution or solutions for each open sentence.

- $5 + \overset{a}{\square} = 9$ $5, 4, 3, 2, 1, 0$ $2 + \overset{c}{\square} < 8$ See below.
- $7 - 5 = \overset{2}{\square}$ $\overset{8, 9}{\square} - 4 > 3$ $\square + 3 \neq 8$ See below.
- $\overset{4}{\square} + 2 = 6$ $6 - 4 = \overset{2}{\square}$ $4 + \square \neq 7$ See below.

- 18 **Oral**
- 1c $0, 1, 2, 3, 4, 5$
 - 2c $0, 1, 2, 3, 4, 6, 7, 8, 9$
 - 3c $0, 1, 2, 4, 5, 6, 7, 8, 9$

Written Copy. Use $\{1, 3, 5, 7, 9\}$ as the replacement set. Find the solution or solutions for each open sentence.

- $6 + \overset{a}{\square} = 9$ $7 > \overset{b}{\square}$ $2 + \overset{c}{\square} < 7$
- $7 - 4 = \overset{3}{\square}$ $\square - 4 = 5$ $\square + 2 \neq 7$
- $\overset{3}{\square} + 3 = 6$ $6 - 5 = \overset{1}{\square}$ $4 + \overset{1}{\square} = 5$

- *1 set A
*2 The numerals for the numbers in set A were used to replace \square in $\square+2=5$.
*3 set B
*4 The numerals for the numbers in set B were used to replace \square in $\square+2>3$.

Solution Sets

To solve an open sentence means to find all the possible solutions. What is a solution? ^{*1} All the solutions are usually named as members of a set called the **solution set**. For example, if $\{0,1,2,3,4,5,6\}$ is the replacement set for the open sentence $2+\square < 5$, the *solution set* is $\{0,1,2\}$. Why? $2+\square < 5$ becomes a true sentence only when \square is replaced by 0, 1, or 2.

Is $2+\square < 5$ a true sentence when \square is replaced by 0? By 1? By 2? Is $2+\square < 5$ a true sentence for any other number in the replacement set? **Yes; Yes; Yes; No**

If a replacement set is not named for an open sentence, then you may use the set of all numbers as the replacement set.

Oral Answer these questions.

1. Is the solution set a subset of the replacement set? Is this always so? Why? **Yes; Yes; Each solution is a member of the replacement set.**
2. Can a solution set contain only one member? **Yes**
3. Can a solution set be empty? **Yes**

Written Use the set named below as the replacement set. Find the solution set for each open sentence.

$\{0,1,2,3,4,5,6,7,8,9\}$

a

b

1. $5+\square = 6$

$\{1\}$

2. $5+\square > 6$

$\{2,3,4,5,6,7,8,9\}$

3. $5+\square < 6$

$\{0\}$

4. $5+\square \neq 6$

$\{0,2,3,4,5,6,7,8,9\}$

$\square + 5 = 6$

$\{1\}$

$\square + 5 > 6$

$\{2,3,4,5,6,7,8,9\}$

$\square + 5 < 6$

$\{0\}$

$\square + 5 \neq 6$

$\{0,2,3,4,5,6,7,8,9\}$

Can you do this? In each exercise below, there is given part of an open sentence and its solution set. Copy and complete each open sentence by replacing \square with the correct symbol ($=$, \neq , $<$, or $>$). Use $\{2,3,4,5,6\}$ as the replacement set.

Open sentence	Solution set
1. $2+\square \square 6$	$\{4\}$
2. $2+\square \square 6$	$\{2,3\}$
3. $2+\square \square 6$	$\{5,6\}$
4. $2+\square \square 6$	$\{2,3,5,6\}$
5. $7-\square \square 1$	$\{6\}$
6. $7-\square \square 1$	$\{\}$
7. $7-\square \square 1$	$\{2,3,4,5\}$

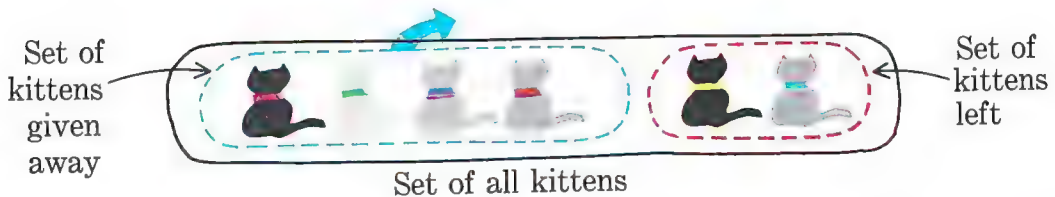
^{*1} A solution of an open sentence is a member of the replacement set for which the open sentence is true.

Solving Problems

Scott had 6 kittens. He gave 4 of the kittens away. How many kittens did Scott have left? **2**

You can solve a problem such as this by using an open sentence. But first, think of the sets which the problem tells about. Thinking about the sets will help you decide which open sentence you should use.

For this problem you may think about the sets this way.

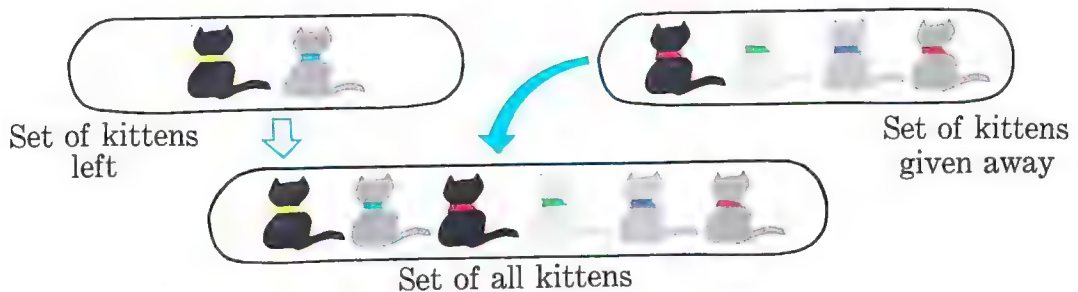


Is a subset being removed from a set? Can you write the following subtraction sentence about the numbers of the sets? **Yes; Yes**

$$6 - 4 = \square$$

Now solve the open sentence, and write the answer by naming the number of kittens left. Is the answer *2 kittens left*? **2; Yes**

For this problem you may also think about the sets this way.



Is one set being joined to another? Can you write the following open addition sentence about the numbers of the sets? **Yes; Yes**

$$\square + 4 = 6$$

Oral Answer these questions.

1. Can all three of the following open sentences be used to solve the problem on page 20? **Yes**

$$6 - 4 = \square$$

$$\square + 4 = 6$$

$$4 + \square = 6$$

2. How would you think about the sets in the problem before writing each open sentence in *Oral* 1?

Answers for Oral 2-9 on p. T21.

Tell how you would think about sets to solve each problem below.

3. Laura had 5 scarves. She gave 2 to her sister. How many scarves did Laura have left? $5 - 2 = \square$;

3 scarves

4. Steve has tropical fish. He has 3 guppies and 2 angelfish. How many fish does he have in all? $3 + 2 = \square$;

5 fish

5. There were 8 boys playing ball. Then 2 of the boys left. How many of the boys remained? $8 - 2 = \square$;

6 boys

6. Marsha had 6 phonograph records. Then her mother gave Marsha 2 more records. How many records did Marsha have in all? $6 + 2 = \square$;

8 records

7. There were 9 children at a party. Only 4 of the children were girls. How many were boys? $9 - 4 = \square$;

5 boys

8. Jimmy had 6 kittens. He gave 2 of them away. How many kittens did Jimmy have left? $6 - 2 = \square$;

4 kittens

9. There were 6 girls jumping rope. Then 3 of them went home. How many of the girls remained? $6 - 3 = \square$;
3 girls

Written Write an open sentence for each problem in *Oral* 4-9. Then solve each open sentence and write the answer to each problem. **See beneath each problem in Oral.**

Another way Write a different open sentence for each problem in *Oral* 4-9. Then solve each open sentence and write the answer to each problem. Check to see that your second answer agrees with your first answer for each problem. **See below.**

Can you do this? Many story problems may be written for an open sentence. For example, the two story problems below may be written for $4 + \square = 7$.

Doris picked 7 roses. She placed 4 of the roses in a blue vase and all of the rest in a white vase. How many roses did she place in the white vase? **3 roses**

There were 7 ducks on a pond. Then 4 of the ducks flew away. How many ducks were left? **3 ducks**

Write two story problems for each open sentence below.

Answers will vary.

a

b

1. $\square + 2 = 6$

$8 - \square = 2$

2. $4 + \square = 5$

$\square - 3 = 5$

Another Way

4. $2 + 3 = \square$; 5

5. $8 = \square + 2$; 6

6. $2 + 6 = \square$; 8

7. $9 = \square + 4$; 5

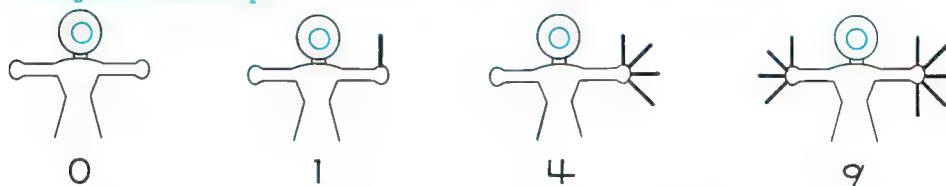
8. $2 + \square = 6$; 4

9. $3 + \square = 6$; 3

Decimal Numerals

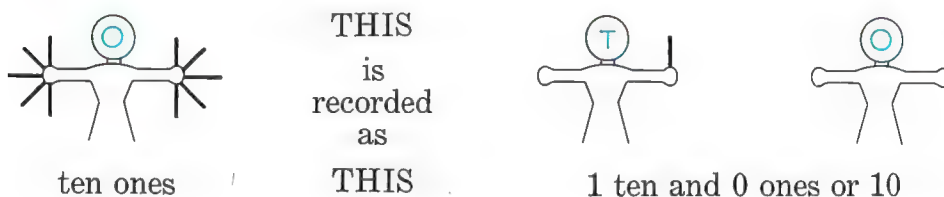
The ten numerals 0,1,2,3,4,5,6,7,8 and 9 are called **digits**. These are the only symbols we use in our system for naming numbers. This system is called the **decimal system**. *Decimal* means *ten*.

To explain how our system of decimal numerals works, we can use **counting men**. A counting man can record numbers by holding up fingers. Look at the following pictures. How can the same counting man record different numbers by holding up certain fingers? **The number recorded is the number of fingers held up.**



The counting man shown above in four different positions is called the **Ones man**. The ones man records ones only. Suppose the ones man has recorded *ten ones*. Can he record one more? Why? **No; He has used all of his fingers.**

To record numbers greater than ten, a second counting man acts as a **Memory man** for the ones man. This second man is called the **Tens** or **T man**. The tens man uses one of his fingers to record as *one ten* each record of *ten ones* made by the ones man.



Since the tens man keeps a record of each *ten ones* recorded by the ones man, the ones man is never left with all fingers extended. As soon as the ones man has recorded *ten ones*, he returns to the closed fists position.

Oral Answer these questions.

1. Suppose the tens man has recorded one ten and the ones man has returned to the closed fists position. Can the ones man begin to count again? **Yes**

2. When the ones man has recorded ten ones for a second time, what must the tens man do? How many fingers will the tens man show then? **Extend another finger; 2**

3. When the ones man has recorded ten ones for the ninth time, what must the tens man do? How many fingers will the tens man show then? **Extend another finger; 9**

4. Why is a numeral like 10 called a two-digit numeral? **Two place-value positions are used.**

Written For each exercise, name the number in 3 ways as shown below.

<i>Tens man</i>		<i>Ones man</i>
seven fingers		three fingers
7 tens	plus	3 ones
70	+	3
		73

<i>Tens man</i>		<i>Ones man</i>
1. one finger		two fingers
1 ten plus 2 ones; 10+2; 12		
2. two fingers		one finger
2 tens plus 1 one; 20+1; 21		
3. three fingers		seven fingers
3 tens plus 7 ones; 30+7; 37		
4. seven fingers		three fingers
7 tens plus 3 ones; 70+3; 73		

5. five fingers zero fingers
5 tens plus 0 ones; 50+0; 50

6. zero fingers five fingers
0 tens plus 5 ones; 0+5; 5

7. one finger zero fingers
1 ten plus 0 ones; 10+0; 10

8. zero fingers one finger
0 tens plus 1 one; 0+1; 1

Copy. Write the decimal numeral for each of the following.

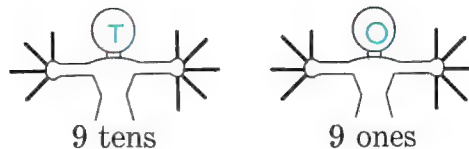
- | | |
|-------------------------------|-----------|
| 9. four tens plus three ones | 43 |
| 10. three tens plus four ones | 34 |
| 11. five tens plus one one | 51 |
| 12. one ten plus five ones | 15 |
| 13. nine tens plus zero ones | 90 |
| 14. eight tens plus five ones | 85 |
| 15. one ten plus zero ones | 10 |
| 16. zero tens plus one one | 1 |

Copy the equations below. Replace each \square with the correct numeral.

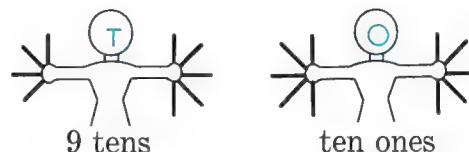
- | <i>a</i> | <i>b</i> |
|-------------------------|---------------------|
| 17. $35 = 30 + \square$ | $35 = \square + 5$ |
| 18. $53 = 50 + \square$ | $53 = \square + 3$ |
| 19. $71 = 70 + \square$ | $71 = \square + 1$ |
| 20. $17 = \square + 7$ | $17 = 10 + \square$ |
| 21. $20 = 20 + \square$ | $20 = \square + 0$ |
| 22. $29 = 20 + \square$ | $\square + 9 = 29$ |
| 23. $39 = 30 + \square$ | $9 + \square = 39$ |

Three-Digit Numerals

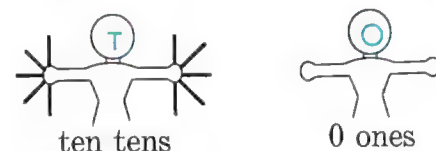
Suppose the tens man and the ones man have recorded 9 tens and 9 ones as shown at the right.



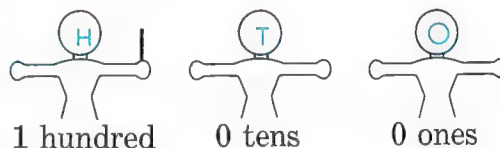
If the ones man records one more, he will show *ten ones*. But, the tens man will record the *ten ones* as *one ten*.



Now the T man has all his fingers extended. But, just as the ones man is never left with all fingers out, the T man must be returned to the closed fists position also. Why? *so he is ready to record another count*



Therefore, a third man is used. He records the *ten tens* as *one ten-ten* or *one hundred*. This third man is called the **Hundreds** or **H man**.



Oral Answer each of the following questions.

1. Suppose the H man has recorded one hundred and the T man and ones man have returned to the closed fists position. Can the T man and ones man begin to record numbers again? **Yes**

2. Suppose the H man shows 1 finger, the T man shows 9 fingers, and the ones man shows 9 fingers. If one more is to be recorded, what changes will the men make? What

will be their final position? What number will be named? **See below.**

3. Suppose the H man shows 2 fingers, the T man shows 9 fingers, and the ones man shows 9 fingers. If one more is to be recorded, what changes will the men make? What will be their final position? **See below.**

4. If the three men continue counting, what will be their final position before a fourth counting man is needed? **H man 9 fingers; T man 9 fingers; Ones man 9 fingers**

- 24 Oral** 2. The ones man extends another finger. Then the ones man closes hands and the T man extends another finger. The T man then closes hands and the H man extends another finger. **H man 2 fingers; T man 0 fingers; Ones man 0 fingers.**
3. **See Oral 2; H man 3 fingers; T man 0 fingers; Ones man 0 fingers**

Written When the H man shows *six* fingers, the T man shows *four* fingers, and the ones man shows *two* fingers, the three men together show:

6 hundreds plus 4 tens plus 2 ones

$$\begin{array}{r} 600 \quad + \quad 40 \quad + \quad 2 \\ \hline 642 \end{array}$$

Each exercise below tells the number of fingers shown by an H man, a T man, and a ones man. For each exercise name the number in the three ways shown above.

<i>H man</i>	<i>T man</i>	<i>Ones man</i>
1. six <i>See above.</i>	four	two
2. four	six	two
<i>Answers for Oral 2-6 on p. T25.</i>		
3. two	four	six
4. two	six	four
5. five	zero	one
6. eight	eight	eight

Copy the equations below. Replace each \square with the correct numeral.

$$7. 193 = 100 + 90 + \square^3$$

$$8. 193 = 100 + \square^{90} + 3$$

$$9. 193 = \square^{100} + 90 + 3$$

$$10. 293 = 200 + \square^{90} + 3$$

$$11. 239 = 200 + 30 + \square^9$$

$$12. 923 = \square^{900} + 20 + 3$$

Copy the following sentences. Replace each \bullet with the correct symbol ($=$, $<$, $>$).

$$13. 100 \bullet 100 + 1$$

$$14. 932 \bullet 900 + 30 + 2$$

$$15. 932 \bullet 900 + 30 + 3$$

$$16. 563 \bullet 600 + 30 + 5$$

$$17. 732 \bullet 700 + 30 + 2$$

$$18. 789 \bullet 788$$

$$19. 123 \bullet 321$$

$$20. 169 \bullet 196$$

Can you do this? Write the numerals described in the following.

1. Using each of the digits 1, 2, and 3 once and only once, write as many three-digit numerals as possible. **123, 132, 231, 213, 312, 321**

2. Using each of the digits 0, 8, and 9 once and only once, write as many three-digit numerals as possible. **980, 908, 809, 890**

See p. T25 for note on 089 and 098.

3. Name the greatest number it is possible to name by using each of the digits 5, 7, and 1 once and only once. **751**

4. Name the least number it is possible to name by using each of the digits 5, 7, and 1 once and only once. **157**

Different Names for the Same Number

We have already named the same number in different ways. Let us review some of these ways now.

six hundreds plus two tens plus seven ones \leftarrow **Number Word**
 $600+20+7 \leftarrow$ **Expanded Notation**
 $627 \leftarrow$ **Decimal Numeral**

The **number word** names the number in words as *so many hundreds plus so many tens plus so many ones*. The number word above has a shorter form which we will discuss later.

The **expanded notation** names the number with numerals and plus signs as *so many hundreds plus so many tens plus so many ones*. The number of each—hundreds, tens, and ones—is never greater than 9.

The **decimal numeral** names the number with digits placed one after the other. The digit on the right is understood to name ones. The digit second from the right is understood to name tens. We agree to call the decimal numeral the **simplest numeral** for a number.

Oral Tell if each number *is* or *is not* named in expanded notation. If it is not, explain why. denotes "Is in expanded notation."

- | | a | b |
|---------------|-------------|-----|
| 1. $300+50+2$ | $200+150+2$ | |
| | $300+50+2$ | |
| 2. $400+70+1$ | $400+60+11$ | |
| | $400+70+1$ | |
| 3. $750+9+2$ | $700+60+1$ | |
| | $700+60+1$ | |
| 4. $300+50+0$ | $300+50$ | |
| | $300+50+0$ | |
| 5. $500+0+9$ | $400+100+9$ | |
| | $500+0+9$ | |

Written Name each of the following in expanded notation.

- | | a | b | c |
|----|------------|------------|------------|
| 1. | 429 | 249 | 924 |
| | $400+20+9$ | $200+40+9$ | $900+20+4$ |
| 2. | 444 | 333 | 230 |
| | $400+40+4$ | $300+30+3$ | $200+30+0$ |

Give the simplest numeral for each number named below.

- | | a | b | c |
|----|------------|------------|---------|
| 3. | $600+80+9$ | $800+60+9$ | $900+9$ |
| | 689 | 869 | 909 |

Egyptian Numerals

We can better understand our system of decimal numerals by studying other systems. A system of numerals which was used earlier than the decimal system is the **Egyptian system**. A few basic Egyptian symbols are shown below.

<i>Egyptian Symbol</i>	<i>Meaning</i>	<i>Object Pictured</i>
I	one	a staff
∩	ten	a heel bone
☉	hundred	a coil of rope

To name numbers in the Egyptian system the basic symbols may simply be repeated. For example, to name 2, the symbol for one is written twice as II. To name 20, the symbol for ten is written twice as ∩∩. Does ∩∩II name 22? Why? Does ☉☉ name 200? Why? Does ☉☉∩∩II name 222? Why?

Yes; $10+10+1+1=22$; Yes; $100+100=200$; Yes; $100+100+10+10+1+1=222$

Oral Answer the following questions.

1. Why would IIIIIIIIIII not be the best Egyptian numeral for 10?
too long, hard to recognize

2. In the Egyptian numeral ∩∩, what number does the first ∩ name? What number does the second ∩ name?
ten; ten

3. In the decimal numeral 22, what number does the first 2 name? What number does the second 2 name?
twenty; two

4. 67 and ∩∩∩∩∩∩IIIIIIII name the same number. Which numeral do you prefer? Why? Which numeral is easier to read? Why? 67; It is easier to write; 67; It is shorter.

Written 6b. ☉☉☉☉∩∩∩∩∩∩IIII

7b. ☉☉☉☉☉☉☉☉☉∩∩∩∩∩∩∩IIIIIIII

Written Write an Egyptian numeral for each decimal numeral and a decimal numeral for each Egyptian numeral below.

a	b
1. ☉☉∩IIII	23
2. ☉☉∩IIII	32
3. ☉☉☉∩∩	123
4. ☉∩∩II	333
5. ☉∩∩∩IIII	405
6. ☉☉☉☉∩∩II	483
7. ☉∩∩∩∩II	See below.
	999
	See below.
	27

Roman Numerals

Another ancient system of numerals is the **Roman system**. Numbers are named by simply repeating the basic symbols. To name 3, the symbol for one is written three times as III. However, a symbol is not repeated more than three times in succession in a single numeral.

IV means $5-1$ or 4

VI means $5+1$ or 6

What does IX mean? What does XI mean? $10-1$ or 9; $10+1$ or 11

The symbols V and L are used only once in a numeral.

Basic Roman Symbols

<i>Roman Symbol</i>	<i>Meaning</i>
I	one
V	five
X	ten
L	fifty
C	hundred

Study the following to learn how Roman numerals are used.

<i>Decimal Numeral</i>	<i>Roman Numeral</i>	<i>Decimal Numeral</i>	<i>Roman Numeral</i>	<i>Decimal Numeral</i>	<i>Roman Numeral</i>
1	I	11	XI	40	XL
2	II	12	XII	41	XL I
3	III	13	XIII	44	XLIV
4	IV	14	XIV	45	XL V
5	V	15	XV	49	XLIX
6	VI	19	XIX	50	L
7	VII	20	XX	51	L I
8	VIII	24	XXIV	60	LX
9	IX	25	XXV	61	LX I
10	X	29	XXIX	90	XC

Oral Answer these questions.

- Does XXX mean $10+10+10$? **Yes**
- Is the numeral VV needed? **No; X is the basic symbol for ten.**
- Does VI mean $5+1$ or $5-1$? **$5+1$**
- Does XL mean $50+10$ or $50-10$? **$50-10$**
- Does XC mean $100+10$ or $100-10$? **$100-10$**

6. Does CX mean 100+10 or 100-10? **100+10**

7. Does a Roman numeral indicate subtraction whenever I precedes V or X and whenever X precedes L or C? **Yes**

8. Think about Roman numeral LXXVIII. In reading from left to right, does each new symbol name a lesser number? Does this numeral indicate addition only? Tell the decimal numeral for LXXVIII.

Yes; Yes; 78

9. When the same symbol is repeated in a Roman numeral, such as X in XXX, does each of the symbols name the same number? **Yes**

10. How many times is X repeated in succession in XXXIX? Is this an acceptable Roman numeral? **3; Yes**

11. Both 88 and LXXXVIII name the same number. Which numeral do you prefer? Why? Which numeral is easier to read? Why? **88; It's shorter; 88; You need not add.**

Written Write a decimal numeral for each Roman numeral.

a	b	c
1. II 2	XX 20	XXX 30
2. VI 6	XI 11	XXI 21
3. IV 4	IX 9	XIX 19
4. LX 60	XL 40	XC 90
5. CX 110	XLVII 47	LXVIII 68

6. XLIX 49	LXVI 66	CXLI 141
7. CCC 300	CCCXC 390	CCXCIX 299
8. CCXIV 214	CCLXX 270	CCXLIX 249

Write a Roman numeral for each decimal numeral.

a	b	c
9. 8 VIII	9 IX	5 V
10. 13 XIII	14 XIV	17 XVII
11. 22 XXII	24 XXIV	29 XXIX
12. 31 XXXI	34 XXXIV	38 XXXVIII
13. 40 XL	44 XLIV	49 XLIX
14. 384 CCCLXXXIV	154 CLIV	169 CLXIX
15. 378 CCCLXXVIII	162 CLXII	399 CCCXCIX

Another way In some uses of Roman numerals a basic symbol may be repeated *four* times in succession in a single numeral. Perhaps you have seen 4 named as IIII on a clockface. With this other way, instead of naming 40 as XL, we can name it as XXXX. Using this rule for repeating a symbol, write another Roman numeral for each of the following.

a	b	c
1. XIV XIIII	IX VIIII	CXL CXXXX
2. XC LXXXX	XLV XXXV	CXC CLXXXX
3. XCIX LXXXXVIIII	XLIX XXXXVIII	XLIV XXXXIIII

Number Words

You already know how to read and write many number words. But you may not have noticed the patterns in number words. You can find these patterns by comparing expanded notation and number words.

$13 = 10 + 3$	thirteen	← “teen” pattern
$34 = 30 + 4$	thirty-four	← “ty” pattern
$100 = 100 + 0 + 0$	one hundred	} ← _____ hundred _____ pattern
$305 = 300 + 0 + 5$	three hundred five	
$956 = 900 + 50 + 6$	nine hundred fifty-six	

Oral Answer these questions.

1. In the “teen” number words, what does “teen” name? What does the other part of the “teen” number word name? **ten; ones**

2. In the “ty” number words, what does the part before the hyphen name? The part after the hyphen? **the number of tens; the number of ones**

3. In the “ty” number words, can the part before the hyphen be a shortened form of such number words as *two tens* or *three tens*? **Yes**

4. Of what is forty the shortened form? Fifty? Sixty? Seventy? Eighty? Ninety? **4 tens; 5 tens; 6 tens; 7 tens; 8 tens; 9 tens**

5. In the “_____ hundred _____” number words, must the first blank always be filled? What set of number words may be used to fill it? **Yes; one through nine**

30

6. In the “_____ hundred _____” number words, must the second blank always be filled? What set of number words may be used to fill it? **Yes; one through ninety-nine**

Written Write the number word for each of the following.

Answers given for part a only.

	a	b	c	d
1.	14 fourteen	15	18	19
2.	21 twenty-one	47	56	99
3.	134 one hundred thirty-four	262	379	333
4.	483 four hundred eighty-three	505	617	707
5.	719 seven hundred nineteen	801	912	111

Tell how The number words eleven and twelve come from the Old High German words *einlif* and *zwelif*. If *ein* means *one*, *zwe* means *two*, and *lif* means *beyond ten*, tell how eleven and twelve fit the “teen” pattern.

Einlif means *one and ten*. **Zwelif** means *two and ten*.

Checkup Time

The numerals in () tell the pages where you can turn for help.

Important Ideas

1. All sets that can be matched one-to-one have the same number. (7)
2. The union of sets may be used to explain addition. (10)
3. Removing a subset may be used to explain subtraction. (13)
4. Thinking about the sets will help you decide which open sentence to write for a story problem. (20)
5. By agreement, a decimal numeral is the simplest numeral. (26)

Words to Know

1. Set, subset (5)
2. Equivalent sets, non-equivalent sets (6, 14)
3. Number, numeral, number word (7,8)
4. Is equal to ($=$), is not equal to (\neq) (8,14)
5. Unnamed addend (12)
6. Is greater than ($>$), is less than ($<$) (15)
7. Replacement set, solution set (18,19)
8. Expanded notation (26)

Questions to Discuss

Answers on p. T31.

1. How are $3 + \square = 5$ and $5 - 3 = \square$ similar? (13)
2. When is a number sentence an equation? An inequality? (16)
3. When is a number sentence open? Closed? (17)
4. Suppose $\{0,1,2,3,4,5\}$ is the replacement set for an open sentence. Can $\{0,3,6\}$ be the solution set? Why? (19)
5. How is repeating a digit in a decimal numeral, such as 33, different from repeating a symbol in a Roman numeral, such as XX? (28)

Written Practice

Solve each open sentence. Let the replacement set be $\{0,1,2,3,4,5,6\}$.

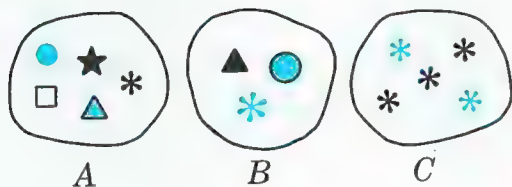
- | | | |
|----------------------|-------------------|------|
| 1. $\square < 5$ | 4 + $\square = 9$ | (18) |
| 2. $3 + \square < 7$ | 9 - 4 = \square | (18) |

Write the expanded notation for each of the following.

- | a | b | c | |
|----------|----------|---------|------|
| 3. 17 | 34 | 99 | (26) |
| 10+7 | 30+4 | 90+9 | |
| 4. 321 | 231 | 406 | (26) |
| 300+20+1 | 200+30+1 | 400+0+6 | |

Self-Evaluation

Part 1 Use the sets pictured below. Copy the sentences below. Replace each \bullet with the correct symbol ($=$ or \neq).



- $n(A) \neq n(B)$ $n(B) \neq n(A)$
- $n(A) = n(C)$ $n(B) \neq n(C)$

Part 2 Use the sets pictured in Part 1. Copy the sentences in Part 1. Replace each \bullet with the correct symbol ($=$, $<$, or $>$).
 1. $n(A) > n(B)$; $n(B) < n(A)$
 2. $n(A) = n(C)$; $n(B) < n(C)$

Part 3 Copy each sentence below. Write E before each equation. Write I before each inequality.

- | a | b |
|--------------------|-----------------|
| 1. E $3+5=8$ | I $7-1<6$ |
| 2. I $\square+3>9$ | E $5+\square=9$ |
| 3. I $2-2\neq5$ | E $7+0=\square$ |

Part 4 Use the sentences from Part 3. Write *Closed and true* after each true sentence. Write *Closed and false* after each false sentence. Write *Open* after each sentence which is neither true nor false. See below.

32 **Part 4** 1. closed and true; closed and false 2. open; open 3. closed and true; open

Part 8 1. 01; 01111111; 000111; 0011111111

2. 0000111111; 0000111111; 0000000011; 001111

Part 9 1. XI; XIX; XXIV; XXIX

2. CXXVII; CCCVII; CLXXIII; CCIV

Part 5 Solve each open sentence below. Use the following as the replacement set.

$\{0,1,2,3,4,5,6,7,8,9\}$

- | | |
|----------------------|-------------------|
| 1. $\square > 6$ | $3 + \square = 7$ |
| 2. $2 + \square < 8$ | $7 - 3 = \square$ |

Part 6 Read each problem carefully. Write an open sentence for it. Solve the open sentence. Write an answer for the problem.

1. Beth's father took 8 photographs with his new camera. Only 5 of the photographs were good. How many photographs were not good?

$8-5=\square$; 3 photographs

2. A hen has been sitting on some eggs. So far, 4 of the eggs have hatched. There still are 2 eggs to be hatched. How many eggs has the hen been trying to hatch?

$2+4=\square$; 6 eggs

Part 7 Write the expanded notation for each of the following.

- | a | b | c | d |
|------------|------------|------------|-----------|
| 1. 11 | 19 | 24 | 29 |
| $10+1$ | $10+9$ | $20+4$ | $20+9$ |
| 2. 137 | 317 | 173 | 204 |
| $100+30+7$ | $300+10+7$ | $100+70+3$ | $200+0+4$ |

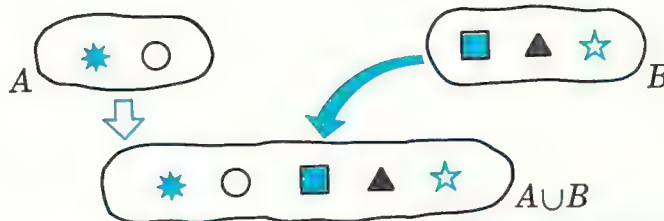
Part 8 Write an Egyptian numeral for each number named in Part 7. See below.

Part 9 Write a Roman numeral for each number named in Part 7. See below.

Chapter 2

PROPERTIES OF ADDITION AND SUBTRACTION

Sums



See the sets above.

Which objects are members of set A ? Of set B ? Of set $A \cup B$? Does set $A \cup B$ contain all the objects that are in set A and set B ? **Yes**

The union of sets may be used to explain the addition of numbers. The number of set B added to the number of set A is equal to the number of set $A \cup B$. This may be written as follows.

$$\begin{array}{r} n(A) + n(B) = n(A \cup B) \\ 2 + 3 = 5 \end{array}$$

We call $2+3$ an **addition numeral**. The numbers 2 and 3 are called **addends**. The number 5 is called the **sum**.

Addition is an operation on two numbers resulting in a third number called the *sum*.

Oral Read each addition numeral below. Name each addend. See representative answers for 1 below.

- | | | |
|-------------|----------|----------|
| a | b | c |
| 1. $3+2=5$ | $4+6=10$ | $6+4=10$ |
| 2. $8+7=15$ | $6+7=13$ | $9+5=14$ |
| 3. $2+9=11$ | $8+4=12$ | $8+8=16$ |

Written Do the following.

1-3. Write the simplest numeral for each sum in Oral 1-3. See Oral.

4. Write as many addition numerals as you can for each number named below. Many possible correct answers. 3, 5, 9, 10
For example, for 3 we may write $0+3$, $3+0$, $1+2$, $2+1$ or $1+1+1$. **33**

- Oral**
- three plus two; addends 3 and 2
 - four plus six; addends 4 and 6
 - six plus four; addends 6 and 4

The Commutative Property of Addition



Set B is joined to set A.
 $n(A) + n(B) = n(A \cup B)$



Set A is joined to set B.
 $n(B) + n(A) = n(B \cup A)$

Do $A \cup B$ and $B \cup A$ contain the same objects? Does the order of joining A and B change the resulting set? Is $n(A \cup B)$ equal to $n(B \cup A)$? **Yes; No; Yes**

$$n(A) + n(B) = n(B) + n(A)$$

$$3 + 4 = 4 + 3$$

Changing the order of two addends does not change the sum. We call this idea the **commutative property of addition**. Or we say *addition is commutative*.

Oral Using the commutative property of addition, replace each — so that each sentence below becomes true.

1. $9 + 8 = \underline{8} + \underline{9}$

2. $6 + 5 = \underline{5} + \underline{6}$

3. $8 + 7 = \underline{7} + \underline{8}$

4. $6 + 16 = \underline{16} + \underline{6}$

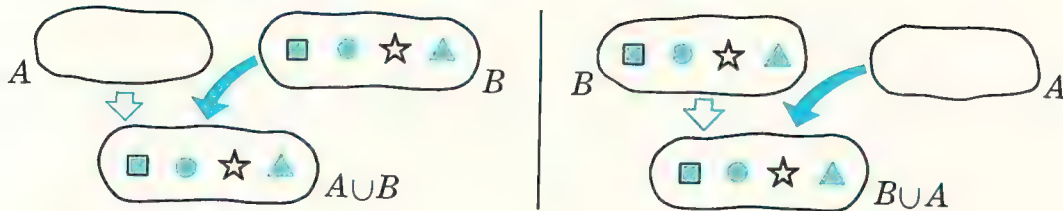
For each number named in rows 5 and 6, state two addition numerals that illustrate the commutative property of addition. **Answers will vary. Typical answers only are 34 given.**

	<i>a</i>	<i>b</i>	<i>c</i>
5.	6	12	18
	5+1; 1+5	9+3; 3+9	8+10; 10+8
6.	15	14	16
	10+5; 5+10	9+5; 5+9	11+5; 5+11

Written Copy. Replace each \square by the simplest numeral to make each sentence true.

	<i>a</i>	<i>b</i>
1.	$\square + 7 = 7 + 9$	$0 + 19 = 19 + \square$
2.	$12 + \square = 9 + 12$	$14 + 8 = \square + 14$
3.	$\square + 12 = 12 + 13$	$0 + 7 = \square + 0$
4.	$11 + 9 = 9 + \square$	$4 + \square = 6 + 4$

The Identity Number of Addition



What is $n(A)$? $n(B)$? $n(A \cup B)$? $n(B \cup A)$? 0; 4; 4; 4

We can write addition sentences for the joining of each pair of sets above.

$$n(A) + n(B) = n(A \cup B)$$

$$0 + 4 = 4$$

$$n(B) + n(A) = n(B \cup A)$$

$$4 + 0 = 4$$

When zero is added to any number, what can you say about the sum? When any number is added to zero, what can you say about the sum? *The sum is that number; The sum is that number.*

We call *zero* the **identity number of addition** because the sum of zero and any number is that number.

Oral What numeral should replace each \square to make the sentences true?

- | a | b |
|-------------------------------------|----------------------------------|
| 1. $13 + \overset{0}{\square} = 13$ | $\overset{2}{\square} + 17 = 19$ |
| 2. $\overset{0}{\square} + 10 = 10$ | $0 + \overset{18}{\square} = 18$ |
| 3. $6 + 5 = \overset{11}{\square}$ | $2 + \overset{9}{\square} = 11$ |
| 4. $\overset{0}{\square} + 9 = 9$ | $3 + 9 = \overset{12}{\square}$ |
| 5. $7 + \overset{0}{\square} = 7$ | $4 + 9 = \overset{13}{\square}$ |

6. Using *Oral* 1–5, tell which sentences use the identity number of addition. **1a; 2a; 2b; 4a; 5a**

Written Study these sets.



Write each addition sentence below by using numerals.

- $n(A) + n(C) = n(A \cup C)$ **3+0=3**
- $n(C) + n(B) = n(C \cup B)$ **0+2=2**
- $n(A) + n(B) = n(A \cup B)$ **3+2=5**
- $n(C) + n(A) = n(C \cup A)$ **0+3=3**
- $n(B) + n(A) = n(B \cup A)$ **2+3=5**
- $n(B) + n(C) = n(B \cup C)$ **2+0=2**

Differences



For the sets shown above, if subset B is removed from set $A \cup B$, which subset remains? We can say this as follows.

$$n(A \cup B) - n(B) = n(A)$$

What number is $n(A \cup B)$? $n(B)$? $n(A)$? **4; 1; 3**

The subtraction sentence can now be written as follows.

$$4 - 1 = 3$$

We call $4 - 1$ a **subtraction numeral**. We read $4 - 1$ as *four minus one*. The number 3 is called the **difference**.

Subtraction is an operation on two numbers resulting in a third number called the *difference*.

Oral Read each subtraction numeral below. Name each difference as a simplest numeral.

- | a | b | c |
|----------------------|-------------------|-------------------|
| 1. $9 - 6$ 3 | $8 - 2$ 6 | $15 - 8$ 7 |
| 2. $12 - 8$ 4 | $14 - 6$ 8 | $15 - 6$ 9 |
| 3. $16 - 8$ 8 | $15 - 9$ 6 | $14 - 7$ 7 |
| 4. $17 - 9$ 8 | $14 - 5$ 9 | $16 - 9$ 7 |
| 5. $13 - 8$ 5 | $12 - 7$ 5 | $18 - 9$ 9 |
| 6. $14 - 9$ 5 | $13 - 7$ 6 | $13 - 5$ 8 |

Written Find each difference.

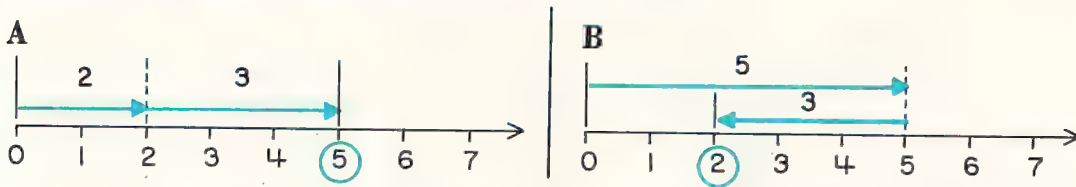
- | a | b |
|----------------------------------|-------------------------------|
| 1. $28 - 7 = \square$ 21 | $44 - 13 = \square$ 31 |
| 2. $37 - 16 = \square$ 21 | $26 - 15 = \square$ 11 |
| 3. $48 - 16 = \square$ 32 | $31 - 11 = \square$ 20 |

Write 3 subtraction numerals for each number named below.

Answers will vary.

- | a | b | c | d |
|------|-----|-----|-----|
| 4. 3 | 5 | 9 | 4 |
| 5. 7 | 6 | 2 | 1 |

Addition and Subtraction on a Number Line



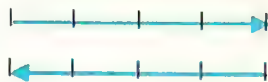
In **A**, think of starting at 0 and moving 2 segments to the right. Then move 3 more segments to the right. At which mark on the number line do you stop? 5 These 2 moves show the following addition.

$$2 + 3 = 5$$

The upper arrow in **B** shows a move of how many segments from 0? 5 In which direction? The lower arrow shows a move of how many segments from 5? 3 In which direction? These 2 moves together show the following subtraction.

$$5 - 3 = 2$$

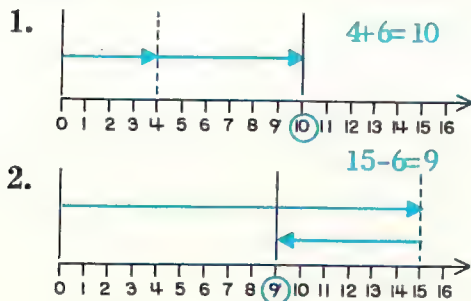
The number of segments in the length of a number line arrow represents the number. The direction of the arrow represents an operation.



shows addition of 4

shows subtraction of 4

Oral Tell the addition or subtraction shown on each number line below.



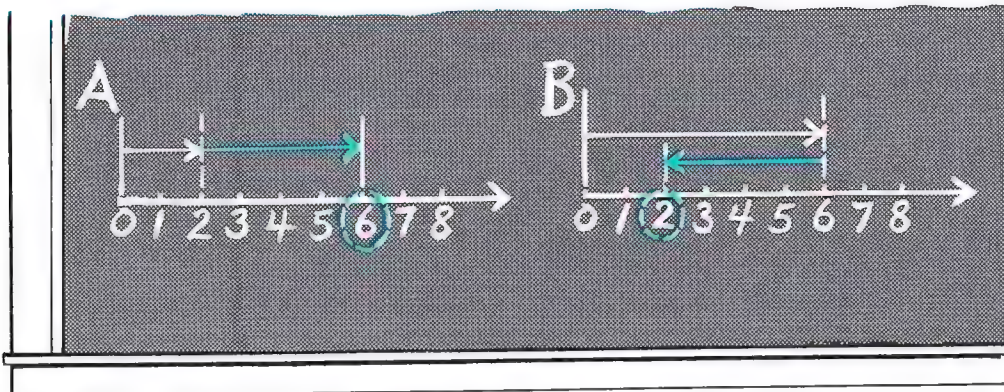
Written Make a number line drawing for each addition or subtraction sentence below.

- | a | b |
|-----------------|--------------|
| 1. $5 + 7 = 12$ | $12 - 7 = 5$ |
| 2. $7 + 5 = 12$ | $12 - 5 = 7$ |

Tell why When a number line arrow is 9 segments long and points to the left, what whole number must we start from to end at zero? Why?
9; $9 - 9 = 0$

- *1 to the right
- *2 to the left

Addition and Subtraction as Inverse Operations



In A, think of starting at 0 and moving 2 segments to the right. Then move 4 more segments to the right. The colored arrow shows a move from 2 to 6. What addition is shown on this number line? $2+4=6$

In B, the upper arrow shows a move of how many segments from 0? ^{*1} In which direction? The lower arrow shows a move of how many segments from 6? ^{*2} In which direction? The colored arrow shows a move from 6 to 2. What subtraction is shown on this number line? $6-4=2$

Compare the lengths of the colored arrows. Compare the directions of the colored arrows. Does the colored arrow in A undo what the colored arrow in B does? Yes

Does subtracting 4 from 6 undo adding 4 to 2? Does adding 4 to 2 undo subtracting 4 from 6? We can state this as follows.

Yes

$$\text{If } 2+4=6, \text{ then } 6-4=2.$$

$$\text{If } 4+2=6, \text{ then } 6-2=4.$$

$$\text{If } 6-4=2, \text{ then } 2+4=6.$$

$$\text{If } 6-2=4, \text{ then } 4+2=6.$$

Since addition and subtraction undo each other, they are called **inverse operations**.

Oral How could you undo each of the following?

1. adding 5 **subtracting 5**
2. subtracting 7 **adding 7**
3. adding 12 **subtracting 12**
4. adding 19 **subtracting 19**
5. subtracting 25 **adding 25**

Replace the \square with a numeral to make each sentence true.

6. If $4+8=12$, then $12-8=\square$. **4**
7. If $6+9=15$, then $15-9=\square$. **6**
8. If $17-8=9$, then $9+8=\square$. **17**
9. If $16-9=7$, then $7+9=\square$. **16**
10. If $14-5=9$, then $9+\square=14$. **5**
11. If $8+7=15$, then $15-\square=8$. **7**
12. If $9+5=14$, then $14-\square=9$. **5**
13. If $18-9=9$, then $9+9=\square$. **18**
14. If $7+8=15$, then $\square=15-8$. **7**
15. If $14-9=5$, then $\square=5+9$. **14**

Written Write a subtraction sentence for each of the following.

- | | <i>a</i> | <i>b</i> |
|----|-------------------------------|----------------------------------|
| 1. | $9-1=8$
$8+1=9$
$8-0=8$ | $10-4=6$
$6+4=10$
$13-6=7$ |
| 2. | $8+0=8$
$17-8=9$ | $7+6=13$
$15-9=6$ |
| 3. | $9+8=17$ | $6+9=15$ |

Tell why 1. Commute the addends in $7+9=16$ to obtain $9+7=16$. Use the inverse operation of addition to obtain $16-9=7$ and **39** $16-7=9$.

3. Subtraction is not commutative; When 0 is subtracted from a number, the result is that number.

4. Commute the addends to obtain $18+17=35$. Use the inverse operation of addition to obtain $35-18=17$ and $35-17=18$.

- | | | |
|----|----------------------|----------------------|
| 4. | $8+7=15$
$15-7=8$ | $5+6=11$
$11-6=5$ |
| 5. | $9+4=13$
$13-4=9$ | $8+5=13$
$13-5=8$ |
| 6. | $4+7=11$
$11-7=4$ | $8+6=14$
$14-6=8$ |

Write an addition sentence for each of the following.

- | | <i>a</i> | <i>b</i> |
|-----|----------------------|----------------------|
| 7. | $14-8=6$
$6+8=14$ | $15-9=6$
$6+9=15$ |
| 8. | $9-0=9$
$9+0=9$ | $7-1=6$
$6+1=7$ |
| 9. | $16-7=9$
$9+7=16$ | $17-8=9$
$9+8=17$ |
| 10. | $14-5=9$
$9+5=14$ | $15-6=9$
$9+6=15$ |
| 11. | $13-7=6$
$6+7=13$ | $12-3=9$
$9+3=12$ |

Tell why Do the following.

1. When you have learned an addition fact like $7+9=16$, you need not learn $16-7=\square$ or $16-9=\square$. Why? **See below.**

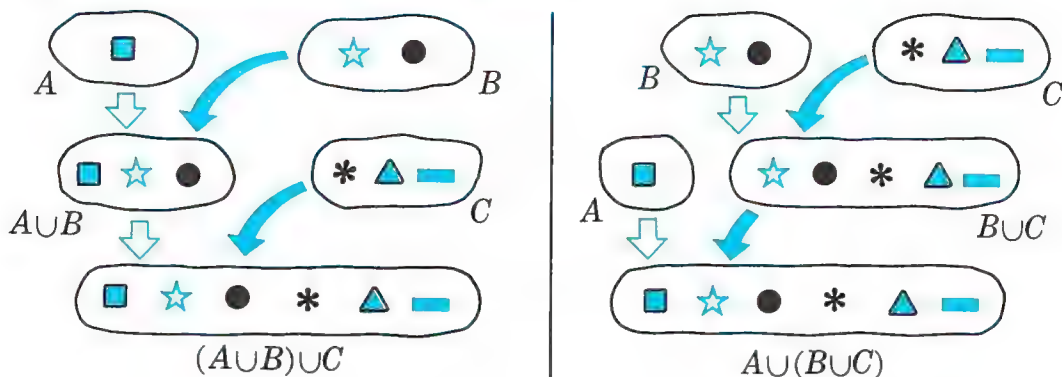
2. When you add 39 and 42 to get 81, you can check the addition by subtracting 42 from 81. Why?

Subtracting 42 undoes adding 42.

3. When 0 is one of two whole number addends, the other addend is the sum, like in $4+0=0+4=4$. Why is it not possible to make a similar statement for zero in subtraction? What statement can you make about zero in subtraction that will always be true? **See below.**

4. If $17+18=35$, then $35-18=17$ and $35-17=18$. Why? **See below.**

The Associative Property of Addition



The pictures above show two different ways of joining the same three sets. Study the set pictures carefully. On the left, set B is joined to set A to form $A \cup B$. Then set C is joined to set $A \cup B$ to form $(A \cup B) \cup C$. The $()$ in $(A \cup B) \cup C$ show which two sets are joined first.

On the right, set C is joined to set B to form $B \cup C$. Then $B \cup C$ is joined to set A to form $A \cup (B \cup C)$.

Do $(A \cup B) \cup C$ and $A \cup (B \cup C)$ have the same members? Does the way in which the sets are joined change the final result?

Yes; No

What number is $n(A)$? $n(B)$? $n(C)$? We can write an addition sentence for each joining of sets above. The $()$ tell which two numbers are added first.

$$\begin{array}{r} (1+2)+3=1+(2+3) \\ 3 \quad +3=1+ \quad 5 \\ 6=6 \end{array}$$

Does the way in which we group or associate the addends change the sum? Is $(1+2)+3=1+(2+3)$ a true sentence? No; Yes

The way in which we group addends does not change the sum. We call this idea the **associative property of addition**. Or we say *addition is associative*.

Oral Replace each \square with a numeral that makes each sentence true.

1. $(7+2)+8=7+(2+8)=\square$ ¹⁷
2. $(6+3)+7=6+(3+7)=\square$ ¹⁶
3. $(8+1)+9=8+(\square+9)=18$ ¹
4. $(6+5)+5=\square+(5+5)=16$ ⁶
5. $(9+3)+7=9+(3+\square)=19$ ⁷
6. $(8+\square)+2=8+(8+2)=18$ ⁸
7. $(\square+9)+1=4+(9+1)=14$ ⁴
8. $(8+7)+3=8+(7+3)=\square$ ¹⁸
9. $(7+6)+4=7+(6+4)=\square$ ¹⁷

Tell the two ways you can add each of the following without changing the order of the addends.

- | a | b |
|--------------------------------------|----------------------------------|
| 10. $8+4+6$
$(8+4)+6$; $8+(4+6)$ | $7+3+2$
$(7+3)+2$; $7+(3+2)$ |
| 11. $9+1+5$
$(9+1)+5$; $9+(1+5)$ | $8+2+7$
$(8+2)+7$; $8+(2+7)$ |
| 12. $6+4+3$
$(6+4)+3$; $6+(4+3)$ | $3+7+5$
$(3+7)+5$; $3+(7+5)$ |
| 13. $9+2+8$
$(9+2)+8$; $9+(2+8)$ | $6+4+7$
$(6+4)+7$; $6+(4+7)$ |
| 14. $8+8+2$
$(8+8)+2$; $8+(8+2)$ | $6+6+4$
$(6+6)+4$; $6+(6+4)$ |
| 15. $7+7+3$
$(7+7)+3$; $7+(7+3)$ | $9+9+1$
$(9+9)+1$; $9+(9+1)$ |
| 16. $9+1+9$
$(9+1)+9$; $9+(1+9)$ | $1+9+9$
$(1+9)+9$; $1+(9+9)$ |
| 17. $2+3+7$
$(2+3)+7$; $2+(3+7)$ | $3+7+7$
$(3+7)+7$; $3+(7+7)$ |
| 18. $7+3+7$
$(7+3)+7$; $7+(3+7)$ | $4+6+7$
$(4+6)+7$; $4+(6+7)$ |
| 19. $7+6+4$
$(7+6)+4$; $7+(6+4)$ | $7+4+6$
$(7+4)+6$; $7+(4+6)$ |

Written Copy. Use the method below to show that each of the following is a true sentence.

$$\begin{array}{r} (1+2)+3=1+(2+3) \\ 3 \quad +3=1+ \quad 5 \\ 6=6 \end{array}$$

1. $(2+3)+4=2+(3+4)$ $5+4=2+7$
 $9=9$
2. $(3+4)+5=3+(4+5)$ $7+5=3+9$
 $12=12$
3. $(4+5)+3=4+(5+3)$ $9+3=4+8$
 $12=12$
4. $(3+2)+4=3+(2+4)$ $5+4=3+6$
 $9=9$
5. $(8+2)+6=8+(2+6)$ $10+6=8+8$
 $16=16$
6. $(7+3)+4=7+(3+4)$ $10+4=7+7$
 $14=14$
7. $6+(4+5)=(6+4)+5$ $6+9=10+5$
 $15=15$

The associative property of addition makes adding 3 or more numbers easy. Use the associative property to find each sum.

- | a | b |
|-------------------------------------|---------------------------------|
| 8. $(7+2)+8=\square$ ¹⁷ | $7+(2+8)=\square$ ¹⁷ |
| 9. $4+(6+8)=\square$ ¹⁸ | $(4+6)+8=\square$ ¹⁸ |
| 10. $(4+9)+1=\square$ ¹⁴ | $4+(9+1)=\square$ ¹⁴ |
| 11. $(9+2)+8=\square$ ¹⁹ | $9+(2+8)=\square$ ¹⁹ |

Can you do this? Without changing the order of the addends, place parentheses to show in how many different ways we may solve this open sentence.

$$\begin{array}{l} 2+3+4+5=\square \\ ((2+3)+4)+5, (2+(3+4))+5, \\ 2+(3+(4+5)), 2+((3+4)+5), 41 \\ (2+3)+(4+5) \end{array}$$

Addition and Subtraction Table

+/_	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

$5+6=11$

$15-8=7$

Do the colored arrows in the table above show addition or subtraction? Do the gray arrows show addition or subtraction?
addition; subtraction

To find the simplest numeral for $5+6$, find 5 in the left column and 6 in the top row. Where will you find the sum?
where the 5 row and 6 column intersect

To find the simplest numeral for $15-8$, find the 8 in the top row. Why 8? Follow the 8 column down until you reach 15. Why 15? Follow this row to the left until you reach the first column. What numeral do you find there? Do $15-8$ and 7 name the same number? 7; Yes

Oral Tell the sum or difference in each of the following. Check your answer by using the table.

a

$$1. \quad 4+7=\square$$

$$2. \quad 6+5=\square$$

$$3. \quad 8+7=\square$$

b

$$8+5=\square$$

$$9+4=\square$$

$$\square=6+7$$

$$4. \quad \square=8+6$$

$$5. \quad \square=14-6$$

$$6. \quad 17-6=\square$$

$$7. \quad 14-8=\square$$

$$8. \quad 15-6=\square$$

$$\square=13-5$$

$$4+9=\square$$

$$3+9=\square$$

$$12-5=\square$$

$$\square=14-5$$

42

*1 8 is to be subtracted.

*2 8 is subtracted from 15.

Written Copy. Replace each \square to make each sentence true.

1. If $6+8=14$, then $14-8=\square$. ⁶
2. If $8+4=12$, then $12-4=\square$. ⁸
3. If $14-6=8$, then $8+\square=14$. ⁶
4. If $15-9=6$, then $6+9=\square$. ¹⁵
5. If $\square+9=14$, then $14-9=5$. ⁵
6. If $8+\square=15$, then $15-7=8$. ⁷
7. If $13-7=\square$, then $6+7=13$. ⁶
8. If $16-7=\square$, then $9+7=16$. ⁹
9. If $5+6=11$, then $\square-6=5$. ¹¹
10. If $6+5=11$, then $11-\square=6$. ⁵

Copy. Find the sum or difference in each of the following.

- | <i>a</i>
12 | <i>b</i>
15 |
|--------------------|----------------|
| 11. $8+4=\square$ | $9+6=\square$ |
| 12. $7+7=\square$ | $5+8=\square$ |
| 13. $6+5=\square$ | $6+6=\square$ |
| 14. $13-7=\square$ | $12-5=\square$ |
| 15. $14-7=\square$ | $12-3=\square$ |
| 16. $13-9=\square$ | $11-9=\square$ |

Can you do this? Discover two patterns in the table on page 42. One has to do with corresponding

rows and columns. The other has to do with a diagonal from upper right to lower left. See Mathematical Background page T42.

Another way We can solve $6+5=\square$ as follows.

$$\begin{aligned}
 6+5 &= 6+(4+1) \leftarrow \text{How was 5 renamed? as } 4+1 \\
 &= (6+4)+1 \leftarrow \text{What property is used here? associative} \\
 &= 10+1 \\
 &= 11
 \end{aligned}$$

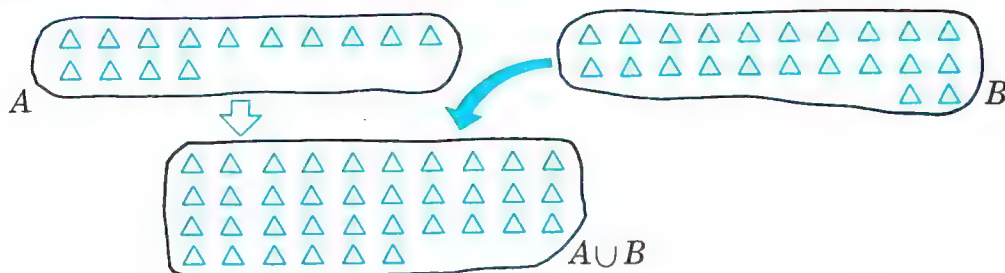
We can solve $16-7=\square$ as follows.

$$\begin{aligned}
 16-7 &= (6+10)-7 \leftarrow \text{How was 16 renamed? as } 6+10 \\
 &= 6+(10-7) \\
 &= 6+3 \\
 &= 9
 \end{aligned}$$

Find the sum or difference for each of the following as shown above.

- | <i>a</i>
12 | <i>b</i>
12 |
|--------------------|----------------|
| 1. $8+4=\square$ | $9+3=\square$ |
| 2. $9+7=\square$ | $14-6=\square$ |
| 3. $12-9=\square$ | $8+5=\square$ |
| 4. $7+5=\square$ | $5+9=\square$ |
| 5. $13-8=\square$ | $14-9=\square$ |
| 6. $5+6=\square$ | $5+8=\square$ |
| 7. $8+7=\square$ | $15-8=\square$ |
| 8. $7+9=\square$ | $16-9=\square$ |
| 9. $4+7=\square$ | $3+8=\square$ |
| 10. $11-9=\square$ | $12-8=\square$ |
| 11. $8+6=\square$ | $13-7=\square$ |

Addition Using Two-Digit and Three-Digit Numerals



How is the open sentence $14+22=\square$ related to the set picture above? To find the simplest numeral for $14+22$ we can write the following.

$$\begin{array}{r} 14 \\ +22 \\ \hline \end{array}$$

Could we just think about expanded notation and write place-value numerals? By thinking of 14 and 22 in expanded notation, we can add the tens and ones separately.

Think

Write

$$\begin{array}{r} 14 \\ +22 \\ \hline \end{array}$$

$$\begin{array}{r} 10+4 \\ + (20+2) \\ \hline 30+6 \end{array}$$

$$\begin{array}{r|l} & 0 \\ 14 & \\ +22 & \\ \hline 36 & \end{array}$$

$$\begin{array}{r} 14 \\ +22 \\ \hline 36 \end{array}$$

Add the ones.

Add the tens.

Does this method also hold for 3-digit numerals? Yes

Think

Write

$$\begin{array}{r} 158 \\ +431 \\ \hline \end{array}$$

$$\begin{array}{r} 100+50+8 \\ + (400+30+1) \\ \hline 500+80+9 \end{array}$$

$$\begin{array}{r|ll} H & T & O \\ 158 & & \\ +431 & & \\ \hline 589 & & \end{array}$$

$$\begin{array}{r} 158 \\ +431 \\ \hline 589 \end{array}$$

Add the ones.

Add the tens.

Add the hundreds.

44

*1 $n(\underline{A})+n(\underline{B})=n(\underline{A \cup B})$ and $n(\underline{A})=14$, $n(\underline{B})=22$

*2 Yes

Oral Name each of the following in expanded notation. **Representative answers only**

- | | <i>a</i> | <i>b</i> | <i>c</i> |
|----|-----------------|----------|----------|
| 1. | 27
20+7 | 52 | 64 |
| 2. | 44
40+4 | 53 | 32 |
| 3. | 274
200+70+4 | 165 | 136 |
| 4. | 365
300+60+5 | 234 | 283 |

Tell how to find each sum below.

See **representative answers below.**

- | | <i>a</i> | <i>b</i> | <i>c</i> |
|----|-------------|-------------|-------------|
| 5. | 27
+52 | 44
+53 | 64
+32 |
| 6. | 204
+165 | 365
+234 | 136
+253 |

Written Copy. Find each sum.

- | | <i>a</i> | <i>b</i> | <i>c</i> |
|----|--------------------|--------------------|--------------------|
| 1. | 42
+37
79 | 58
+41
99 | 46
+43
89 |
| 2. | 38
+51
89 | 25
+32
57 | 146
+213
359 |
| 3. | 32
+45
77 | 22
+36
58 | 224
+531
755 |
| 4. | 61
+34
95 | 125
+213
338 | 405
+312
717 |
| 5. | 128
+760
888 | 132
+432
564 | 141
+243
384 |
| 6. | 481
+417
898 | 293
+106
399 | 381
+404
785 |

- | | | | |
|----|--------------------|--------------------|--------------------|
| 7. | 365
+234
599 | 269
+230
499 | 181
+408
589 |
| 8. | 703
+186
889 | 421
+378
799 | 630
+258
888 |

Write an open sentence for each problem. Solve the open sentence. Answer the problem.

9. George had 15 cents. His father gave him 43 cents. How many cents does George have now? $15+43=\square$; **58 cents**

10. Sue weighed 82 pounds on her eighth birthday. Since then she has gained 15 pounds. How much does she weigh now? $82+15=\square$; **97 pounds**

11. Tom had 152 marbles. His brother gave him 123 marbles. How many marbles does he have now? $152+123=\square$; **275 marbles**

12. John picked 148 quarts of strawberries in one week. The next week he picked 141 quarts. How many quarts of berries did he pick altogether? $148+141=\square$; **289 quarts**

13. Sam weighed 123 pounds last year. This year he gained 16 pounds. How much does he weigh now? $123+16=\square$; **139 pounds**

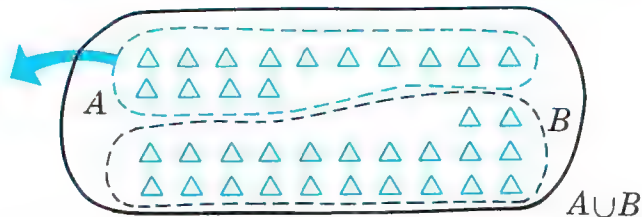
14. Mary's father bought a stove for 245 dollars. He also bought a radio for 44 dollars. How many dollars did he spend altogether? $245+44=\square$; **289 dollars**

15. Margie made 125 cookies. Carolyn made 104 cookies. Together they made how many cookies? $125+105=\square$; **229 cookies**

Oral 5a. Add 7 and 2. Then add 2T and 5T.

6a. Add 4 and 5. Add 0T and 6T. Then add 2H and 1H.

Subtraction Using Two-Digit and Three-Digit Numerals



How is the open sentence $36 - 14 = \square$ related to the set picture above? To find the simplest numeral for $36 - 14$ we can write the following.

$$\begin{array}{r} 36 \\ -14 \\ \hline \end{array} \quad \begin{array}{l} \text{minuend} \\ \text{subtrahend} \end{array}$$

Could we just think about expanded notation and write place-value numerals? By thinking of 36 and 14 in expanded notation, we discover that we can subtract the tens and ones separately.

Think

Write

$$\begin{array}{r} 36 \\ -14 \\ \hline \end{array} \quad \begin{array}{r} 30+6 \\ -(10+4) \\ \hline 20+2 \end{array} \quad \begin{array}{r} \text{T} | \text{O} \\ 36 \\ -14 \\ \hline 22 \end{array} \quad \begin{array}{r} 36 \\ -14 \\ \hline 22 \end{array}$$

Subtract the ones.

Subtract the tens.

The $-$ sign in $-(10+4)$ means to subtract 10 and subtract 4.

Does this method also hold for three-digit numerals? Yes

Think

Write

$$\begin{array}{r} 589 \\ -431 \\ \hline \end{array} \quad \begin{array}{r} 500+80+9 \\ -(400+30+1) \\ \hline 100+50+8 \end{array} \quad \begin{array}{r} \text{H} | \text{T} | \text{O} \\ 589 \\ -431 \\ \hline 158 \end{array} \quad \begin{array}{r} 589 \\ -431 \\ \hline 158 \end{array}$$

Subtract the ones.

Subtract the tens.

Subtract the hundreds.

46

*1 $n(\underline{A \cup B}) - n(\underline{A}) = n(\underline{B})$ and $n(\underline{A \cup B}) = 36$, $n(\underline{A}) = 14$

*2 Yes

Oral Name each of the following in expanded notation. **Representative answers only**

	<i>a</i>	<i>b</i>	<i>c</i>
1.	48 <i>40+8</i>	32	84
2.	76 <i>70+6</i>	51	31
3.	481 <i>400+80+1</i>	271	468
4.	948 <i>900+40+8</i>	527	234

Explain how you would find each difference below. **See representative answers below.**

	<i>a</i>	<i>b</i>	<i>c</i>
5.	$\begin{array}{r} 48 \\ -32 \\ \hline \end{array}$	$\begin{array}{r} 76 \\ -51 \\ \hline \end{array}$	$\begin{array}{r} 84 \\ -31 \\ \hline \end{array}$
6.	$\begin{array}{r} 481 \\ -271 \\ \hline \end{array}$	$\begin{array}{r} 948 \\ -527 \\ \hline \end{array}$	$\begin{array}{r} 468 \\ -234 \\ \hline \end{array}$

7. Tell what the open sentences would be for Oral 5-6. **See below.**

Written Copy. Find each difference.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 29 \\ -18 \\ \hline 11 \end{array}$	$\begin{array}{r} 43 \\ -21 \\ \hline 22 \end{array}$	$\begin{array}{r} 76 \\ -22 \\ \hline 54 \end{array}$
2.	$\begin{array}{r} 48 \\ -14 \\ \hline 34 \end{array}$	$\begin{array}{r} 98 \\ -48 \\ \hline 50 \end{array}$	$\begin{array}{r} 72 \\ -42 \\ \hline 30 \end{array}$
3.	$\begin{array}{r} 84 \\ -51 \\ \hline 33 \end{array}$	$\begin{array}{r} 68 \\ -27 \\ \hline 41 \end{array}$	$\begin{array}{r} 42 \\ -10 \\ \hline 32 \end{array}$
4.	$\begin{array}{r} 92 \\ -31 \\ \hline 61 \end{array}$	$\begin{array}{r} 29 \\ -16 \\ \hline 13 \end{array}$	$\begin{array}{r} 65 \\ -31 \\ \hline 34 \end{array}$
5.	$\begin{array}{r} 619 \\ -408 \\ \hline 211 \end{array}$	$\begin{array}{r} 720 \\ -310 \\ \hline 410 \end{array}$	$\begin{array}{r} 809 \\ -408 \\ \hline 401 \end{array}$

6.	$\begin{array}{r} 431 \\ -130 \\ \hline 301 \end{array}$	$\begin{array}{r} 742 \\ -341 \\ \hline 401 \end{array}$	$\begin{array}{r} 838 \\ -638 \\ \hline 200 \end{array}$
7.	$\begin{array}{r} 249 \\ -102 \\ \hline 147 \end{array}$	$\begin{array}{r} 961 \\ -720 \\ \hline 241 \end{array}$	$\begin{array}{r} 438 \\ -106 \\ \hline 332 \end{array}$
8.	$\begin{array}{r} 627 \\ -205 \\ \hline 422 \end{array}$	$\begin{array}{r} 721 \\ -321 \\ \hline 400 \end{array}$	$\begin{array}{r} 843 \\ -243 \\ \hline 600 \end{array}$
9.	$\begin{array}{r} 784 \\ -682 \\ \hline 102 \end{array}$	$\begin{array}{r} 935 \\ -634 \\ \hline 301 \end{array}$	$\begin{array}{r} 872 \\ -422 \\ \hline 450 \end{array}$
10.	$\begin{array}{r} 791 \\ -490 \\ \hline 301 \end{array}$	$\begin{array}{r} 249 \\ -131 \\ \hline 118 \end{array}$	$\begin{array}{r} 327 \\ -102 \\ \hline 225 \end{array}$

Write an open subtraction sentence for each problem below. Solve the open sentence. Answer the problem.

11. Mary had 48 records. She gave 15 records to her sister. How many records does she still have? **48-15=□; 33 records**

12. Bill weighed 95 pounds last year. Since then he lost 12 pounds. How much does Bill weigh now? **95-12=□; 83 pounds**

13. Tom had 275 marbles. He gave 110 of them to his friend Jerry. How many marbles does he still have? **275-110=□; 165 marbles**

14. Mary went to visit her aunt. Her aunt lives 248 miles away. Mary drove 225 miles and then stopped for gasoline. How many more miles did she have to drive? **248-225=□; 23 miles**

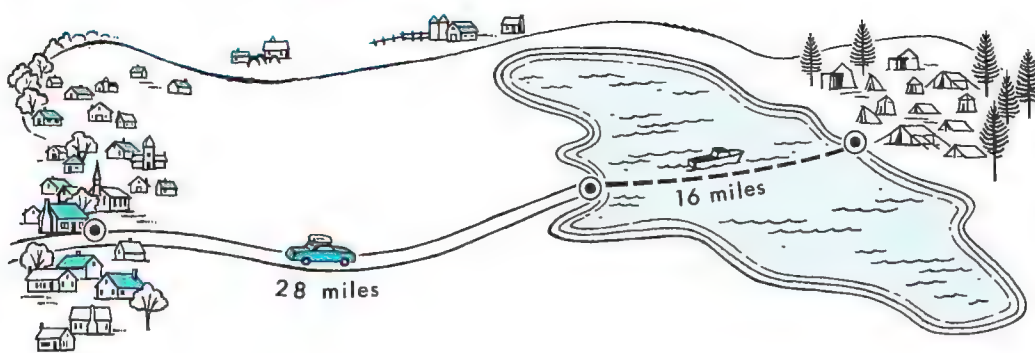
15. Sam and his father were going fishing 163 miles from home. They rode for 52 miles. How much farther did they have to go? **163-52=□; 111 miles**

Oral 5a. Subtract 2 from 8. Subtract 3T from 4T.

6a. Subtract 1 from 1. Subtract 7T from 8T. Subtract 2H from 4H.

7. **48-32=□; 76-51=□; 84-31=□; 481-271=□; 948-527=□; 468-234=□**

Renaming Ones in Addition



William and his father went fishing at Lake Troy. How far did they travel in reaching their campsite?

Write an open sentence for this problem. To solve $28 + 16 = \square$ we can think about the addition in expanded notation.

Think

$$\begin{array}{r} 28 \\ +16 \\ \hline \end{array}$$

$$\begin{array}{r} 20 + 8 \\ + (10 + 6) \\ \hline \end{array}$$

$$30 + 14 = 30 + (10 + 4) \leftarrow \text{How did we rename 14? as } 10 + 4$$

$$= (30 + 10) + 4 \leftarrow \text{What property of addition has been used? associative}$$

$$= 40 + 4$$

$$= 44$$

Could we just think about expanded notation and write place-value numerals? How does expanded notation help?

Yes; You can add the ones and the tens separately.

Think

$$\begin{array}{r} 28 \\ +16 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ 20 + 8 \\ + (10 + 6) \\ \hline 40 + 14 = 44 \end{array}$$

$$\begin{array}{r|l} 1 & 0 \\ 1 & 28 \\ +1 & 6 \\ \hline 4 & 4 \end{array}$$

Write

$$\begin{array}{r} 28 \\ +16 \\ \hline 44 \end{array}$$

$$8 + 6 = 14 = 1\text{T} + 4$$

$$10 + 20 + 10 = 40 = 4\text{T}$$

Since $8 + 6 = 1\text{T} + 4$, where do we record the 1T ? The 4 ? in the tens column; in the ones position in the numeral for the sum

Oral Explain how to find each sum below. Tell what numeral should replace each . Add the ones, add the tens, and add the hundreds.

a	b	c
1. $\begin{array}{r} \text{H} \text{T} \text{O} \\ 64 \\ +28 \\ \hline 92 \end{array}$	$\begin{array}{r} \text{H} \text{T} \text{O} \\ 38 \\ +47 \\ \hline 85 \end{array}$	$\begin{array}{r} \text{H} \text{T} \text{O} \\ 342 \\ +239 \\ \hline 581 \end{array}$
2. $\begin{array}{r} \text{H} \text{T} \text{O} \\ 46 \\ +28 \\ \hline 74 \end{array}$	$\begin{array}{r} \text{H} \text{T} \text{O} \\ 37 \\ +39 \\ \hline 76 \end{array}$	$\begin{array}{r} \text{H} \text{T} \text{O} \\ 369 \\ +413 \\ \hline 782 \end{array}$

Written Copy. Find each sum.

a	b	c
1. $\begin{array}{r} 68 \\ +17 \\ \hline 85 \end{array}$	$\begin{array}{r} 72 \\ +19 \\ \hline 91 \end{array}$	$\begin{array}{r} 19 \\ +36 \\ \hline 55 \end{array}$
2. $\begin{array}{r} 44 \\ +38 \\ \hline 82 \end{array}$	$\begin{array}{r} 48 \\ +17 \\ \hline 65 \end{array}$	$\begin{array}{r} 56 \\ +19 \\ \hline 75 \end{array}$
3. $\begin{array}{r} 33 \\ +28 \\ \hline 61 \end{array}$	$\begin{array}{r} 26 \\ +18 \\ \hline 44 \end{array}$	$\begin{array}{r} 17 \\ +44 \\ \hline 61 \end{array}$
4. $\begin{array}{r} 245 \\ +139 \\ \hline 384 \end{array}$	$\begin{array}{r} 625 \\ +137 \\ \hline 762 \end{array}$	$\begin{array}{r} 487 \\ +108 \\ \hline 595 \end{array}$
5. $\begin{array}{r} 624 \\ +149 \\ \hline 773 \end{array}$	$\begin{array}{r} 383 \\ +209 \\ \hline 592 \end{array}$	$\begin{array}{r} 267 \\ +427 \\ \hline 694 \end{array}$
6. $\begin{array}{r} 325 \\ +268 \\ \hline 593 \end{array}$	$\begin{array}{r} 274 \\ +108 \\ \hline 382 \end{array}$	$\begin{array}{r} 138 \\ +448 \\ \hline 586 \end{array}$
7. $\begin{array}{r} 428 \\ +234 \\ \hline 662 \end{array}$	$\begin{array}{r} 724 \\ +108 \\ \hline 832 \end{array}$	$\begin{array}{r} 318 \\ +448 \\ \hline 766 \end{array}$
8. $\begin{array}{r} 635 \\ +149 \\ \hline 784 \end{array}$	$\begin{array}{r} 356 \\ +218 \\ \hline 574 \end{array}$	$\begin{array}{r} 653 \\ +219 \\ \hline 872 \end{array}$

Write an open sentence for each problem. Solve the open sentence. Answer the problem.

9. In George's class there are 28 pupils. In Tom's class there are 27 pupils. How many pupils are there in these 2 classes? $28+27=\square$; 55 pupils

10. Bill sold 34 newspapers. Jim sold 28 newspapers. Together they sold how many newspapers? $34+28=\square$; 62 newspapers

11. Sally made 48 cookies. Ann made 36 cookies. Together they made how many cookies? $48+36=\square$; 84 cookies

12. Mary's dad worked 128 hours on their house. Her brother worked 104 hours on the house. Together they worked how many hours? $128+104=\square$; 232 hours

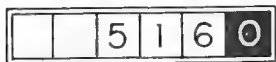
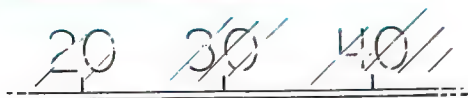
13. Jim had 136 marbles. His brother gave him 45 more. How many marbles does Jim have now? $136+45=\square$; 181 marbles

14. Warren drove 152 miles on Monday. On Tuesday he drove 128 miles. How many miles did he drive altogether? $152+128=\square$; 280 miles

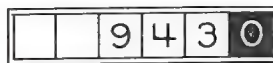
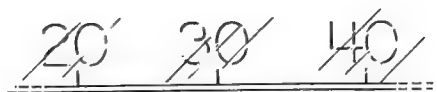
15. Tom counted 132 boys at the picnic. Joe counted 159 girls. Altogether they counted how many people at the picnic? $132+159=\square$; 291 people

16. Fred sold 248 papers on Sunday. The Sunday before he sold 227 papers. How many papers did he sell in the last two Sundays? $248+227=\square$; 475 papers

Renaming Tens in Subtraction



Odometer reading on Monday



Odometer reading on Thursday

This car was driven how many miles?

Write an open subtraction sentence for this problem. To solve the open sentence think about the subtraction as shown below.

$$\begin{array}{r} 943 \\ -516 \\ \hline \end{array}$$

No *1

Can you subtract in every place-value position? Why? To make subtraction possible in the ones column we can rename 943. We can think about the subtraction as shown below.

A	B	C	D
$\begin{array}{r} 943 \\ -516 \\ \hline \end{array}$	$\begin{array}{r} 900+40+3 \\ -(500+10+6) \\ \hline \end{array}$	$\begin{array}{r} 900+30+13 \\ -(500+10+6) \\ \hline 400+20+7 \end{array}$	$\begin{array}{r l} \text{H} & \text{T} & \text{O} \\ 3 & 13 & \\ \hline 9 & 4 & 3 \\ -5 & 1 & 6 \\ \hline 4 & 2 & 7 \end{array}$

Explain the change in going from A to B. Why is $900+40+3$ not useful for doing the subtraction in B? Explain the change in going from B to C. Why is $900+30+13$ useful for doing the subtraction? How is the change from B to C shown in the grid form in D? We can show the renaming in simpler column form as shown below.

D	E	F
$\begin{array}{r l} \text{H} & \text{T} & \text{O} \\ 3 & 13 & \\ \hline 9 & 4 & 3 \\ -5 & 1 & 6 \\ \hline 4 & 2 & 7 \end{array}$	$\begin{array}{r} 3 \ 13 \\ 943 \\ -516 \\ \hline 427 \end{array}$	$\begin{array}{r} 943 \\ -516 \\ \hline 427 \end{array}$

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- *1 In ones position $3 < 6$.
- *2 943 and 516 are changed to expanded notation.
- *3 $900+40+3$ is renamed as $900+30+13$.

- *4 Subtraction can be performed in each place-value position.
- *5 by the reminder numerals above 943

Oral Answer each of the following questions.

1. Explain the change in going from D to E on page 50. **The place-value grid is omitted.**

2. Explain the change in going from E to F on page 50. **The remainder numerals are omitted.**

3. How many miles was this car driven between Monday and Thursday? **427 miles**

In which of the following must you rename the minuend so that you can subtract in the ones column?

4b, 5a, 5c, 6c, 7a, 7b

	a	b	c
4.	$\begin{array}{r} 28 \\ -14 \\ \hline \end{array}$	$\begin{array}{r} 64 \\ -36 \\ \hline \end{array}$	$\begin{array}{r} 38 \\ -28 \\ \hline \end{array}$
5.	$\begin{array}{r} 73 \\ -24 \\ \hline \end{array}$	$\begin{array}{r} 69 \\ -28 \\ \hline \end{array}$	$\begin{array}{r} 34 \\ -18 \\ \hline \end{array}$
6.	$\begin{array}{r} 264 \\ -132 \\ \hline \end{array}$	$\begin{array}{r} 379 \\ -229 \\ \hline \end{array}$	$\begin{array}{r} 168 \\ -129 \\ \hline \end{array}$
7.	$\begin{array}{r} 737 \\ -619 \\ \hline \end{array}$	$\begin{array}{r} 366 \\ -138 \\ \hline \end{array}$	$\begin{array}{r} 435 \\ -225 \\ \hline \end{array}$

Tell the expanded notation that you would use for each minuend in doing each subtraction below. **See below.**

	a	b	c
8.	$\begin{array}{r} 64 \\ -36 \\ \hline \end{array}$	$\begin{array}{r} 73 \\ -24 \\ \hline \end{array}$	$\begin{array}{r} 34 \\ -18 \\ \hline \end{array}$
9.	$\begin{array}{r} 168 \\ -129 \\ \hline \end{array}$	$\begin{array}{r} 737 \\ -619 \\ \hline \end{array}$	$\begin{array}{r} 366 \\ -138 \\ \hline \end{array}$

$$\begin{array}{r} 10. \quad 482 \\ -114 \\ \hline \end{array} \quad \begin{array}{r} 395 \\ -218 \\ \hline \end{array} \quad \begin{array}{r} 743 \\ -416 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad 183 \\ -119 \\ \hline \end{array} \quad \begin{array}{r} 831 \\ -129 \\ \hline \end{array} \quad \begin{array}{r} 462 \\ -138 \\ \hline \end{array}$$

Written Find each difference.

	a	b	c
1.	$\begin{array}{r} 64 \\ -36 \\ \hline 28 \end{array}$	$\begin{array}{r} 73 \\ -24 \\ \hline 49 \end{array}$	$\begin{array}{r} 34 \\ -18 \\ \hline 16 \end{array}$
2.	$\begin{array}{r} 45 \\ -28 \\ \hline 17 \end{array}$	$\begin{array}{r} 62 \\ -17 \\ \hline 45 \end{array}$	$\begin{array}{r} 81 \\ -29 \\ \hline 52 \end{array}$
3.	$\begin{array}{r} 92 \\ -38 \\ \hline 54 \end{array}$	$\begin{array}{r} 76 \\ -18 \\ \hline 58 \end{array}$	$\begin{array}{r} 66 \\ -27 \\ \hline 39 \end{array}$
4.	$\begin{array}{r} 443 \\ -217 \\ \hline 226 \end{array}$	$\begin{array}{r} 338 \\ -129 \\ \hline 209 \end{array}$	$\begin{array}{r} 414 \\ -308 \\ \hline 106 \end{array}$
5.	$\begin{array}{r} 246 \\ -137 \\ \hline 109 \end{array}$	$\begin{array}{r} 382 \\ -178 \\ \hline 204 \end{array}$	$\begin{array}{r} 483 \\ -129 \\ \hline 354 \end{array}$
6.	$\begin{array}{r} 431 \\ -117 \\ \hline 314 \end{array}$	$\begin{array}{r} 232 \\ -116 \\ \hline 116 \end{array}$	$\begin{array}{r} 194 \\ -67 \\ \hline 127 \end{array}$
7.	$\begin{array}{r} 341 \\ -126 \\ \hline 215 \end{array}$	$\begin{array}{r} 832 \\ -418 \\ \hline 414 \end{array}$	$\begin{array}{r} 914 \\ -309 \\ \hline 605 \end{array}$

Write an open sentence for each problem. Solve the open sentence. Answer the problem.

8. Don and Lee together caught 62 fish. Don caught 38 of them. How many of them did Lee catch?

$$62 - 38 = \square; 24 \text{ fish}$$

9. Fred and Joe together drove 386 miles. Fred drove 158 miles. How many miles did Joe drive?

$$386 - 158 = \square; 228 \text{ miles}$$

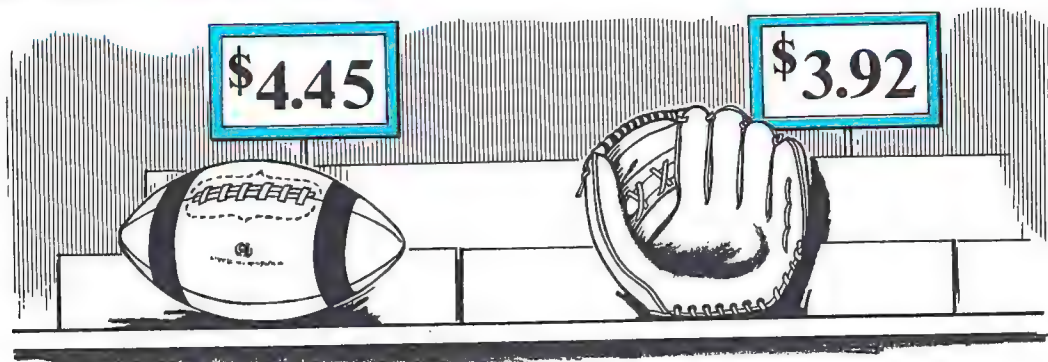
Oral 8a. $50+14$; b. $60+13$; c. $20+14$

9a. $100+50+18$; b. $700+20+17$; c. $300+50+16$

10a. $400+70+12$; b. $300+80+15$; c. $700+30+13$

11a. $100+70+13$; b. $800+20+11$; c. $400+50+12$

Renaming Tens in Addition



George's father bought a football and a glove for his son's birthday. How much did he spend for both articles?

An open sentence for this problem is $445 + 392 = \square$. To solve the open sentence think about the addition as shown below.

$\begin{array}{r} 445 \\ + 392 \\ \hline \end{array}$	$\begin{array}{r} 400 + 40 + 5 \\ + (300 + 90 + 2) \\ \hline 700 + 130 + 7 \end{array}$	$\begin{array}{r} \overset{100}{400} + 40 + 5 \\ + (300 + 90 + 2) \\ \hline 800 + 130 + 7 = 837 \end{array}$
	<p>The greatest number we should name in the tens column is 90. Why? ^{#1} Is 130 greater than 90? Yes</p>	<p>We can think of 130 as $100 + 30$. Where do we write the 100? The 30? See the display above.</p>

We can show this renaming in column form.

Think		Write												
$\begin{array}{r} 445 \\ + 392 \\ \hline \end{array}$ $\begin{array}{r} \overset{100}{400} + 40 + 5 \\ + (300 + 90 + 2) \\ \hline 800 + 130 + 7 \end{array}$	<table border="1" style="border-collapse: collapse; margin: auto;"> <tr> <td style="padding: 2px;">H</td> <td style="padding: 2px;">T</td> <td style="padding: 2px;">O</td> </tr> <tr> <td style="padding: 2px;">1</td> <td style="padding: 2px;">4</td> <td style="padding: 2px;">4</td> </tr> <tr> <td style="padding: 2px;">4</td> <td style="padding: 2px;">4</td> <td style="padding: 2px;">5</td> </tr> <tr> <td style="padding: 2px;">8</td> <td style="padding: 2px;">3</td> <td style="padding: 2px;">7</td> </tr> </table> <div style="margin-top: 10px;"> $5 + 2 = 7$ $4T + 9T = 13T = 1H + 3T$ $1H + 4H + 3H = 8H$ </div>	H	T	O	1	4	4	4	4	5	8	3	7	$\begin{array}{r} 445 \\ + 392 \\ \hline 837 \end{array}$
H	T	O												
1	4	4												
4	4	5												
8	3	7												

Since $4T + 9T = 1H + 3T$, where is the 1H recorded? The 3T? **in the hundreds column; in the tens place of the numeral for the sum**

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^{#1} In expanded notation the number of ones, the number of tens, the number of hundreds, and so on is always less than 10.

Oral Tell which numeral should replace each below.

1.

$$\begin{array}{r} 492 \\ +345 \\ \hline \end{array}$$

H	T	O
1 	4	9
+3	4	5

2.

$$\begin{array}{r} 695 \\ +281 \\ \hline \end{array}$$

H	T	O
1 	6	9
+2	8	1

3.

$$\begin{array}{r} 432 \\ +494 \\ \hline \end{array}$$

H	T	O
1 	4	3
+4	9	4

4.

$$\begin{array}{r} 741 \\ +174 \\ \hline \end{array}$$

H	T	O
1 	7	4
+1	7	4

Written Copy. Find each sum.

	a	b	c
1.	$\begin{array}{r} 385 \\ +464 \\ \hline 849 \end{array}$	$\begin{array}{r} 294 \\ +143 \\ \hline 437 \end{array}$	$\begin{array}{r} 198 \\ +190 \\ \hline 388 \end{array}$
2.	$\begin{array}{r} 788 \\ +160 \\ \hline 948 \end{array}$	$\begin{array}{r} 349 \\ +280 \\ \hline 629 \end{array}$	$\begin{array}{r} 165 \\ +328 \\ \hline 493 \end{array}$
3.	$\begin{array}{r} 148 \\ +271 \\ \hline 419 \end{array}$	$\begin{array}{r} 135 \\ +272 \\ \hline 407 \end{array}$	$\begin{array}{r} 642 \\ +197 \\ \hline 839 \end{array}$
4.	$\begin{array}{r} 725 \\ +255 \\ \hline 980 \end{array}$	$\begin{array}{r} 328 \\ +292 \\ \hline 620 \end{array}$	$\begin{array}{r} 463 \\ +237 \\ \hline 700 \end{array}$
5.	$\begin{array}{r} 639 \\ +170 \\ \hline 809 \end{array}$	$\begin{array}{r} 493 \\ +243 \\ \hline 736 \end{array}$	$\begin{array}{r} 621 \\ +193 \\ \hline 814 \end{array}$
6.	$\begin{array}{r} 274 \\ +541 \\ \hline 815 \end{array}$	$\begin{array}{r} 348 \\ +391 \\ \hline 739 \end{array}$	$\begin{array}{r} 632 \\ +174 \\ \hline 806 \end{array}$

7.	$\begin{array}{r} 632 \\ +296 \\ \hline 928 \end{array}$	$\begin{array}{r} 438 \\ +291 \\ \hline 729 \end{array}$	$\begin{array}{r} 324 \\ +283 \\ \hline 607 \end{array}$
8.	$\begin{array}{r} 369 \\ +140 \\ \hline 509 \end{array}$	$\begin{array}{r} 348 \\ +291 \\ \hline 639 \end{array}$	$\begin{array}{r} 483 \\ +294 \\ \hline 777 \end{array}$
9.	$\begin{array}{r} 234 \\ +395 \\ \hline 629 \end{array}$	$\begin{array}{r} 624 \\ +193 \\ \hline 817 \end{array}$	$\begin{array}{r} 632 \\ +176 \\ \hline 808 \end{array}$
10.	$\begin{array}{r} 426 \\ +392 \\ \hline 818 \end{array}$	$\begin{array}{r} 634 \\ +172 \\ \hline 806 \end{array}$	$\begin{array}{r} 652 \\ +185 \\ \hline 837 \end{array}$

Write an open sentence for each problem. Solve the open sentence. Answer the problem.

11. Mary bought a sofa for 495 dollars. She also bought a rug for 233 dollars. How much did she spend? $495+233=\square$; 728 dollars

12. There are 124 fourth graders at the Shaffer School. There are 192 pupils in the fourth grade at the Smith School. Altogether there are how many fourth graders in these two schools? $124+192=\square$; 316 fourth graders

13. Glenn had 184 pennies. His father gave him 31 more. How many pennies does Glenn have now? $184+31=\square$; 215 pennies

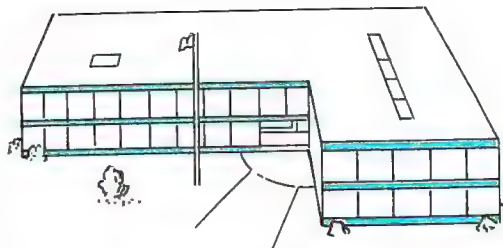
14. There are 542 houses in Center-ville. There are 382 houses in New Salem. Altogether there are how many houses in the 2 towns? $542+382=\square$; 924 houses

15. Mr. Robertson and his family were on a trip. On Monday they drove 422 miles. On Tuesday they drove 396 miles. How far did they drive in the two days? $422+396=\square$; 818 miles

Renaming Hundreds in Subtraction



Old School- 421 pupils



New School-512 pupils

Write an open sentence to show how many more pupils are in the new school than in the old school. To solve $512 - 421 = \square$, think about the subtraction as shown below.

$$\begin{array}{r} 512 \\ -421 \\ \hline \end{array}$$

Can you subtract in every place-value position? Why? To make $10 < 20$. subtraction possible in the tens column, what can we do to the minuend? We can think about the subtraction as shown below. Rename 512

A	B	C	D
$\begin{array}{r} 512 \\ -421 \\ \hline \end{array}$	$\begin{array}{r} 500+10+2 \\ -(400+20+1) \\ \hline \end{array}$	$\begin{array}{r} 400+110+2 \\ -(400+20+1) \\ \hline 90+1 \end{array}$	$\begin{array}{r l l} \text{H} & \text{T} & \text{O} \\ 4 & 11 & 2 \\ \hline 5 & 1 & 2 \\ -4 & 2 & 1 \\ \hline 9 & 1 & \end{array}$

Explain the change in going from **A** to **B**. Why is $500+10+2$ not useful for doing the subtraction in **B**? Explain the change in going from **B** to **C**. Why is $400+110+2$ useful for doing the subtraction? How is the change from **B** to **C** shown in the grid form in **D**? We can show the renaming in simpler column form.

D	E	F
$\begin{array}{r l l} \text{H} & \text{T} & \text{O} \\ 4 & 11 & 2 \\ \hline 5 & 1 & 2 \\ -4 & 2 & 1 \\ \hline 9 & 1 & \end{array}$	$\begin{array}{r} 411 \\ 512 \\ -421 \\ \hline 91 \end{array}$	$\begin{array}{r} 512 \\ -421 \\ \hline 91 \end{array}$

- 54
- *1 512 and 421 are named in expanded notation.
 - *2 In tens column, $10 < 20$.
 - *3 $500+10+2$ is renamed as $400+110+2$.
 - *4 Can subtract in each place-value position.
 - *5 by the reminder numerals above 512

Oral Answer each of the following questions.

1. Explain the change in going from D to E on page 54. **The place-value grid is omitted.**

2. Explain the change in going from E to F on page 54. **The remainder numerals are omitted.**

3. How many more pupils attend the new school than attend the old school? **91 pupils**

In which of the following must you rename the minuend so that you can subtract in the tens column? **4a, 4b, 4c, 5c, 6a, 6c, 7a, 7b, 7c**

	<i>a</i>	<i>b</i>	<i>c</i>
4.	$\begin{array}{r} 411 \\ -280 \\ \hline \end{array}$	$\begin{array}{r} 424 \\ -239 \\ \hline \end{array}$	$\begin{array}{r} 862 \\ -291 \\ \hline \end{array}$
5.	$\begin{array}{r} 438 \\ -137 \\ \hline \end{array}$	$\begin{array}{r} 348 \\ -139 \\ \hline \end{array}$	$\begin{array}{r} 384 \\ -192 \\ \hline \end{array}$
6.	$\begin{array}{r} 624 \\ -484 \\ \hline \end{array}$	$\begin{array}{r} 379 \\ -274 \\ \hline \end{array}$	$\begin{array}{r} 618 \\ -492 \\ \hline \end{array}$
7.	$\begin{array}{r} 377 \\ -179 \\ \hline \end{array}$	$\begin{array}{r} 636 \\ -396 \\ \hline \end{array}$	$\begin{array}{r} 345 \\ -290 \\ \hline \end{array}$

Tell the expanded notation that you would use for each minuend in doing each subtraction below.

See below.

	<i>a</i>	<i>b</i>	<i>c</i>
8.	$\begin{array}{r} 428 \\ -135 \\ \hline \end{array}$	$\begin{array}{r} 249 \\ -187 \\ \hline \end{array}$	$\begin{array}{r} 635 \\ -192 \\ \hline \end{array}$
9.	$\begin{array}{r} 942 \\ -661 \\ \hline \end{array}$	$\begin{array}{r} 482 \\ -291 \\ \hline \end{array}$	$\begin{array}{r} 353 \\ -182 \\ \hline \end{array}$

$$\begin{array}{r} 10. \quad 861 \\ \quad -270 \\ \hline \end{array} \quad \begin{array}{r} 377 \\ -185 \\ \hline \end{array} \quad \begin{array}{r} 636 \\ -196 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad 284 \\ \quad -193 \\ \hline \end{array} \quad \begin{array}{r} 539 \\ -182 \\ \hline \end{array} \quad \begin{array}{r} 347 \\ -176 \\ \hline \end{array}$$

Written Copy. Find each difference.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 464 \\ -173 \\ \hline 291 \end{array}$	$\begin{array}{r} 372 \\ -182 \\ \hline 190 \end{array}$	$\begin{array}{r} 334 \\ -172 \\ \hline 162 \end{array}$
2.	$\begin{array}{r} 642 \\ -281 \\ \hline 361 \end{array}$	$\begin{array}{r} 738 \\ -271 \\ \hline 467 \end{array}$	$\begin{array}{r} 341 \\ -150 \\ \hline 191 \end{array}$
3.	$\begin{array}{r} 455 \\ -262 \\ \hline 193 \end{array}$	$\begin{array}{r} 627 \\ -362 \\ \hline 265 \end{array}$	$\begin{array}{r} 811 \\ -550 \\ \hline 261 \end{array}$
4.	$\begin{array}{r} 246 \\ -151 \\ \hline 95 \end{array}$	$\begin{array}{r} 382 \\ -190 \\ \hline 192 \end{array}$	$\begin{array}{r} 483 \\ -191 \\ \hline 292 \end{array}$
5.	$\begin{array}{r} 437 \\ -271 \\ \hline 166 \end{array}$	$\begin{array}{r} 238 \\ -164 \\ \hline 74 \end{array}$	$\begin{array}{r} 164 \\ -72 \\ \hline 92 \end{array}$
6.	$\begin{array}{r} 149 \\ -58 \\ \hline 91 \end{array}$	$\begin{array}{r} 167 \\ -92 \\ \hline 75 \end{array}$	$\begin{array}{r} 176 \\ -83 \\ \hline 93 \end{array}$

Write an open sentence for each problem. Solve the open sentence. Answer the problem.

7. In one hour 452 cars passed the corner of 1st and Main going east; 280 cars passed going west. How many more cars went east?

$$452 - 280 = \square; 172 \text{ cars}$$

8. June's book has 148 pages in it. She has read 63 pages. How many more pages does she have to read?

$$148 - 63 = \square; 85 \text{ pages}$$

- Oral** 8a. $300 + 120 + 8$; b. $100 + 140 + 9$; c. $500 + 130 + 5$
 9a. $800 + 140 + 2$; b. $300 + 180 + 2$; c. $200 + 150 + 3$
 10a. $700 + 160 + 1$; b. $200 + 170 + 7$; c. $500 + 130 + 6$
 11a. $100 + 180 + 4$; b. $400 + 130 + 9$; c. $200 + 140 + 7$

Solving Problems

Write an open sentence for each problem. Solve the open sentence. Answer the problem.

1. Margie had 25 cents. Her mother gave her 23 cents. How much money did Margie have then?
 $25+23=\square$; 48 cents

2. On the sixteenth day of May Glenn said, "In 12 days I will have a birthday." What is the date of his birthday? $16+12=\square$; May 28

3. Georgia wants to buy a doll dress which costs 98 cents. She has 42 cents. How much money does she still need? $98-42=\square$; 56 cents

4. John had 159 review problems to solve. He solved 124 of them. How many more does he still need to solve? $159-124=\square$; 35 problems

5. John has 48 cents. He needs 47 cents more to buy a game. How much does the game cost?
 $48+47=\square$; 95 cents

6. Tim weighs 94 pounds. Ralph weighs 89 pounds. Tim weighs how many more pounds than Ralph?
 $94-89=\square$; 5 pounds

7. Donna weighed 69 pounds a year ago. Now she weighs 77 pounds. She has gained how many pounds?
 $77-69=\square$; 8 pounds

8. There are 14 boys and 17 girls in a class. In all, how many pupils are there in the class? $14+17=\square$;
31 pupils

9. There are 28 pupils in a class. Thirteen of them are girls. How many boys are there in the class?
 $28-13=\square$; 15 boys

10. The sum of two numbers is 482. One of the addends is 195. What is the other addend? $\square+195=482$; 287

11. Carol has 42 cents in all. She has a quarter and some pennies. How many pennies does she have?
 $42-25=\square$; 17 pennies

12. George had 148 football cards. His friend gave him 28 more. How many football cards does he have now?
 $148+28=\square$; 176 football cards

13. A newsboy had 254 newspapers. He sold 138 of them. How many did he still have? $254-138=\square$;
116 newspapers

14. A lion weighs 345 pounds. Its mate weighs 284 pounds. What is the difference of their weights?
 $345-284=\square$; 61 pounds

15. A lion weighs 327 pounds. Its mate weighs 281 pounds. Together they weigh how many pounds?
 $327+281=\square$; 608 pounds

16. John and Glenn went bowling. John bowled 152 and Glenn bowled 138. How much better is John's score than Glenn's score?
 $152-138=\square$; 14 points or pins

Can you do this? Write a word problem for each of the following open sentences. **Answers will vary.**

$$\begin{array}{c} a \\ 48+41=\square \end{array}$$

$$\begin{array}{c} b \\ 327-145=\square \end{array}$$

Checkup Time

The numerals in () tell the pages where you can turn for help.

Important Ideas

1. Addition is an operation on two numbers resulting in a third number called the sum. (33)

2. When adding two or more addends, changing the order of the addends does not change the sum. We say addition is commutative. (34)

3. Zero is the identity number of addition. (35)

4. Subtraction is an operation on two numbers resulting in a third number called the difference. (36)

5. Addition and subtraction are inverse operations. (38)

Words to Know

1. Sum, addend, addition numeral (33)

2. Commutative property of addition (34)

3. Identity number of addition (35)

4. Subtraction numeral, difference (36)

5. Inverse operations (38)

6. Associative property of addition (40)

Questions to Discuss

See page T57.

1. How is the union of two sets used to explain addition? (33)

2. How is the commutative property of addition useful in learning basic addition facts? (34)

3. Why are subtraction and addition called inverse operations? (38)

4. When joining 2 or more sets, why doesn't it matter which sets we join first? (40)

Written Practice

Copy. Find each sum or difference.

	<i>a</i>	<i>b</i>
1.	$\begin{array}{r} 37 \\ -16 \\ \hline 21 \end{array} \quad (36)$	$\begin{array}{r} 26 \\ -13 \\ \hline 13 \end{array} \quad (36)$
2.	$\begin{array}{r} 42 \\ +34 \\ \hline 76 \end{array} \quad (45)$	$\begin{array}{r} 56 \\ +42 \\ \hline 98 \end{array} \quad (45)$
3.	$\begin{array}{r} 356 \\ +233 \\ \hline 589 \end{array} \quad (45)$	$\begin{array}{r} 360 \\ +238 \\ \hline 598 \end{array} \quad (45)$
4.	$\begin{array}{r} 481 \\ -130 \\ \hline 351 \end{array} \quad (47)$	$\begin{array}{r} 249 \\ -132 \\ \hline 117 \end{array} \quad (47)$
5.	$\begin{array}{r} 273 \\ +109 \\ \hline 382 \end{array} \quad (49)$	$\begin{array}{r} 287 \\ -118 \\ \hline 169 \end{array} \quad (51)$
6.	$\begin{array}{r} 245 \\ +183 \\ \hline 428 \end{array} \quad (53)$	$\begin{array}{r} 249 \\ -175 \\ \hline 74 \end{array} \quad (55)$

Self-Evaluation

Part 1 Copy. Find each sum or difference.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 14 \\ +13 \\ \hline 27 \end{array}$	$\begin{array}{r} 16 \\ +15 \\ \hline 31 \end{array}$	$\begin{array}{r} 142 \\ +280 \\ \hline 422 \end{array}$
2.	$\begin{array}{r} 39 \\ -24 \\ \hline 15 \end{array}$	$\begin{array}{r} 42 \\ -19 \\ \hline 23 \end{array}$	$\begin{array}{r} 436 \\ -128 \\ \hline 308 \end{array}$
3.	$\begin{array}{r} 56 \\ +38 \\ \hline 94 \end{array}$	$\begin{array}{r} 524 \\ -283 \\ \hline 241 \end{array}$	$\begin{array}{r} 431 \\ -148 \\ \hline 283 \end{array}$

Part 2 Replace the \square with a numeral to make true sentences.

- If $3+9=12$, then $12-9=\square$.
- If $6+8=14$, then $14-8=\square$.
- If $17-9=8$, then $8+9=\square$.
- If $14-5=9$, then $9+\square=14$.
- If $7+8=15$, then $\square=15-8$.

Part 3 Copy. Find each sum.

	<i>a</i>	<i>b</i>
1.	$8+6+2=\square$	$6+5+4=\square$
2.	$8+5+5=\square$	$5+7+5=\square$
3.	$14+9+6=\square$	$3+9+1=\square$
4.	$12+9+8=\square$	$1+8+9=\square$
5.	$7+6+4=\square$	$6+7+4=\square$
6.	$5+9+1=\square$	$9+2+8=\square$

Part 4 Copy. Find each sum or difference.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 16 \\ +19 \\ \hline 35 \end{array}$	$\begin{array}{r} 15 \\ +28 \\ \hline 43 \end{array}$	$\begin{array}{r} 25 \\ +46 \\ \hline 71 \end{array}$
2.	$\begin{array}{r} 63 \\ -18 \\ \hline 45 \end{array}$	$\begin{array}{r} 81 \\ -28 \\ \hline 53 \end{array}$	$\begin{array}{r} 93 \\ -16 \\ \hline 77 \end{array}$
3.	$\begin{array}{r} 147 \\ +431 \\ \hline 578 \end{array}$	$\begin{array}{r} 682 \\ -351 \\ \hline 331 \end{array}$	$\begin{array}{r} 345 \\ -243 \\ \hline 102 \end{array}$
4.	$\begin{array}{r} 18 \\ +44 \\ \hline 62 \end{array}$	$\begin{array}{r} 487 \\ +108 \\ \hline 595 \end{array}$	$\begin{array}{r} 267 \\ +427 \\ \hline 694 \end{array}$
5.	$\begin{array}{r} 35 \\ -18 \\ \hline 17 \end{array}$	$\begin{array}{r} 395 \\ -219 \\ \hline 176 \end{array}$	$\begin{array}{r} 743 \\ -417 \\ \hline 326 \end{array}$
6.	$\begin{array}{r} 385 \\ +463 \\ \hline 848 \end{array}$	$\begin{array}{r} 293 \\ +143 \\ \hline 436 \end{array}$	$\begin{array}{r} 639 \\ -248 \\ \hline 391 \end{array}$
7.	$\begin{array}{r} 428 \\ -236 \\ \hline 192 \end{array}$	$\begin{array}{r} 626 \\ +282 \\ \hline 908 \end{array}$	$\begin{array}{r} 465 \\ -372 \\ \hline 93 \end{array}$

Part 5 Write an open sentence for each problem. Solve the open sentence. Answer the problem.

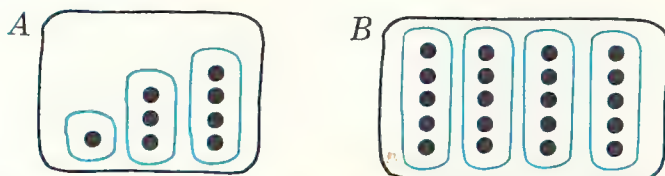
1. George worked 129 review problems last week. This week he worked 125 problems. How many problems has he worked? $129+125=\square$;
254 problems

2. Marty and Sue have to bake 262 cookies for a birthday party. They have already baked 181 cookies. How many more cookies do they need to bake? $262-181=\square$;
81 cookies

Chapter 3

PROPERTIES OF MULTIPLICATION AND DIVISION

Sets and Subsets



In set A , how many subsets are shown? What is the number of each subset? Are they equivalent subsets? What is the number of set A ? **3; 1, 3, and 4; No; 8**

In set B , how many subsets are shown? What is the number of each subset? Are they equivalent subsets? The number of set B can be expressed in two ways. **4; 5; Yes**

$$5+5+5+5=20$$

$$4 \times 5 = 20$$

Does 4×5 name the same number as $5+5+5+5$? How many times is 5 used as an addend in $5+5+5+5$? **Yes; 4**

The operation indicated by the sign \times is called **multiplication**.

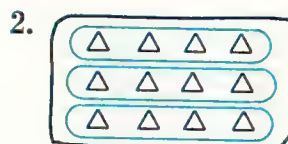
Which is the easier way to name the number of set B , $5+5+5+5$ or 4×5 ? Why? **4×5 ; It is shorter.**

Oral Tell a closed addition and a closed multiplication sentence for each of the following.



$$3+3+3+3=12;$$

$$4 \times 3 = 12$$



$$4+4+4=12;$$

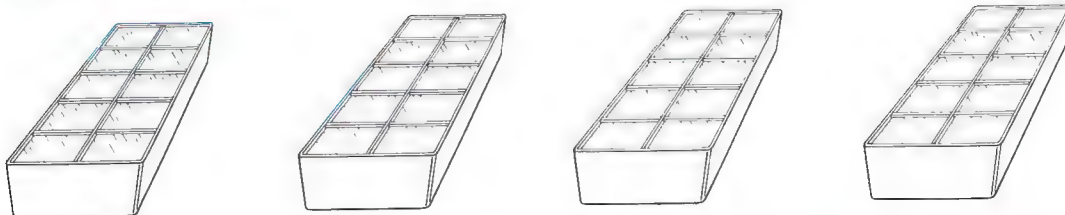
$$3 \times 4 = 12$$



$$2+2+2+2+2=10;$$

$$5 \times 2 = 10$$

Repeated Addition



How many ice cubes can be made in each of these trays? **10**
 How many ice cubes can be made in all 4 trays? **40**

You can express this in two ways as follows.

$$10+10+10+10=\square$$

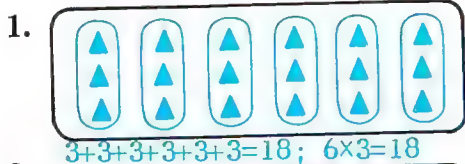
$$4\times 10=\square$$

Do $10+10+10+10$ and 4×10 name the same number? What is that number? How many times is 10 used as an addend in $10+10+10+10$? What does the 4 tell you in 4×10 ? What does the 10 tell you in 4×10 ? **Yes; 40; 4; the number of sets; the number of each set**

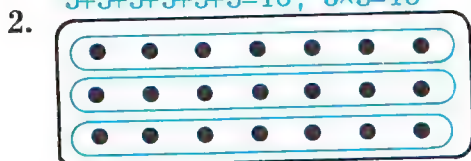
How are 4×10 and $10+10+10+10$ related to each other? Which of them would you prefer to use? **Both name the same number; 4×10**

Multiplication is a short way of finding a sum when all addends are the same.

Oral Tell an addition and a multiplication sentence for each of the following.



$$3+3+3+3+3+3=18; 6\times 3=18$$



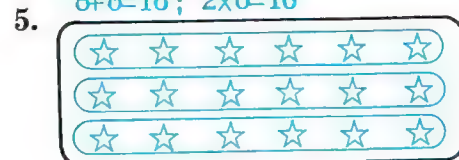
$$7+7+7=21; 3\times 7=21$$



$$2+2+2+2+2+2+2+2=16; 8\times 2=16$$



$$8+8=16; 2\times 8=16$$



$$6+6+6=18; 3\times 6=18$$

Tell an addition numeral and a multiplication numeral for the number of each set below.

6. A set consisting of 3 equivalent subsets of 5 members each $5+5+5$;
 3×5

7. A set consisting of 5 equivalent subsets of 3 members each $3+3+3+3+3$;
 5×3

8. A set consisting of 6 equivalent subsets of 4 members each
 $4+4+4+4+4+4$; 6×4

9. A set consisting of 4 equivalent subsets of 6 members each $6+6+6+6$;
 4×6

10. A set consisting of 2 equivalent subsets of 7 members each $7+7$;
 2×7

11. A set consisting of 7 equivalent subsets of 2 members each
 $2+2+2+2+2+2+2$; 7×2

Written Copy. Replace each \square with the simplest numeral. Then write a multiplication sentence for each addition.

- | a | b |
|--|---|
| 1. $2+2+2=\square$ 6
$3 \times 2=6$ | $5+5=\square$ 10
$2 \times 5=10$ |
| 2. $4+4+4+4=\square$ 16
$4 \times 4=16$ | $5+5+5=\square$ 15
$3 \times 5=15$ |
| 3. $6+6+6=\square$ 18
$3 \times 6=18$ | $7+7+7=\square$ 21
$3 \times 7=21$ |
| 4. $3+3=\square$ 6
$2 \times 3=6$ | $9+9=\square$ 18
$2 \times 9=18$ |
| 5. $8+8=\square$ 16
$2 \times 8=16$ | $2+2+2+2=\square$ 8
$4 \times 2=8$ |
| 6. $4+4=\square$ 8
$2 \times 4=8$ | $6+6+6+6=\square$ 24
$4 \times 6=24$ |
| 7. $7+7+7+7=\square$ 28
$4 \times 7=28$ | $9+9+9=\square$ 27
$3 \times 9=27$ |
| 8. $8+8+8=\square$ 24
$3 \times 8=24$ | $1+1+1=\square$ 3
$3 \times 1=3$ |

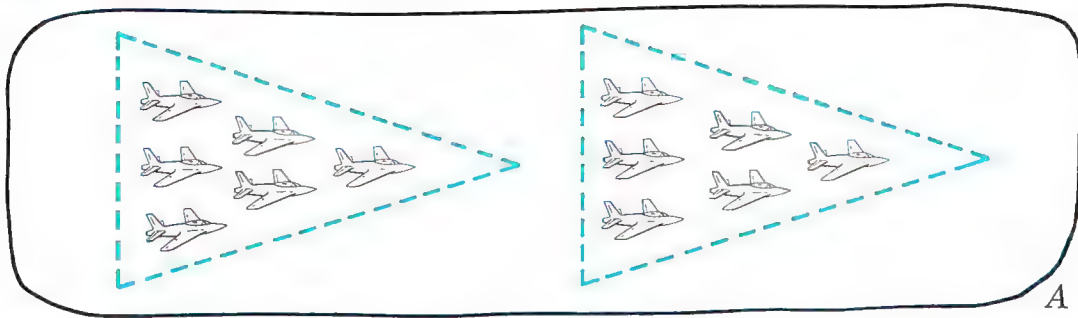
Copy. Replace each \square by the simplest numeral. Then write an addition sentence for each multiplication. **Addition sentences shown for column a only.**

- | a | b |
|---|----------------------------|
| 9. $6 \times 4=\square$ 24
$4+4+4+4+4+4=24$ | $4 \times 6=\square$
56 |
| 10. $7 \times 8=\square$ 56
$8+8+8+8+8+8+8=56$ | $8 \times 7=\square$
27 |
| 11. $9 \times 3=\square$ 27
$3+3+3+3+3+3+3+3+3=27$ | $3 \times 9=\square$
10 |
| 12. $2 \times 5=\square$ 10
$5+5=10$ | $5 \times 2=\square$
16 |
| 13. $2 \times 8=\square$ 16
$8+8=16$ | $8 \times 2=\square$
28 |
| 14. $4 \times 7=\square$ 28
$7+7+7+7=28$ | $7 \times 4=\square$
21 |
| 15. $3 \times 7=\square$ 21
$7+7+7=21$ | $7 \times 3=\square$
18 |
| 16. $3 \times 6=\square$ 18
$6+6+6=18$ | $6 \times 3=\square$
32 |
| 17. $4 \times 8=\square$ 32
$8+8+8+8=32$ | $8 \times 4=\square$
18 |
| 18. $2 \times 9=\square$ 18
$9+9=18$ | $9 \times 2=\square$
36 |
| 19. $4 \times 9=\square$ 36
$9+9+9+9=36$ | $9 \times 4=\square$
40 |
| 20. $8 \times 5=\square$ 40
$5+5+5+5+5+5+5+5=40$ | $5 \times 8=\square$
48 |
| 21. $8 \times 6=\square$ 48
$6+6+6+6+6+6+6+6=48$ | $6 \times 8=\square$
15 |
| 22. $5 \times 3=\square$ 15
$3+3+3+3+3=15$ | $3 \times 5=\square$ |

Can you do this? Draw a set picture for each of the following.

- | a | b |
|-------------------------|----------------------|
| 1. $3 \times 7=\square$ | $7+7+7=\square$ |
| 2. $6+6+6+6=\square$ | $4 \times 6=\square$ |
| 3. $7 \times 3=\square$ | $6 \times 4=\square$ |

Multiplication



In set A above, how many subsets are shown? What is the number of each subset? Are they equivalent subsets? 2; 6; Yes

We can express the number of set A as follows.

$$2 \times 6$$

We call a numeral like 2×6 a **multiplication numeral**. Each of the two numbers in 2×6 is called a **factor**.

What is the first factor in 2×6 ? What does it tell you? What is the second factor? What does it tell you? 2; the number of equivalent subsets; 6; the number of each subset

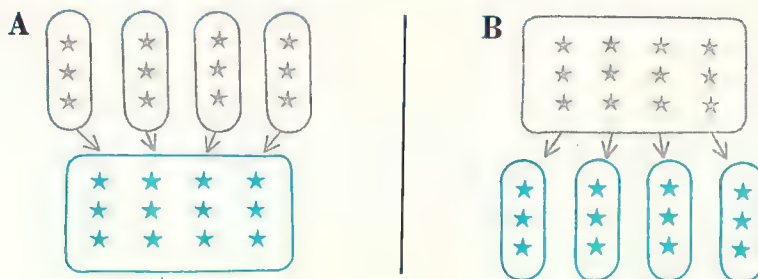
Multiplication is an operation on two numbers resulting in a third number called the **product**.

Oral What number is named by each of the following?

Written Using whole numbers as factors, name each number below as a product of two factors in as many different ways as you can. See page T62.

	<i>a</i>	<i>b</i>	<i>c</i>		<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
1.	8×4 32	4×8 32	6×8 48	1.	6	7	8	10
2.	8×6 48	9×7 63	7×9 63	2.	36	18	20	24
3.	9×5 45	5×9 45	4×9 36	3.	27	28	32	40
4.	6×7 42	3×9 27	5×8 40	4.	16	15	14	21
5.	4×7 28	7×8 56	8×3 24					

Separating a Set into Equivalent Subsets

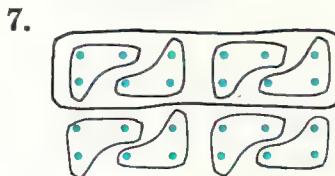
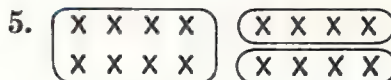


In **A**, 4 sets of 3 members each are joined to form one set of 12 members. How do the arrows show this joining? **They show that the 4 sets are combined into a single set.**

In **B**, think of starting with the set of 12 members that was formed in **A**. What do the arrows in **B** show? How many subsets are formed? What is the number of each subset? Are the subsets equivalent? **The set of 12 members is separated into 4 subsets; 4; 3; Yes**

Does separating the set of 12 in **B** undo the joining of sets in **A**? Does joining the sets in **A** undo the separating of the set of 12 in **B**? **Yes; Yes**

Oral Tell what you could do with each set to obtain the subsets shown. Then tell what you could do with the subsets to obtain the set.
See page T63.



Division



Think of the picture as showing a set of 12 being separated into equivalent subsets of 3 members each. How many subsets are formed? **4**

Thinking of the numbers of the sets above, we can write the following sentences.

$$12 \div 3 = 4$$

Twelve divided by three is equal to four.

The sign \div in $12 \div 3 = 4$ indicates an operation called **division**. The number 4 is called the **quotient**. We call the symbol $12 \div 3$ a **division numeral**.

*Division is an operation on two numbers resulting in a third number called the **quotient**.*

Oral Tell a division sentence for each set picture below.

1. $6 \div 3 = 2$

2. $10 \div 2 = 5$

3. $6 \div 3 = 2$

Written Copy. Then find each quotient.

a	b
1. $18 \div 3 = \boxed{6}$	$12 \div 2 = \boxed{6}$

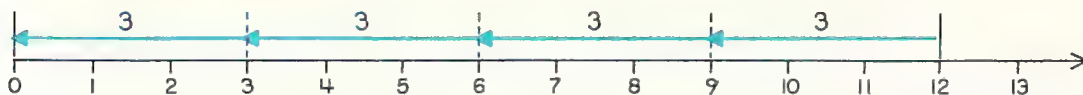
2. $21 \div 3 = \boxed{7}$	$36 \div 4 = \boxed{9}$
----------------------------	-------------------------

3. $48 \div 6 = \boxed{8}$	$48 \div 8 = \boxed{6}$
----------------------------	-------------------------

Draw a set picture for each of the following.

a	b
4. $12 \div 4 = \boxed{}$	$12 \div 2 = \boxed{}$


Repeated Subtraction

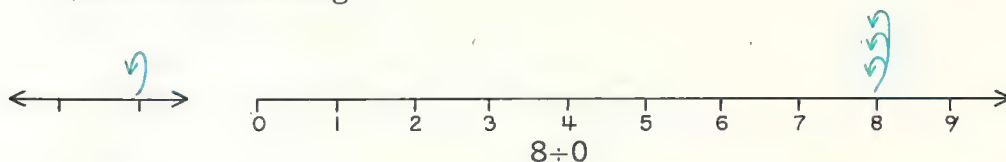


Start at 12 on the number line above. How many times can you subtract 3 before reaching 0? We can express the repeated subtraction by the following division sentence. **4**

$$12 \div 3 = 4$$

Division can be thought of as repeated subtraction.

The  on the first number line below shows how we might think of subtracting 0.



How many subtractions of 0 are shown on the number line above $8 \div 0$? Would you still be at 8? Could you think of subtracting 0 from 8 more times? Could you ever reach 0 by repeated subtraction of 0 from 8? Try this for $7 \div 0$. **3; Yes; Yes; No**

Division by zero is meaningless. We never divide by zero.

Oral To answer each question below, think of subtracting until the final difference is zero.

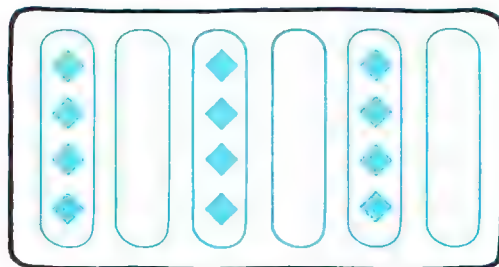
1. Start with 15. How many times can you subtract 5? Express this as a division sentence. **3; $15 \div 5 = 3$**

2. Start with 15. How many times can you subtract 3? Express this as a division sentence. **5; $15 \div 3 = 5$**

Written Use repeated subtraction on a number line to find the simplest numeral for each quotient.

	<i>a</i>	<i>b</i>
1.	$14 \div 2 = \boxed{7}$	$18 \div 3 = \boxed{6}$
2.	$14 \div 7 = \boxed{2}$	$18 \div 6 = \boxed{3}$
3.	$24 \div 6 = \boxed{4}$	$24 \div 4 = \boxed{6}$
4.	$21 \div 7 = \boxed{3}$	$21 \div 3 = \boxed{7}$

Multiplication and Division as Inverse Operations



Think about the picture as representing 6 subsets of 4 members. These 6 subsets are being joined to form a larger set. What multiplication sentence can you write for this? $6 \times 4 = 24$

Think about the picture as representing a set of 24 members separated into equivalent subsets containing 4 members each. What division sentence can you write for this? $24 \div 4 = 6$

Does dividing by 4 undo multiplying by 4? Does multiplying by 4 undo dividing by 4? ^{Yes} These ideas can be expressed as follows.

If $6 \times 4 = 24$, then $24 \div 4 = 6$.

If $24 \div 4 = 6$, then $6 \times 4 = 24$.

Since multiplication and division undo each other, they are called **inverse operations**.

Oral What numeral should replace the \square in each of the following?

1. If $3 \times 4 = 12$, then $12 \div 4 = \square$. ³

2. If $6 \times 4 = 24$, then $24 \div 4 = \square$. ⁶

3. If $6 \times 8 = 48$, then $48 \div 8 = \square$. ⁶

4. If $7 \times 8 = 56$, then $56 \div 8 = \square$. ⁷

5. If $9 \times 7 = 63$, then $63 \div 7 = \square$. ⁹

6. If $8 \times 9 = 72$, then $72 \div 9 = \square$. ⁸

7. If $56 \div 8 = 7$, then $\square \times 8 = 56$. ⁷

8. If $45 \div 9 = 5$, then $5 \times 9 = \square$. ⁴⁵

9. If $42 \div 7 = 6$, then $6 \times \square = 42$. ⁷

10. If $42 \div 6 = 7$, then $\square \times 6 = 42$. ⁷

11. If $40 \div 8 = 5$, then $5 \times 8 = \square$. ⁴⁰

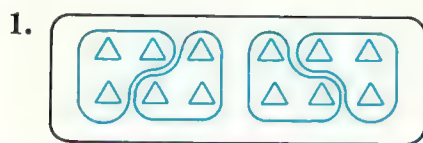
12. If $36 \div 9 = 4$, then $\square \times 9 = 36$. ⁴

13. If $81 \div 9 = 9$, then $9 \times \square = 81$. ⁹

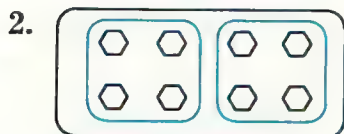
Tell the inverse for each of the following.

- | <i>a</i> | <i>b</i> |
|--|--------------------------------------|
| 14. $2 \times 4 = 8$
$8 \div 4 = 2$ | $9 \div 3 = 3$
$3 \times 3 = 9$ |
| 15. $9 \times 8 = 72$
$72 \div 8 = 9$ | $45 \div 9 = 5$
$5 \times 9 = 45$ |
| 16. $8 \times 7 = 56$
$56 \div 7 = 8$ | $64 \div 8 = 8$
$8 \times 8 = 64$ |
| 17. $8 \times 4 = 32$
$32 \div 4 = 8$ | $28 \div 7 = 4$
$4 \times 7 = 28$ |
| 18. $7 \times 6 = 42$
$42 \div 6 = 7$ | $40 \div 8 = 5$
$5 \times 8 = 40$ |
| 19. $8 \times 3 = 24$
$24 \div 3 = 8$ | $48 \div 6 = 8$
$8 \times 6 = 48$ |
| 20. $2 \times 9 = 18$
$18 \div 9 = 2$ | $42 \div 7 = 6$
$6 \times 7 = 42$ |

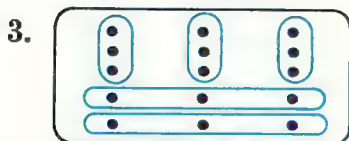
Written Write a multiplication and a division sentence for each set picture below.



$4 \times 3 = 12$; $12 \div 3 = 4$



$2 \times 4 = 8$; $8 \div 4 = 2$



$5 \times 3 = 15$; $15 \div 3 = 5$



$3 \times 3 = 9$; $9 \div 3 = 3$

Draw a set picture to show each multiplication or division.

- | <i>a</i> | <i>b</i> |
|----------------------|-----------------|
| 5. $8 \times 3 = 24$ | $20 \div 5 = 4$ |
| 6. $4 \times 7 = 28$ | $24 \div 6 = 4$ |
| 7. $5 \times 4 = 20$ | $36 \div 4 = 9$ |

Use repeated addition or repeated subtraction on a number line to show each of the following.

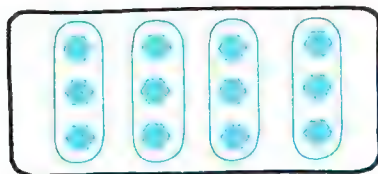
- | <i>a</i> | <i>b</i> |
|-----------------------|-----------------|
| 8. $6 \times 4 = 24$ | $24 \div 4 = 6$ |
| 9. $5 \times 4 = 20$ | $20 \div 4 = 5$ |
| 10. $3 \times 8 = 24$ | $24 \div 8 = 3$ |

Copy. Find each quotient or product.

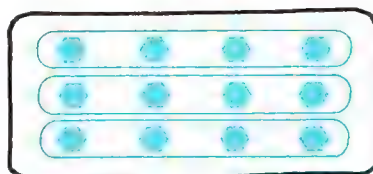
11. If $6 \times 7 = 42$, then $42 \div 7 = \square$. 6
12. If $8 \times 3 = 24$, then $24 \div 3 = \square$. 8
13. If $54 \div 6 = 9$, then $9 \times 6 = \square$. 54
14. If $6 \times 6 = 36$, then $36 \div 6 = \square$. 6
15. If $4 \times 9 = 36$, then $36 \div 9 = \square$. 4
16. If $108 \div 9 = 12$, then $12 \times 9 = \square$. 108
17. If $84 \div 12 = 7$, then $7 \times 12 = \square$. 84
18. If $13 \times 5 = 65$, then $65 \div 5 = \square$. 13
19. If $14 \times 7 = 98$, then $98 \div 7 = \square$. 14
20. If $15 \times 7 = 105$, then $105 \div 7 = \square$. 15

Multiplication is Commutative

The following pictures show two ways of arranging the members of set S .



How many subsets are shown? How many members are there in each subset? What multiplication numeral names the number of set S ? 4×3



How many subsets are shown? How many members are there in each subset? What multiplication numeral names the number of set S ? 3×4

Do 4×3 and 3×4 name the same number? How do you know? We can express this as follows. **Yes; $4 \times 3 = 12$ and $3 \times 4 = 12$**

$$3 \times 4 = 4 \times 3$$

Does commuting or changing the order of the factors change the product? **No**

Changing the order of two factors does not change the product. We call this idea the **commutative property of multiplication**. Or we say **multiplication is commutative**.

Oral Commute the factors to form another name for each number below. Then tell the simplest numeral for each number.

	a	b	c
1.	6×8 8×6 ; 48	7×9 9×7 ; 63	9×8 8×9 ; 72
2.	6×7 7×6 ; 42	9×5 5×9 ; 45	8×4 4×8 ; 32
3.	6×9 9×6 ; 54	3×7 7×3 ; 21	6×3 3×6 ; 18
4.	4×5 5×4 ; 20	2×9 9×2 ; 18	8×2 2×8 ; 16

Written Write two multiplication numerals for each number named below. Use the same two factors for both multiplication numerals. **Answers will vary. Typical answers given below.**

	a	b	c	d
1.	8 2×4 4×2	6 2×3 3×2	28 4×7 7×4	16 8×2 2×8
2.	36 4×9 9×4	12 2×6 6×2	13 1×13 13×1	27 3×9 9×3
3.	9 1×9 9×1	10 2×5 5×2	14 2×7 7×2	18 3×6 6×3
4.	24 4×6 6×4	20 4×5 5×4	32 4×8 8×4	40 5×8 8×5

Zero and One as Factors

A		B	
$0 \times 1 = 0$	$1 \times 0 = 0$	$1 \times 1 = 1$	
$0 \times 2 = 0$	$2 \times 0 = 0$	$1 \times 2 = 2$	$2 \times 1 = 2$
$0 \times 3 = 0$	$3 \times 0 = 0$	$1 \times 3 = 3$	$3 \times 1 = 3$
$0 \times 4 = 0$	$4 \times 0 = 0$	$1 \times 4 = 4$	$4 \times 1 = 4$

In **A**, what number is named by each of the multiplication numerals? Is zero a factor in each of the multiplication sentences? When one of the two factors is zero, what can you say about the product? **0; Yes; The product is 0.**

The product of zero and *any number* is zero.

In **B**, what number is named by 1×2 ? By 2×1 ? By 1×3 ? By 3×1 ? Is the number one a factor in each of the multiplication sentences? When one of two factors is the number one, what can you say about the product? **2; 2; 3; 3; Yes; The product is the other factor.**

When one of two factors is the number one, the product is identical to the other factor. We call the number one **the identity number of multiplication**.

Oral What numeral should replace the \square in each sentence below?

- | a | b |
|---------------------------|------------------------|
| 1. $7 \times \square = 7$ | $7 \times \square = 0$ |
| 2. $\square \times 8 = 0$ | $\square \times 8 = 8$ |

Answer the following.

- What do we call the number one in multiplication? **the identity number**
- When zero is one of two factors what can you say about the product? **The product is 0.**

Written Copy. Find the product.

- | a | b |
|----------------------------|-------------------------|
| 1. $8 \times 4 = \square$ | $\square = 9 \times 1$ |
| 2. $8 \times 0 = \square$ | $\square = 4 \times 9$ |
| 3. $6 \times 9 = \square$ | $\square = 0 \times 9$ |
| 4. $6 \times 0 = \square$ | $\square = 1 \times 6$ |
| 5. $8 \times 7 = \square$ | $\square = 7 \times 1$ |
| 6. $18 \times 1 = \square$ | $\square = 0 \times 4$ |
| 7. $16 \times 1 = \square$ | $\square = 1 \times 35$ |
| 8. $1 \times 24 = \square$ | $\square = 12 \times 0$ |

Multiplication Table

The table below shows how we can arrange the multiplication numerals for the multiples of the first ten whole numbers.

×	0	1	2	3	4	5	6	7	8	9
0	0×0	0×1	0×2	0×3	0×4	0×5	0×6	0×7	0×8	0×9
1	1×0	1×1	1×2	1×3	1×4	1×5	1×6	1×7	1×8	1×9
2	2×0	2×1	2×2	2×3	2×4	2×5	2×6	2×7	2×8	2×9
3	3×0	3×1	3×2	3×3	3×4	3×5	3×6	3×7	3×8	3×9
4	4×0	4×1	4×2	4×3	4×4	4×5	4×6	4×7	4×8	4×9
5	5×0	5×1	5×2	5×3	5×4	5×5	5×6	5×7	5×8	5×9
6	6×0	6×1	6×2	6×3	6×4	6×5	6×6	6×7	6×8	6×9
7	7×0	7×1	7×2	7×3	7×4	7×5	7×6	7×7	7×8	7×9
8	8×0	8×1	8×2	8×3	8×4	8×5	8×6	8×7	8×8	8×9
9	9×0	9×1	9×2	9×3	9×4	9×5	9×6	9×7	9×8	9×9

The gray arrows above show where to write 6×3 in the table. What is the first factor in 6×3 ? Does the arrow start at the 6 in the left column or the top row? What is the second factor in 6×3 ? Does the arrow start at the 3 in the left column or the top row? How can you decide where to write 6×3 in the table? **6; the left column; 3; the top row; where the 6-row and the 3-column meet**

Look at the blue arrows in the table. How can you decide where to write 3×6 in the table? **Write 3×6 in the space where the 3-row and 6-column meet.**

Do 6×3 and 3×6 name the same number? How do you know? If you know that $6 \times 3 = 18$, do you have to multiply 3 by 6 to learn that $3 \times 6 = 18$? Why or why not? **Yes; Multiplication is commutative; No; Since $6 \times 3 = 3 \times 6$ and $6 \times 3 = 18$, then $3 \times 6 = 18$.**

If you know each product shown in black in the table, do you also know each product shown in blue in the table? Do you need to memorize all the products named in the table? Why or why not? **Yes; No; If you know those in black you already know those in blue because multiplication is commutative.**

Oral Answer the following questions about the table on page 70.

1. If you know that a multiple of 7 ends in 3, tell which multiple it is. **63**

2. If a multiple of 9 ends in 4, which multiple is it? **54**

3. What multiple of 7 ends in 8? **28**

4. What does each multiple of 5 end in? **0 or 5**

5. Which multiples of 6 end in 4? **24, 54**

6. Which two multiples of 2 end in 4? **4, 14**

7. Which multiple of 3 ends in 7? **27**

8. Which two multiples of 4 end in 6? **16, 36**

9. Which multiple of 7 ends in 4? **14**

Tell the simplest numeral for each of the following.

	<i>a</i>	<i>b</i>	<i>c</i>
10.	3×8 24	3×4 12	4×9 36
11.	4×7 28	6×9 54	0×3 0
12.	9×7 63	8×9 72	4×5 20
13.	0×4 0	2×9 18	3×7 21
14.	2×8 16	5×8 40	6×5 30
15.	7×4 28	7×7 49	4×4 16

Written Copy. Replace each \square with the simplest numeral.

1. $4 \times 7 = \overset{a}{\square} \times 4 = 28$ $7 \times 8 = \overset{b}{\square} \times 7 = 56$
2. $3 \times 6 = 6 \times 3 = \overset{18}{\square}$ $2 \times 5 = 5 \times 2 = \overset{10}{\square}$
3. $4 \times 9 = 9 \times \overset{4}{\square} = 36$ $1 \times \overset{8}{\square} = 8 \times 1 = 8$
any numeral
4. $0 \times 6 = 6 \times \overset{9}{\square} = 0$ $0 \times 2 = \overset{5}{\square} \times 0 = 0$
5. $1 \times 9 = \overset{3}{\square} \times 1 = 9$ $4 \times 5 = \square \times 4 = \overset{45}{20}$
6. $8 \times \overset{9}{\square} = 3 \times 8 = 24$ $5 \times 9 = 9 \times 5 = \overset{2}{\square}$
7. $\square \times 7 = 7 \times 9 = 63$ $8 \times \square = 2 \times 8 = 16$
8. $5 \times 3 = 3 \times \overset{15}{\square} = 15$ $9 \times 3 = 3 \times 9 = \overset{27}{\square}$
9. $6 \times \overset{8}{\square} = 8 \times 6 = 48$ $7 \times 4 = \overset{4}{\square} \times 7 = 28$

Copy. Make a table as shown below. Complete the table by writing the simplest numeral for each product.

\times	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Divisors, Dividends, Quotients



Johnny arranged a dozen straws in 4 rows as shown above. How many straws are in each row? 3

Either open sentence below can be used to solve the problem given above.

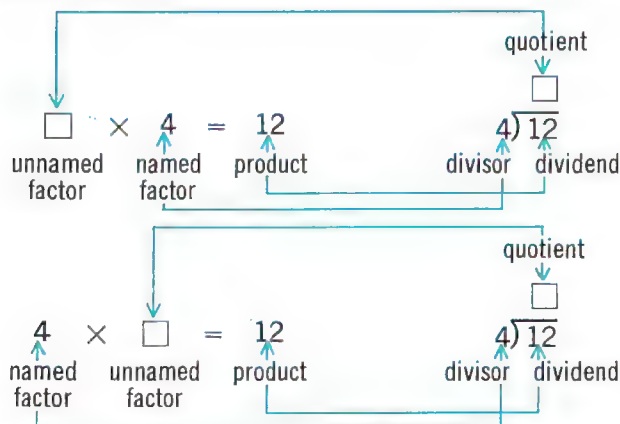
$$4 \times \square = 12 \quad \text{or} \quad 12 \div 4 = \square \quad \text{\textcolor{teal}{*1}}$$

How do you think about the problem to write $4 \times \square = 12$? To write $12 \div 4 = \square$? Another way to express $12 \div 4 = \square$ is shown below.

$$4 \overline{)12}$$

We call 4 the **divisor**. The *divisor* is the number by which we divide. We call 12 the **dividend**. The *dividend* is the number which is to be divided.

We change from an open multiplication sentence to the $\overline{)}$ form as shown below.



What does the named factor become? What does the unnamed factor become? What does the product become? **the divisor; the quotient; the dividend**

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***1** 4 equivalent sets are joined to form a set of 12.

***2** A set of 12 is separated into 4 equivalent subsets.

Oral Tell which division numeral in column **B** matches each $\overline{\hspace{1cm}}$ form in column **A**.

- | A | B |
|-------------------------------|----------------|
| 1. $4\overline{)36}$ c | a. $48 \div 8$ |
| 2. $8\overline{)48}$ a | b. $48 \div 6$ |
| 3. $6\overline{)48}$ b | c. $36 \div 4$ |
| 4. $7\overline{)56}$ e | d. $36 \div 9$ |
| 5. $9\overline{)36}$ d | e. $56 \div 7$ |
| 6. $8\overline{)56}$ f | f. $56 \div 8$ |

Written Write each of the following division numerals in the $\overline{\hspace{1cm}}$ form. Then find each quotient.

- | a | b | c |
|---|--------------------------------------|--------------------------------------|
| 1. $12 \div 3$
$3\overline{)12}$; 4 | $21 \div 3$
$3\overline{)21}$; 7 | $36 \div 4$
$4\overline{)36}$; 9 |
| 2. $24 \div 6$
$6\overline{)24}$; 4 | $24 \div 4$
$4\overline{)24}$; 6 | $36 \div 9$
$9\overline{)36}$; 4 |
| 3. $63 \div 9$
$9\overline{)63}$; 7 | $63 \div 7$
$7\overline{)63}$; 9 | $36 \div 6$
$6\overline{)36}$; 6 |
| 4. $56 \div 7$
$7\overline{)56}$; 8 | $48 \div 6$
$6\overline{)48}$; 8 | $8 \div 4$
$4\overline{)8}$; 2 |
| 5. $64 \div 8$
$8\overline{)64}$; 8 | $81 \div 9$
$9\overline{)81}$; 9 | $49 \div 7$
$7\overline{)49}$; 7 |
| 6. $40 \div 8$
$8\overline{)40}$; 5 | $40 \div 5$
$5\overline{)40}$; 8 | $45 \div 9$
$9\overline{)45}$; 5 |
| 7. $45 \div 5$
$5\overline{)45}$; 9 | $28 \div 4$
$4\overline{)28}$; 7 | $28 \div 7$
$7\overline{)28}$; 4 |
| 8. $72 \div 9$
$9\overline{)72}$; 8 | $0 \div 4$
$4\overline{)0}$; 0 | $0 \div 8$
$8\overline{)0}$; 0 |
| 9. $54 \div 9$
$9\overline{)54}$; 6 | $72 \div 8$
$8\overline{)72}$; 9 | $27 \div 9$
$9\overline{)27}$; 3 |

Write an open division sentence for each problem. Then use the $\overline{\hspace{1cm}}$ form to find the quotient. Answer the problem.

10. Eighty-one chairs were arranged in 9 rows. Each row had the same number of chairs. How many chairs were in each row? $81 \div 9 = \square$; **9 chairs**

11. While shopping, Ann's mother bought 8 cans of green beans for 72 cents. What was the price of just one can? $72 \div 8 = \square$; **9 cents**

12. Wally's teacher gave 36 sheets of construction paper to the 6 pupils on her decorating committee. Each pupil got an equal number of sheets. How many sheets was that?

$36 \div 6 = \square$; **6 sheets**

13. Altogether 4 boys picked 24 apples from a tree. Each boy picked the same number of apples. How many apples did each boy pick?

$24 \div 4 = \square$; **6 apples**

14. Jeff and five friends caught a total of 54 fish. If each boy kept the same number of fish, how many fish did each boy keep? $54 \div 6 = \square$; **9 fish**

Can you do this? Compose a story problem for each open sentence below. **Answers will vary.**

- | a | b |
|----------------------------|-------------------------|
| 1. $\square \times 9 = 45$ | $\square \times 8 = 48$ |
| 2. $9 \times \square = 27$ | $15 \div 3 = \square$ |
| 3. $12 \div 6 = \square$ | $36 \div 9 = \square$ |

Multiplication-Division Table

\times / \div	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

$$20 \div 4 = \square$$

$$42 \div 6 = \square$$

The gray arrows above show how to solve $20 \div 4 = \square$ by using the multiplication table. Find 4 in the top row. Is 4 the dividend or the divisor in $20 \div 4 = \square$? *1 Follow down the 4-column until you reach 20. Is 20 the dividend or the divisor in $20 \div 4 = \square$? *2 Then follow this row to the left. The first numeral (on the left) in this row names the quotient. What is it? 5

Look at the blue arrows in the table. How can you use the table to solve $42 \div 6 = \square$? *3

Since division undoes multiplication, the multiplication table can be used as a division table.

Use the table above to solve the following open sentences.

$$9 \div 9 = \square \quad 8 \div 8 = \square \quad 7 \div 7 = \square \quad 6 \div 6 = \square \quad 5 \div 5 = \square$$

What is the quotient for each open sentence above? In $9 \div 9 = \square$, what is the dividend? What is the divisor? In $8 \div 8 = \square$, $7 \div 7 = \square$, $6 \div 6 = \square$, and $5 \div 5 = \square$ is the dividend the same as the divisor?

What is your conclusion about such quotients? 1; 9; 9; Yes; If the dividend and divisor are the same (but not 0) the quotient is 1.

If any whole number, except zero, is divided by itself, the quotient is one.

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*1 the divisor

*2 the dividend

*3 Follow down the 6-column until you reach 42. Follow this row to the left. The first numeral on the left in this row names the quotient.

Oral Answer the following questions.

1. Can the table at the top of page 72 be used to solve open multiplication sentences? **Yes**

2. How could you use the table to solve $4 \times 5 = \square$? **Find the entry in the 4-row and the 5-column.**

3. Why is the 0-column left out of the table? **Division by 0 is meaningless.**

Using the table, how would you find the simplest numeral for each product below? **Answers will be similar to those for 4a. and 4b. below.**

4. $4 \times 9 = \square$ $36 \div 9 = \square$

5. $6 \times 5 = \square$ $30 \div 5 = \square$

6. $7 \times 8 = \square$ $56 \div 8 = \square$

7. $6 \times 9 = \square$ $54 \div 9 = \square$

8. $8 \times 5 = \square$ $40 \div 5 = \square$

9. $4 \times 7 = \square$ $28 \div 7 = \square$

What numeral should replace the \square in each sentence below?

10. If $8 \times 9 = 72$, then $72 \div 9 = \square$

11. If $35 \div 5 = 7$, then $7 \times 5 = \square$

12. If $6 \times 7 = 42$, then $42 \div 7 = \square$

13. If $8 \times 4 = 32$, then $32 \div 4 = \square$

14. If $63 \div 7 = 9$, then $9 \times 7 = \square$

15. If $9 \times 9 = 81$, then $81 \div 9 = \square$

Written Copy. Replace each \square by the simplest numeral. Check your answer by using the table.

<i>a</i>	<i>b</i>
1. $4 \times 8 = \square$	$32 \div 8 = \square$
2. $3 \times 7 = \square$	$21 \div 7 = \square$
3. $56 \div 7 = \square$	$8 \times 7 = \square$
4. $63 \div 9 = \square$	$7 \times 9 = \square$
5. $64 \div 8 = \square$	$8 \times 8 = \square$
6. $4 \times 6 = \square$	$24 \div 6 = \square$
7. $2 \times 8 = \square$	$16 \div 8 = \square$
8. $9 \times 8 = \square$	$72 \div 8 = \square$

Write each multiplication sentence as a division sentence. Then find each quotient.

<i>a</i>	<i>b</i>
9. $\square \times 8 = 40$ $40 \div 8 = \square ; 5$	$\square \times 6 = 30$ $30 \div 6 = \square ; 5$
10. $\square \times 7 = 56$ $56 \div 7 = \square ; 8$	$\square \times 9 = 81$ $81 \div 9 = \square ; 9$
11. $\square \times 9 = 45$ $45 \div 9 = \square ; 5$	$\square \times 6 = 18$ $18 \div 6 = \square ; 3$
12. $\square \times 5 = 35$ $35 \div 5 = \square ; 7$	$\square \times 5 = 40$ $40 \div 5 = \square ; 8$
13. $\square \times 4 = 28$ $28 \div 4 = \square ; 7$	$\square \times 4 = 24$ $24 \div 4 = \square ; 6$

Can you do this? Compose a story problem for each of the following.

Answers will vary.

<i>a</i>	<i>b</i>
1. $6 \times 5 = \square$	$48 \div 8 = \square$
2. $\square \times 6 = 54$	$9 \times \square = 54$

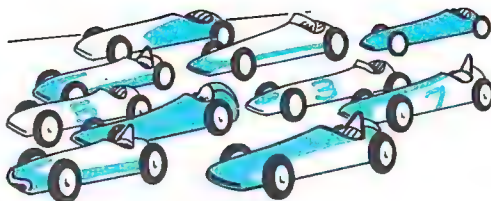
Oral 4a. The simplest numeral for 4×9 appears in the square that is the intersection of the 4-row and 9-column. **75**

4b. Follow down the 9-column until you reach 36. Then follow this row to the left. The first numeral on the left in this row names the quotient.

Finding an Unnamed Factor



Glenn wonders how many pairs of baseball cards he can make from his collection of cards shown on the table above.



Bob wants to separate his set of cars shown above into 5 equivalent subsets. How many cars will there be in each subset?

Glenn's problem can be expressed as follows.

$$\square \times 2 = 10$$

The unnamed factor can be found by division.

$$\text{If } \square \times 2 = 10, \text{ then } 10 \div 2 = \square.$$

What is the simplest numeral for $10 \div 2$? How many pairs can Glenn make from his collection? **5; 5**

Bob's problem can be expressed as follows.

$$5 \times \square = 10$$

You can change $5 \times \square = 10$ into a division sentence.

$$\text{If } 5 \times \square = 10, \text{ then } 10 \div \square = 5.$$

Why is $10 \div \square = 5$ not very helpful? ^{*1} Is $5 \times \square = \square \times 5$ a true sentence? Why? Can you change $5 \times \square = 10$ into $\square \times 5 = 10$? Why? **Yes; Multiplication is commutative; Yes; Commute the factors.**

Now you can change $\square \times 5 = 10$ into a division sentence.

$$\text{If } \square \times 5 = 10, \text{ then } 10 \div 5 = \square.$$

What is the simplest numeral for $10 \div 5$? How many cars will there be in each subset? **2; 2**

Because multiplication is commutative we can write two division sentences for each multiplication sentence and two multiplication sentences for each division sentence.

Oral Think of changing each sentence below into a division sentence and tell the simplest numeral for each \square .

- | a | b |
|------------------------------|---------------------------|
| 1. $\boxed{8} \times 9 = 72$ | $8 \times \boxed{9} = 72$ |
| 2. $6 \times \boxed{8} = 48$ | $\boxed{6} \times 8 = 48$ |
| 3. $7 \times \boxed{6} = 42$ | $\boxed{7} \times 6 = 42$ |
| 4. $\boxed{5} \times 8 = 40$ | $\boxed{7} \times 5 = 35$ |
| 5. $\boxed{5} \times 6 = 30$ | $\boxed{4} \times 7 = 28$ |
| 6. $\boxed{8} \times 7 = 56$ | $\boxed{7} \times 9 = 63$ |
| 7. $9 \times \boxed{7} = 63$ | $7 \times \boxed{5} = 35$ |
| 8. $9 \times \boxed{3} = 27$ | $8 \times \boxed{3} = 24$ |
| 9. $\boxed{8} \times 3 = 24$ | $\boxed{4} \times 9 = 36$ |

Think of changing each sentence below into a multiplication sentence and tell the simplest numeral for each \square .

- | a | b |
|-----------------------------|-------------------------|
| 10. $\boxed{20} \div 4 = 5$ | $\boxed{21} \div 3 = 7$ |
| 11. $\boxed{30} \div 6 = 5$ | $\boxed{49} \div 7 = 7$ |
| 12. $\boxed{32} \div 8 = 4$ | $\boxed{48} \div 6 = 8$ |
| 13. $\boxed{63} \div 9 = 7$ | $\boxed{18} \div 2 = 9$ |
| 14. $\boxed{27} \div 3 = 9$ | $\boxed{24} \div 6 = 4$ |
| 15. $\boxed{40} \div 5 = 8$ | $\boxed{81} \div 9 = 9$ |
| 16. $\boxed{42} \div 7 = 6$ | $\boxed{35} \div 5 = 7$ |
| 17. $\boxed{36} \div 4 = 9$ | $\boxed{28} \div 4 = 7$ |

Written Write an open multiplication sentence for each problem. Then change that sentence into a division sentence. Solve the open sentence. Answer the problem.

1. John has 45 cents. How many 5-cent candy bars can he buy?

$\square \times 5 = 45$; $45 \div 5 = \square$; 9 candy bars

2. Margie planted 4 seeds in each flower pot. She planted 20 seeds in all. How many flower pots did she use? $\square \times 4 = 20$; $20 \div 4 = \square$; 5 flower pots

3. George has 9 guinea pigs. He wants to put 3 guinea pigs in each pen. How many pens does he need for his guinea pigs? $\square \times 3 = 9$; $9 \div 3 = \square$; 3 pens

4. Sally bought 6 cans of soda pop. They cost her 48 cents. What is the cost of one can of soda pop?

$6 \times \square = 48$; $48 \div 6 = \square$; 8 cents

5. There are 35 pupils in a class. They are separated into committees of 7 pupils each. How many committees are formed? $\square \times 7 = 35$; $35 \div 7 = \square$; 5 committees

6. Six boys ate 18 cookies. Each boy ate the same number of cookies. How many cookies did each boy eat? $6 \times \square = 18$; $18 \div 6 = \square$; 3 cookies

7. Sally has 64 pieces of candy to share equally among 8 girls. How many pieces of candy should she give to each girl? $8 \times \square = 64$; $64 \div 8 = \square$; 8 pieces of candy

8. Miss Hoy asked 6 pupils to make 42 posters. If each pupil makes the same number of posters, how many will each pupil make?

$6 \times \square = 42$; $42 \div 6 = \square$; 7 posters

Associative Property of Multiplication

The examples below show two different ways of solving the open sentence $5 \times 3 \times 2 = \square$. To solve this open sentence we must find the product of three numbers. We know that multiplication is an operation on just two numbers. We use this idea and group factors to solve an open sentence containing three factors.

A

$$\begin{aligned} 5 \times 3 \times 2 &= (5 \times 3) \times 2 \\ &= 15 \times 2 \\ &= 30 \end{aligned}$$

B

$$\begin{aligned} 5 \times 3 \times 2 &= 5 \times (3 \times 2) \\ &= 5 \times 6 \\ &= 30 \end{aligned}$$

In **A**, the () indicate that we first consider the factors 5 and 3. What is the simplest numeral for 5×3 ? Do $(5 \times 3) \times 2$ and 15×2 both have the same number of factors? Do they both name the same number? Which number? **15; Yes; Yes; 30**

In **B**, what does it mean to place () around 3×2 ? What is the simplest numeral for 3×2 ? Do $5 \times (3 \times 2)$ and 5×6 both name the same number? Which number? **Perform 3 times 2 first; 6; Yes; 30**

Does it matter which way you associate or group the factors in finding the product of three numbers? Is $(5 \times 3) \times 2 = 5 \times (3 \times 2)$ a true sentence? **No; Yes**

Study how the factors are associated or grouped to find each product below. Then tell which method appears easier. Explain why.

C

$$\begin{aligned} 3 \times 4 \times 5 &= (3 \times 4) \times 5 \\ &= 12 \times 5 \\ &= 60 \end{aligned}$$

D

$$\begin{aligned} 3 \times 4 \times 5 &= 3 \times (4 \times 5) \\ &= 3 \times 20 \\ &= 60 \end{aligned}$$

When multiplying 3 numbers, it does not matter which way you associate or group the factors. We call this idea the **associative property of multiplication**. Or we say that **multiplication is associative**.

Oral What numeral should replace each below?

$$1. \quad 3 \times 2 \times 4 = (3 \times \underline{2}) \times 4 \\ = \underline{6} \times 4 \\ = \underline{24}$$

$$2. \quad 3 \times 2 \times 4 = (3 \times 2) \times \underline{4} \\ = 6 \times \underline{4} \\ = \underline{24}$$

$$3. \quad 4 \times 5 \times 1 = (\underline{4} \times 5) \times 1 \\ = \underline{20} \times 1 \\ = \underline{20}$$

$$4. \quad 4 \times 5 \times 1 = 4 \times (5 \times \underline{1}) \\ = 4 \times \underline{5} \\ = \underline{20}$$

5. Tell why $(2 \times 4) \times 5 = 2 \times (4 \times 5)$ is a true sentence. **Multiplication is associative.**

6. Tell why $3 \times (1 \times 5) = (3 \times 1) \times 5$ is a true sentence. **Multiplication is associative.**

7. Is $(1 \times 5) \times 6 = 1 \times (5 \times 6)$ a true sentence? Why? **Yes; Multiplication is associative.**

8. Is $2 \times (5 \times 6) = (2 \times 5) \times 6$ a true sentence? Why? **Yes; Multiplication is associative.**

9. Which of these would you prefer to solve: $(3 \times 8) \times 5$ or $3 \times (8 \times 5)$? **$3 \times (8 \times 5)$**

10. Which of these would you prefer to solve: $(4 \times 5) \times 3$ or $4 \times (5 \times 3)$? **$(4 \times 5) \times 3$**

Tell the steps taken in solving the multiplication below.

$$11. \quad 3 \times 6 \times 4 = (3 \times 6) \times 4 \quad \text{Assoc. Prop.} \\ = 18 \times 4 \quad 3 \times 6 = 18 \\ = 72 \quad 4 \times 18 = 72$$

Written Copy. Replace the \square by the simplest numeral that makes each sentence true.

1. Since $(4 \times 5) \times 6 = 120$,
then $4 \times (5 \times 6) = \square$. **120**

2. Since $(2 \times 8) \times 3 = 48$,
then $2 \times (8 \times 3) = \square$. **48**

3. Since $(7 \times 5) \times 2 = 70$,
then $7 \times (5 \times 2) = \square$. **70**

4. Since $(6 \times 4) \times 2 = 48$,
then $6 \times (4 \times 2) = \square$. **48**

5. Since $(8 \times 6) \times 5 = 240$,
then $8 \times (6 \times 5) = \square$. **240**

Copy. Find each product.

a	b
6. $(2 \times 4) \times 3 = \square$ 24	$2 \times (4 \times 3) = \square$ 24
7. $(4 \times 0) \times 3 = \square$ 0	$4 \times (0 \times 3) = \square$ 0
8. $(4 \times 1) \times 7 = \square$ 28	$4 \times (1 \times 7) = \square$ 28
9. $(8 \times 5) \times 4 = \square$ 160	$8 \times (5 \times 4) = \square$ 160
10. $(5 \times 6) \times 3 = \square$ 90	$5 \times (6 \times 3) = \square$ 90
11. $(7 \times 3) \times 4 = \square$ 84	$7 \times (3 \times 4) = \square$ 84

Can you do this? Show two different ways to group the factors in each of the following without changing the order of the factors.

See page T79.

a	b
1. $2 \times 3 \times 4 \times 5 = \square$	$3 \times 1 \times 4 \times 2 = \square$
2. $2 \times 5 \times 6 \times 3 = \square$	$6 \times 5 \times 4 \times 3 = \square$

Multiples of 10 and of 100

Multiples of 1	Multiples of 10	Multiples of 100
$2 \times 1 = 2$	$2 \times 10 = 20$	$2 \times 100 = 200$
$3 \times 1 = 3$	$3 \times 10 = 30$	$3 \times 100 = 300$
$4 \times 1 = 4$	$4 \times 10 = 40$	$4 \times 100 = 400$

As shown above, what number is named by 2×1 ?² By 2×10 ?²⁰ By 2×100 ?²⁰⁰ Compare the numerals 2, 20, and 200. How are they alike? How are they different? What pattern appears when we compare 2×1 and 2, 2×10 and 20, 2×100 and 200? Is this pattern shown in all the rows above? **Yes**

Knowing the multiples of 1, of 10, and of 100 helps you find products like those shown below.

$$\begin{aligned}
 3 \times 60 &= 3 \times (6 \times 10) \\
 &= (3 \times 6) \times 10 \\
 &= 18 \times 10 \\
 &= 180
 \end{aligned}$$

$$\begin{aligned}
 3 \times 600 &= 3 \times (6 \times 100) \\
 &= (3 \times 6) \times 100 \\
 &= 18 \times 100 \\
 &= 1800
 \end{aligned}$$

How was 60 renamed? What property of multiplication is used in changing $3 \times (6 \times 10)$ to $(3 \times 6) \times 10$? Do $(3 \times 6) \times 10$ and 18×10 name the same number? How does knowing $18 \times 1 = 18$ help you find $18 \times 10 = 180$? **as 6×10 ; associative; Yes; Since $18 \times 1 = 18$ then $18 \times 1T = 18T$.**

How was 600 renamed? What property of multiplication is used in changing $3 \times (6 \times 100)$ to $(3 \times 6) \times 100$? Do $(3 \times 6) \times 100$ and 18×100 name the same number? How does knowing $18 \times 1 = 18$ help you find $18 \times 100 = 1800$? **as 6×100 ; associative; Yes; Since $18 \times 1 = 18$ then $18 \times 1H = 18H$.**

What pattern appears when we compare 3×6 and 18, 3×60 and 180, and 3×600 and 1800? **See *3 below.**

80

*1 Each contains the digit 2.

*2 They contain different numbers of final 0's.

*3 The number of final 0's in the numerals for both factors is the same as the number of final 0's in the simplest numeral for the product.

Oral Replace each \square by the simplest numeral to make the sentence true.

1. If $6 \times 1 = 6$, then $6 \times 10 = \square$. 60
2. If $6 \times 1 = 6$, then $6 \times 100 = \square$. 600
3. If $7 \times 1 = 7$, then $7 \times 100 = \square$. 700
4. If $7 \times 1 = 7$, then $7 \times 10 = \square$. 70
5. If $2 \times 8 = 16$, then $2 \times 80 = \square$. 160
6. If $2 \times 8 = 16$, then $2 \times 800 = \square$. 1600
7. If $1 \times 5 = 5$, then $10 \times 5 = \square$. 50
8. If $1 \times 5 = 5$, then $100 \times 5 = \square$. 500

Tell the simplest numeral for each product below.

- | a | b |
|---|---|
| 9. $4 \times 10 = \square$ 40 | $\square = 100 \times 4$ 400 |
| 10. $2 \times 10 = \square$ 20 | $\square = 2 \times 100$ 200 |
| 11. $6 \times 20 = \square$ 120 | $\square = 200 \times 6$ 1200 |
| 12. $40 \times 5 = \square$ 200 | $\square = 5 \times 400$ 2000 |
| 13. $9 \times 20 = \square$ 180 | $\square = 200 \times 9$ 1800 |
| 14. $7 \times 80 = \square$ 560 | $\square = 7 \times 800$ 5600 |
| 15. $60 \times 5 = \square$ 300 | $\square = 600 \times 5$ 3000 |
| 16. $9 \times 30 = \square$ 270 | $\square = 9 \times 300$ 2700 |
| 17. $9 \times 40 = \square$ 360 | $\square = 400 \times 9$ 3600 |
| 18. $6 \times 60 = \square$ 360 | $\square = 600 \times 6$ 3600 |
| 19. $7 \times 70 = \square$ 490 | $\square = 7 \times 700$ 4900 |

Written Copy. Find each product.

- | a | b |
|---|---|
| 1. $10 \times 8 = \square$ 80 | $\square = 100 \times 8$ 800 |
| 2. $10 \times 3 = \square$ 30 | $\square = 3 \times 100$ 300 |
| 3. $9 \times 50 = \square$ 450 | $\square = 500 \times 9$ 4500 |
| 4. $60 \times 8 = \square$ 480 | $\square = 8 \times 600$ 4800 |
| 5. $7 \times 40 = \square$ 280 | $\square = 400 \times 7$ 2800 |
| 6. $30 \times 7 = \square$ 210 | $\square = 7 \times 300$ 2100 |
| 7. $5 \times 40 = \square$ 200 | $\square = 400 \times 5$ 2000 |
| 8. $4 \times 90 = \square$ 360 | $\square = 900 \times 4$ 3600 |
| 9. $3 \times 50 = \square$ 150 | $\square = 3 \times 500$ 1500 |
| 10. $6 \times 70 = \square$ 420 | $\square = 700 \times 6$ 4200 |

Write an open sentence for each problem. Solve the open sentence. Answer the problem.

11. There are 30 pupils in the fourth grade. Each pupil made 4 drawings. How many drawings did the pupils make altogether? $30 \times 4 = \square$; 120 drawings

12. Margie has 10 planters each containing 8 plants. How many plants are there in all? $10 \times 8 = \square$; 80 plants

13. Thirty pupils were given 8 sheets of paper each. How many sheets of paper were passed out? $30 \times 8 = \square$; 240 sheets

Tell why If you know $5 \times (3 \times 2) = 30$, then you also know $5 \times (30 \times 2) = 300$. Why is this true? Since 30 is 10 times greater than 3, $5 \times (30 \times 2)$ is 10 times greater than $5 \times (3 \times 2)$. **81**

Dividing Tens and Hundreds

A	B	C
$2 \div 2 = 1$	$20 \div 2 = 10$	$200 \div 2 = 100$
$4 \div 2 = 2$	$40 \div 2 = 20$	$400 \div 2 = 200$
$6 \div 2 = 3$	$60 \div 2 = 30$	$600 \div 2 = 300$
$8 \div 2 = 4$	$80 \div 2 = 40$	$800 \div 2 = 400$

Study the closed sentences above.

Compare the simplest numeral for $20 \div 2$ in **B** to that of $2 \div 2$ in **A**. How many times greater is 10 than 1? What number is being divided in $20 \div 2$? What number is being divided in $2 \div 2$? The number being divided in $20 \div 2$ is how many times greater than the number being divided in $2 \div 2$? What pattern do you discover? 10 times; 20; 2; 10 times; If $a \div b = c$, then $10a \div b = 10c$.

Yes

Can you find a pattern for $200 \div 2$ and $2 \div 2$? We can use these patterns to make dividing tens and hundreds by ones easy. Study these sentences below.

If $2 \div 2 = 1$, then $20 \div 2 = 10$ and $200 \div 2 = 100$.

If $3 \div 3 = 1$, then $30 \div 3 = 10$ and $300 \div 3 = 100$.

If $8 \div 4 = 2$, then $80 \div 4 = 20$ and $800 \div 4 = 200$.

Now use the pattern just discovered to tell what simplest numeral should replace each below so that the sentence becomes true.

If $12 \div 4 = 3$, then $120 \div 4 = \underline{30}$ and $1200 \div 4 = \underline{300}$.

Oral Answer each of the following questions.

1. Tell how knowing $15 \div 3 = 5$ helps you find the quotient named by $150 \div 3$. If $a \div b = c$, then $10a \div b = 10c$.

2. Explain why $280 \div 7 = 40$ since $28 \div 7 = 4$. See Oral 1.

3. Tell how knowing $21 \div 3 = 7$ helps you find the quotient named by $2100 \div 3$. If $a \div b = c$, then $100a \div b = 100c$.

4. Explain why $320 \div 8 = 40$ since $32 \div 8 = 4$. See Oral 1.

5. Explain why $450 \div 9 = 50$ since $45 \div 9 = 5$. See Oral 1.

Replace each \square with the simplest numeral to make the sentence true.

6. If $6 \div 2 = 3$, then $60 \div 2 = \square$. ³⁰
7. If $6 \div 2 = 3$, then $600 \div 2 = \square$. ³⁰⁰
8. If $8 \div 2 = 4$, then $800 \div 2 = \square$. ⁴⁰⁰
9. If $8 \div 2 = 4$, then $80 \div 2 = \square$. ⁴⁰
10. If $24 \div 6 = 4$, then $240 \div 6 = \square$. ⁴⁰⁰
11. If $24 \div 6 = 4$, then $2400 \div 6 = \square$. ⁴⁰⁰
12. If $15 \div 5 = 3$, then $150 \div 5 = \square$. ³⁰
13. If $15 \div 5 = 3$, then $1500 \div 5 = \square$. ³⁰⁰
14. If $30 \div 5 = 6$, then $300 \div 5 = \square$. ⁶⁰

Tell each quotient below.

- | a | b |
|---|---------------------------------------|
| 15. $5 \div 5 = \square$ ¹ | $50 \div 5 = \square$ ¹⁰ |
| 16. $60 \div 3 = \square$ ²⁰ | $600 \div 3 = \square$ ²⁰⁰ |
| 17. $20 \div 2 = \square$ ¹⁰ | $200 \div 2 = \square$ ¹⁰⁰ |
| 18. $40 \div 8 = \square$ ⁵ | $400 \div 8 = \square$ ⁵⁰ |
| 19. $8 \div 8 = \square$ ¹ | $80 \div 8 = \square$ ¹⁰ |
| 20. $40 \div 4 = \square$ ¹⁰ | $400 \div 4 = \square$ ¹⁰⁰ |

Written Copy. Find each quotient.

- | a | b |
|---|--|
| 1. $120 \div 3 = \square$ ⁴⁰ | $1200 \div 3 = \square$ ⁴⁰⁰ |
| 2. $420 \div 6 = \square$ ⁷⁰ | $4200 \div 6 = \square$ ⁷⁰⁰ |
| 3. $240 \div 8 = \square$ ³⁰ | $2400 \div 8 = \square$ ³⁰⁰ |

- | | |
|--|---|
| 4. $700 \div 7 = \square$ ¹⁰⁰ | $7000 \div 7 = \square$ ¹⁰⁰⁰ |
| 5. $300 \div 3 = \square$ ¹⁰⁰ | $3000 \div 3 = \square$ ¹⁰⁰⁰ |
| 6. $160 \div 4 = \square$ ⁴⁰ | $1600 \div 4 = \square$ ⁴⁰⁰ |
| 7. $270 \div 3 = \square$ ⁹⁰ | $2700 \div 3 = \square$ ⁹⁰⁰ |
| 8. $360 \div 4 = \square$ ⁹⁰ | $3600 \div 4 = \square$ ⁹⁰⁰ |
| 9. $36 \div 6 = \square$ ⁶ | $360 \div 6 = \square$ ⁶⁰ |
| 10. $200 \div 4 = \square$ ⁵⁰ | $2000 \div 4 = \square$ ⁵⁰⁰ |

Write an open division sentence for each of the following problems. Solve the open sentence. Answer the problem.

11. Don made a 450-mile automobile trip in 9 hours. He drove the same number of miles each hour. How many miles was that? $450 \div 9 = \square$; ^{50 miles per hour}

12. Beth has 160 pictures. She pasted 8 pictures on each page of her album. How many pages does she have filled in her album? $160 \div 8 = \square$; ^{20 pages}

13. Tom stacked 240 books into stacks. If each stack contained 8 books, how many stacks of books were there altogether? $240 \div 8 = \square$; ^{30 books}

Can you do this? Compose a story problem for each open sentence given below. ^{Answers will vary.}

- | a | b |
|---------------------------|------------------------|
| 1. $320 \div 8 = \square$ | $210 \div 7 = \square$ |
| 2. $420 \div 6 = \square$ | $270 \div 9 = \square$ |

Solving Problems

Write an open multiplication sentence for each problem below. Then change it into an open division sentence. Then solve each open division sentence. Answer the problem.

1. Sue has 6 outfits for each of her dolls. If she has 24 outfits, how many dolls does she have? $\square \times 6 = 24$; $24 \div 6 = \square$; 4 dolls

2. Tim feeds his birds 2 ounces of bird seed each day. How many days will it take to use a 16 ounce box of bird seed? $\square \times 2 = 16$; $16 \div 2 = \square$; 8 days

3. Helen has 64 cents. How many 8-cent cans of soda can she buy?

$$\square \times 8 = 64; 64 \div 8 = \square; 8 \text{ cans}$$

4. A classroom has 35 desks arranged in 5 equal rows. How many desks are in each row? $5 \times \square = 35$; $35 \div 5 = \square$; 7 desks

5. Don and his four friends earned 50 cents carrying leaves to the back yard. If they share the 50 cents equally, how much will each boy get? $5 \times \square = 50$; $50 \div 5 = \square$; 10 cents

6. A rope is 72 feet long. It was cut into pieces each 9 feet long. There are how many pieces of rope in all? $\square \times 9 = 72$; $72 \div 9 = \square$; 8 pieces

7. Fran has 40 records. She stores them in albums. Each album holds 8 records. How many record albums does she need? $\square \times 8 = 40$; $40 \div 8 = \square$; 5 albums

8. A truck hauled 63 tons of top soil. The truck hauled 7 tons on each trip. How many trips were needed to haul the entire 63 tons? $\square \times 7 = 63$; $63 \div 7 = \square$; 9 trips

9. The 32 pupils in a class were separated into 8 committees. Each committee has the same number of pupils. How many pupils were on each committee? $8 \times \square = 32$; $32 \div 8 = \square$; 4 pupils

10. Don raises rabbits. He has 24 of them. He wants to build pens which will hold 4 rabbits each. How many pens must he build? $\square \times 4 = 24$; $24 \div 4 = \square$; 6 pens

11. Cindy bought one-half dozen oranges. She paid 48 cents for them. How much did just one orange cost her? $6 \times \square = 48$; $48 \div 6 = \square$; 8 cents

12. Leon earned 64 dollars in 8 days by working in a grocery store. If he earned the same amount on each of the 8 days, how much did he earn in one day? $8 \times \square = 64$; $64 \div 8 = \square$; 8 dollars

13. Joan picked 36 flowers. She put 9 flowers in each vase. How many vases did she need? $\square \times 9 = 36$; $36 \div 9 = \square$; 4 vases

Can you do this? Compose a story problem for each open sentence below. **Answers will vary.**

a

$$\square \times 9 = 90$$

b

$$6 \times \square = 36$$

Checkup Time

The numerals in () tell the pages where you can turn for help.

Important Ideas

1. Multiplication is a short way of finding a sum when all addends are the same. (60)

2. Division can be thought of as repeated subtraction. (65)

3. Division by zero is meaningless. (65)

4. Multiplication and division are inverse operations. (66)

5. Multiplication is commutative. (68)

6. The number one is the identity number of multiplication. (69)

7. Multiplication is associative. (78)

Words to Know

1. Multiplication (59)

2. Multiplication numeral, factor, product (62)

3. Division, division numeral, quotient (64)

4. Commutative property of multiplication (68)

5. Associative property of multiplication (78)

Questions to Discuss

See page T85.

1. How do you know that $4 \times 8 = 8 + 8 + 8 + 8$? That $5 \times 9 = 9 + 9 + 9 + 9 + 9$? That $3 \times 5 = 5 + 5 + 5$? (60)

2. How do you know that if $6 \times 4 = 24$, then $24 \div 4 = 6$? (66)

3. How do you know that $3 \times 4 = 4 \times 3$? That $4 \times 5 = 5 \times 4$? (68)

4. How do you know that $(2 \times 3) \times 4 = 2 \times (3 \times 4)$? (78)

5. How do you know that if $3 \times 10 = 30$, you also know $3 \times 100 = 300$? (80)

Written Practice

Find each quotient or product.

1. If $7 + 7 + 7 = 21$, then $3 \times 7 = \square$. (61)

2. If $7 \times 6 = 42$, then $42 \div 6 = \square$. (66)

3. If $54 \div 6 = 9$, then $9 \times 6 = \square$. (66)

4. If $28 \div 4 = 7$, then $7 \times 4 = \square$. (66)

5. If $7 \times 8 = 56$, then $8 \times 7 = \square$. (68)

6. If $4 \times 5 = 20$, then $4 \times 50 = \square$. (80)

7. If $5 \times 13 = 65$, then $5 \times 130 = \square$. (80)

8. If $54 \div 6 = 9$, then $540 \div 6 = \square$. (82)

9. If $42 \div 6 = 7$, then $420 \div 6 = \square$. (82)


Self-Evaluation

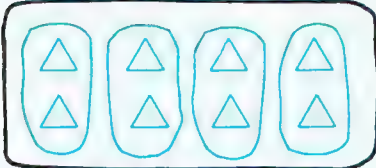
Part 1 Write two addition and two multiplication sentences for each of the following.

1.  $4+4+4=12$;
 $3+3+3+3=12$;
 $3 \times 4=12$;
 $4 \times 3=12$

2.  $2+2+2=6$;
 $3+3=6$;
 $2 \times 3=6$;
 $3 \times 2=6$

Part 2 Write a multiplication and a division sentence for each of the following.

1.  $2 \times 3=6$; $6 \div 3=2$ or $6 \div 2=3$

2.  $4 \times 2=8$; $8 \div 2=4$ or $8 \div 4=2$

Part 3 Write a multiplication sentence for each of the following. Find each product.

- | <i>a</i> | <i>b</i> |
|---|--|
| 1. $2+2+2=\square$
$3 \times 2=\square$; 6 | $3+3+3+3=\square$
$4 \times 3=\square$; 12 |
| 2. $6+6=\square$
$2 \times 6=\square$; 12 | $7+7+7=\square$
$3 \times 7=\square$; 21 |
| 3. $9+9+9=\square$
$3 \times 9=\square$; 27 | $8+8+8+8=\square$
$4 \times 8=\square$; 32 |

Part 4 Write a division sentence for each sentence below. Find each quotient.

- | <i>a</i> | <i>b</i> |
|---|--|
| 1. $\square \times 9=36$
$36 \div 9=\square$; 4 | $\square \times 8=56$
$56 \div 8=\square$; 7 |
| 2. $\square \times 8=64$
$64 \div 8=\square$; 8 | $\square \times 9=72$
$72 \div 9=\square$; 8 |
| 3. $\square \times 8=48$
$48 \div 8=\square$; 6 | $\square \times 7=42$
$42 \div 7=\square$; 6 |
| 4. $\square \times 7=35$
$35 \div 7=\square$; 5 | $\square \times 6=54$
$54 \div 6=\square$; 9 |
| 5. $\square \times 9=54$
$54 \div 9=\square$; 6 | $\square \times 8=40$
$40 \div 8=\square$; 5 |

Part 5 Copy. Solve each open sentence.

- | <i>a</i> | <i>b</i> |
|--|---|
| 1. $3 \times 7=\square$ 21 | $4 \times 8=\square$ 32 |
| 2. $8 \times 7=\square$ 56 | $9 \times 5=\square$ 45 |
| 3. $6 \times 9=\square$ 54 | $1 \times 7=\square$ 7 |
| 4. $0 \times 7=\square$ 0 | $4 \times 9=\square$ 36 |
| 5. $8 \times 6=\square$ 48 | $6 \times 5=\square$ 30 |

Part 6 Copy. Solve each open sentence.

- | <i>a</i> | <i>b</i> |
|--|---|
| 1. $36 \div 9=\square$ 4 | $45 \div 5=\square$ 9 |
| 2. $24 \div 6=\square$ 4 | $0 \div 4=\square$ 0 |
| 3. $56 \div 8=\square$ 7 | $63 \div 9=\square$ 7 |
| 4. $72 \div 8=\square$ 9 | $42 \div 7=\square$ 6 |
| 5. $35 \div 5=\square$ 7 | $40 \div 8=\square$ 5 |

Chapter 4

PATTERNS OF ADDITION AND SUBTRACTION

Place-Value

HTh	TTh	Th	H	T	
Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
100,000	10,000	1,000	100	10	1
2	3	0,	4	5	6

We can name any whole number by its simplest numeral. In forming simplest numerals we use ten symbols and the idea of **place value**. The ten symbols are 0,1,2,3,4,5,6,7,8, and 9. We call these symbols **digits**.

In the numeral 230,456 the 6 names the number of ones. It is called the **ones digit**. Why? ^{*1} The 5 names the number of tens. It is called the **tens digit**. Why? ^{*2} What does the 4 name? The 0? The 3? The 2? *the number of 100's; the number of 1000's; the number of 10000's; the number of 100000's*

In the numeral 230,456 the digit 6 in the ones place has a value of 6×1 or 6. The 5 in the tens place has a value of 5×10 or 50. In the numeral 230,456 what is the value of 4? Of 0? Of 3? Of 2? *4×100 or 400; 0×1000 or 0; 3×10000 or 30000; 2×100000 or 200000*

Oral Tell the value of each digit in each numeral below. *answers for part a only*

- | <i>a</i> | <i>b</i> |
|----------------------------|----------|
| 1. 438 | 384 |
| <i>4H, 3T, 8 ones</i> | |
| 2. 1542 | 5124 |
| <i>1Th, 5H, 4T, 2 ones</i> | |
| 3. 3648 | 4208 |
| <i>3Th, 6H, 4T, 8 ones</i> | |

- | | |
|--|---------|
| 4. 82,431 | 28,346 |
| <i>8TTh, 2Th, 4H, 3T, 1 one</i> | |
| 5. 64,135 | 46,153 |
| <i>6TTh, 4Th, 1H, 3T, 5 ones</i> | |
| 6. 936,450 | 396,540 |
| <i>9HTh, 3TTh, 6Th, 4H, 5T, 0 ones</i> | |
| 7. 384,251 | 834,512 |
| <i>3HTh, 8TTh, 4Th, 2H, 5T, 1 one</i> | |
| 8. 278,593 | 593,278 |
| <i>2HTh, 7TTh, 8Th, 5H, 9T, 3 ones</i> | |

*1 It is in ones position.

*2 It is in tens position.

Four-, Five-, and Six-Digit Numerals

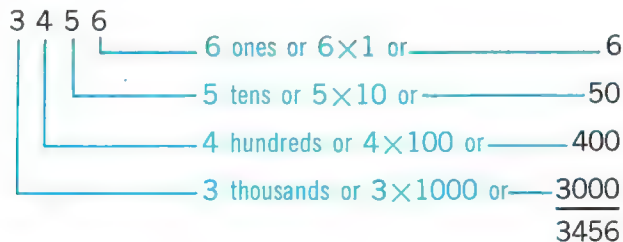
A
3,456

B
34,567

C
345,678

The numeral in A is called a four-digit numeral. Why? What would you call the numeral in B? Why? In C? Why? ^{*1} ^{*2} ^{*3} a six-digit numeral; Six place-value positions are used.

Each digit in each numeral has a value depending on the place it occupies in the numeral.



We can write 3456 in expanded notation as shown below.

$$3456 = 3000 + 400 + 50 + 6$$

$$= (3 \times 1000) + (4 \times 100) + (5 \times 10) + (6 \times 1)$$

How would you write 34,567 in expanded notation? How would you write 345,678 in expanded notation? $30000 + 4000 + 500 + 60 + 7$; $300000 + 40000 + 5000 + 600 + 70 + 8$

We need only ten digits and the idea of place value to name whole numbers.

Oral Answer the following.

1. What is the least number you can name by a 4-digit numeral? A 5-digit numeral? A 6-digit numeral? 1000 ; 10000 ; 100000

2. What is the greatest number you can name by a 4-digit numeral? A 5-digit numeral? A 6-digit numeral? 9999 ; 99999 ; 999999

3. Without repeating a digit, what is the least number you can name by a simplest numeral of 4 digits? Of 5 digits? Of 6 digits? (The numeral cannot begin with 0.) 1023 ; 10234 ; 102345

4. Without repeating a digit, what is the greatest number you can name by a simplest numeral of 4 digits? Of 5 digits? Of 6 digits? 9876 ; 98765 ; 987654

88

*1 Four place-value positions are used.

*2 a five-digit numeral

*3 Five place-value positions are used.

One of the digits in each numeral below is in color. Tell what place-value position that digit occupies. Then tell the value of that digit by a multiplication numeral.

- | <i>a</i> | <i>b</i> |
|------------------------------------|-----------------------------------|
| 5. 3621
T; 2×10 | 36,217
H; 2×100 |
| 6. 362,150
Th; 2×1000 | 621,705
TTh; 2×10000 |
| 7. 432
ones; 2×1 | 248,307
HTh; 2×100000 |
| 8. 426,581
Th; 6×1000 | 3418
Th; 3×1000 |
| 9. 248,346
Th; 8×1000 | 137,952
HTh; 1×100000 |
| 10. 432,109
T; 0×10 | 456,789
TTh; 5×10000 |
| 11. 987,654
ones; 4×1 | 876,354
HTh; 8×100000 |
| 12. 231,046
H; 0×100 | 931,576
T; 7×10 |
| 13. 139,874
Th; 9×1000 | 235,476
H; 4×100 |

Written Do the following.

1–9. Write each numeral in *Oral* 5–13 in expanded notation. See page T89.

Copy. Write the simplest numeral for each of the following.

10. $40000 + 7000 + 300 + 20 + 1$
47,321
11. $60000 + 4000 + 200 + 10 + 3$
64,213
12. $7000 + 300 + 10 + 4$
7314
13. $100000 + 90000 + 9000 + 90 + 9$
199,099
14. $600000 + 80000 + 8000 + 80 + 7$
688,087

15. $400000 + 5000 + 800 + 4$
405,804

16. $30000 + 900 + 60 + 5$
30,965

17. $500000 + 90000 + 50 + 3$
590,053

18. $200000 + 10000 + 300 + 9$
210,309

Copy. Complete each row by naming the number that is 10 more, 100 less, and 1000 more than the number named in the first column.

- | | 10
<i>more</i> | 100
<i>less</i> | 1000
<i>more</i> |
|------------|-------------------|--------------------|---------------------|
| 19. 2640 | 2650 | 2540 | 3640 |
| 20. 22,531 | 22,541 | 22,431 | 23,531 |
| 21. 48,621 | 48,631 | 48,521 | 49,621 |
| 22. 52,738 | 52,748 | 52,638 | 53,738 |
| 23. 48,765 | 48,775 | 48,665 | 49,765 |
| 24. 32,641 | 32,651 | 32,541 | 33,641 |
| 25. 23,461 | 23,471 | 23,361 | 24,461 |

Can you do this? Use each digit in each set below only once. Write the simplest numeral for the least number possible. Then write the simplest numeral for the greatest number possible. See below.

- | <i>a</i> | <i>b</i> |
|---------------------|---------------|
| 1. {0,1,3,5,7} | {0,1,3,5,7,9} |
| 2. {0,2,4,6,8} | {0,2,4,6,8,5} |
| 1a. 10,357; 75,310 | |
| b. 103,579; 975,310 | |
| 2a. 20,468; 86,420 | |
| b. 204,568; 865,420 | |

Number Words

12th	11th	10th	9th	8th	7th	6th	5th	4th	3rd	2nd	1st	ORDER OF DIGITS
		1,	2	3	4,	5	6	7,	8	9	0	DIGITS
Billions			Millions			Thousands			Ones			PERIODS

To read numerals consisting of many digits you may think of groups of three digits at one time. Commas may be used to make the reading easy. How many commas are used in the numeral above? **3** In placing commas we start from the right in the numeral. The first comma is between the 3rd and 4th digits. Between what two digits is the second comma? The third comma? The fourth comma, if needed? **6th and 7th; 9th and 10th; 12th and 13th**

The commas separate the numeral into groups of digits called **periods**. Each period contains at most how many digits? **3** When will a *period* not contain three digits? ***1**

Each of these periods has a name. Tell the name of each period shown above.

The numeral above is read as follows.

1 billion, 234 million, 567 thousand, 890

Oral Answer the following questions.

1. In reading numerals, do you start from the left or from the right? **the left**

2. In placing commas in a numeral, do you start from the left or from the right? **the right**

3. What number is named by the numeral 3427? What number is named by the numeral 3,427? Does the comma change the meaning of a numeral? **3 thousand, 427; 3 thousand, 427; No**

4. Why do we use commas in writing numerals? **to simplify the reading of the numeral**

5. How many commas should you place in a 11-digit numeral? **3**

90

*1 only when it is the first period on the left and the number of digits in the entire numeral is not a multiple of three

Tell the name of the period in color in each numeral below. Then read each numeral. See example on page 90 for reading the numerals.

- | | |
|-------------------------|----------------------------|
| <i>a</i> | <i>b</i> |
| 6. 324,280
thousands | 1,462,028
millions |
| 7. 14,382
ones | 1,982,463,240
billions |
| 8. 23,148
thousands | 924,368,217
millions |
| 9. 408,019
ones | 1,875,233,910
thousands |
| 10. 34,008
thousands | 6,069,051,032
billions |
| 11. 249,021
ones | 9,008,002,001
millions |

Tell where you would place commas in each numeral below. Then read each numeral. See example on page 90 for reading the numerals.

- | | |
|----------------|------------|
| <i>a</i> | <i>b</i> |
| 12. 1468327 | 19763240 |
| 13. 12345678 | 7654321 |
| 14. 4317624 | 63108 |
| 15. 6019004 | 109008017 |
| 16. 1012006148 | 1463245678 |

Written Copy. Use commas to separate each numeral into periods. Then replace each _ with the correct name of a period.

- 1987324
1 __, 987 __, 324
billion thousand

- 86432148
86 __, 432 __, 148
million thousand
- 9318642
9 __, 318 __, 642
million thousand
- 9321498244
9 __, 321 __, 498 __, 244
billion million thousand
- 24683218761
24 __, 683 __, 218 __, 761
billion million thousand
- 13546281980
13 __, 546 __, 281 __, 980
billion million thousand
- 204601009001
204 __, 601 __, 9 __, 1
billion million thousand

Write the simplest numeral for each of the following.

- One million, four hundred twenty thousand, three hundred
1,420,300
- Ten billion, four hundred million, eighty thousand, nine
10,400,080,009
- One hundred billion, thirty-two million, eight thousand, ninety
100,032,008,090
- $(3 \times 10,000) + (0 \times 1000) + (8 \times 100) + (0 \times 10) + (5 \times 1)$
30,805

Tell how How can you tell, without writing such a numeral, the number of periods a 15-digit numeral will have? How many commas it will have? Since $15 \div 3 = 5$, the numeral will contain 5 periods; 4 commas

Rounding Off



Often you cannot or need not be too exact about numbers referring to great distances. Instead of reading or stating the numeral as it is written, you can give an approximate number. The process involved is called **rounding off**.

From the picture above, how far is it from the center of the earth to the center of the moon? The table below shows how to round off 238,450. **238,450 miles**

<i>Precision Stated</i>	<i>Distance</i>
A. To the nearest ten thousand	240,000
B. To the nearest thousand	238,000
C. To the nearest hundred	238,400

To round off 238,450 to the nearest ten thousand, you must decide whether 238,450 is nearer to 230,000 or to 240,000. Why? ^{*1} Is 238,450 nearer to 230,000 or 240,000? ^{*2} Then to the nearest ten thousand 238,450 is what number? **240,000**

To round off 238,450 to the nearest thousand, you must decide whether 238,450 is nearer to 239,000 or to 238,000. Why? ^{*3} Is 238,450 nearer to 238,000 or to 239,000? ^{*4} To the nearest thousand 238,450 is 238,000.

*1 238,450 is between 230,000 and 240,000.

*2 240,000

*3 238,450 is between 238,000 and 239,000.

*4 238,000

To round off 238,450 to the nearest hundred, you must decide whether 238,450 is nearer to 238,400 or to 238,500. Why?^{*1} Is 238,450 nearer to 238,400 or to 238,500? Since 238,450 is as near to 238,400 as to 238,500, you round off "to make even." This means that you make the digit you are concerned with name an even number. In 238,450, the 4 is already even, so 238,450 is rounded off to 238,400.

To round off 476,350 to the nearest hundred, the 3 is odd so you use the next greater even number. What is the next even number?⁴ Then 476,350 is rounded off to 476,400.

Oral Answer each of the following questions.

1. Give an example in which you need not be too exact when referring to large numbers.

Typical answer: number of people in U.S.

2. In rounding off 486 to the nearest hundred, is 486 nearer to 400 or 500? How is 486 named when rounded off to the nearest hundred?**500; 500**

3. In rounding off 4432 to the nearest thousand, is 4432 nearer to 4000 or to 5000? How is 4432 named when rounded off to the nearest thousand? **4000; 4000**

4. In rounding off 4365 to the nearest ten, is 4365 nearer to 4360 or to 4370? Is 6 an even or odd number? How is 4365 named when rounded off to the nearest ten?**as near to one as to the other; even; 4360**

5. In rounding off 3750 to the nearest hundred, is 3750 nearer to 3700 or to 3800? Is 7 an even or odd

number? How is 3750 named when rounded off to the nearest hundred?**as near one as the other; odd; 3800**

Written Copy. Round off each of the following: first to the nearest thousand, then to the nearest hundred, and finally to the nearest ten. See page T93.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	14,874	12,463	184,782
2.	22,750	68,490	293,541
3.	93,360	21,760	148,250
4.	38,450	76,549	384,571
5.	63,554	38,575	649,535
6.	81,635	92,554	735,652

Can you do this? If 60,000 represents a number which has been rounded off to the nearest thousand, what is the least possible number that it could have been? **59,500**

^{*1} 238,450 is between 238,400 and 238,500.

^{*2} as near to one as to the other

Estimating Sums and Differences

A			B		
	Nearest hundred	Nearest ten		Nearest hundred	Nearest ten
$\begin{array}{r} 388 \\ +213 \\ \hline \end{array}$	$\begin{array}{r} 400 \\ +200 \\ \hline 600 \end{array}$	$\begin{array}{r} 390 \\ +210 \\ \hline 600 \end{array}$	$\begin{array}{r} 792 \\ -196 \\ \hline \end{array}$	$\begin{array}{r} 800 \\ -200 \\ \hline 600 \end{array}$	$\begin{array}{r} 790 \\ -200 \\ \hline 590 \end{array}$

In many problems we are able to round off the numbers and give an answer that is near the exact answer. This is called **estimating** an answer. You *estimate* an answer by rounding off to the nearest 10, 100, 1000, or whatever may be useful. The answer you obtain by estimating is called an **approximate** answer.

In A, 388 was rounded off first to 400, then to 390. What was 213 rounded off to? Which addition is easier to solve, $400+200$ or $390+210$? Compute the exact answer. Compare the estimated answers with the exact answer.

In B, 792 was rounded off to 800, then to 790. What was 196 rounded off to? Which subtraction is easier to solve, $800-200$ or $790-200$? Compute the exact answer. Compare the estimated answers with the exact answer.

In some estimating, rounding off to the nearest ten gives an answer that is nearer to the exact answer. But, it usually makes the estimating more difficult. In A, which is more difficult, estimating $388+213$ by rounding off to the nearest hundred or to the nearest ten? Why? In B, which is more difficult, estimating $792-196$ by rounding off to the nearest hundred or to the nearest ten? Why? rounding to nearest ten; $390+210$ requires use of more addition facts than $400+200$; rounding to the nearest ten; $790-200$ requires use of more subtraction facts than $800-200$.

94

- *1 first to 200 and then to 210
- *2 $400+200$
- *3 601

- *4 to 200 both times
- *5 probably $800-200$
- *6 596

Oral Tell how you would round off the numbers to estimate each of the following. Then tell the estimated sum or difference in each case.

- | <i>a</i> | <i>b</i> |
|-----------------------------|-------------------------|
| 1. $97+48$
$100+50=150$ | $97-48$
$100-50=50$ |
| 2. $101+51$
$100+50=150$ | $101-51$
$100-50=50$ |
| 3. $69+42$
$70+40=110$ | $69-42$
$70-40=30$ |
| 4. $82+71$
$80+70=150$ | $82-71$
$80-70=10$ |
| 5. $48+38$
$50+40=90$ | $48-38$
$50-40=10$ |
| 6. $63+42$
$60+40=100$ | $63-42$
$60-40=20$ |
| 7. $49+31$
$50+30=80$ | $49-31$
$50-30=20$ |
| 8. $92+63$
$90+60=150$ | $92-63$
$90-60=30$ |
| 9. $89+38$
$90+40=130$ | $89-38$
$90-40=50$ |

For each of the following questions choose the answer from those that are given.

10. Which sum will give the best estimate for $88+99$?

- a. $80+90$ b. $80+100$ c. $90+100$

11. Which difference will give the best estimate for $88-69$?

- a. $90-60$ b. $90-70$ c. $80-70$

12. Which sum will give the best estimate for $79+48$?

- a. $80+50$ b. $70+40$ c. $80+40$

13. Which difference will give the best estimate for $98-39$?

- a. $100-40$ b. $90-40$ c. $100-30$

14. Which sum will give the best estimate for $894+397$?

- a. $890+390$ b. $900+400$ c. $890+400$

Written Copy. Estimate each sum or difference. Record your estimate. Then find the exact sum or difference. Compare your estimate with the exact answer. **only the exact answers shown**

- | | <i>a</i> | <i>b</i> | <i>c</i> |
|----|--------------------------------|--------------------------------|---|
| 1. | 489
$+308$
<hr/> 797 | 612
$+294$
<hr/> 906 | 709
$+291$
<hr/> 1000 |
| 2. | 489
-294
<hr/> 195 | 796
-188
<hr/> 608 | 924
-195
<hr/> 729 |
| 3. | 611
$+283$
<hr/> 894 | 492
-193
<hr/> 299 | 269
-192
<hr/> 77 |
| 4. | 821
$+178$
<hr/> 999 | 732
$+265$
<hr/> 997 | 441
$+169$
<hr/> 610 |
| 5. | 921
-619
<hr/> 302 | 814
-216
<hr/> 598 | 724
-328
<hr/> 396 |
| 6. | 431
$+278$
<hr/> 709 | 631
-234
<hr/> 397 | 361
$+242$
<hr/> 603 |
| 7. | 298
$+304$
<hr/> 602 | 289
-142
<hr/> 147 | 621
-198
<hr/> 423 |
| 8. | 928
-431
<hr/> 497 | 829
-104
<hr/> 725 | 612
$+198$
<hr/> 810
95 |

Addition (Four- and Five-Digit Numerals)

Think

Write

The diagram illustrates the addition of 3407 and 5382. On the left, the numbers are written in expanded form:

$$\begin{array}{r} 3407 \\ + 5382 \\ \hline \end{array}$$

Below this, the expanded form is shown:

$$\begin{array}{r} 3000 + 400 + 0 + 7 \\ + (5000 + 300 + 80 + 2) \\ \hline 8000 + 700 + 80 + 9 \end{array}$$

On the right, a place value chart is used to show the addition:

Th	H	T	O
3	4	0	7
5	3	8	2
8	7	8	9

Arrows indicate the addition process:

- 7 (ones) + 2 (ones) = 9 (ones)
- 0 (tens) + 8 (tens) = 8 (tens)
- 4 (hundreds) + 3 (hundreds) = 7 (hundreds)
- 3 (thousands) + 5 (thousands) = 8 (thousands)

The final sum is 8789.

Adding with 4- and 5-digit numerals is like adding with 1-, 2-, or 3-digit numerals. You add the ones, add the tens, add the hundreds, and so on, as shown above.

What is the greatest number you can name in each place-value position in the numeral for the sum? **nine times the place value of each position**

In the addition above, was it necessary to rename the sum of the ones? Of the tens? Of the hundreds? Of the thousands? No; No; No; No

To find a sum you need not write the expanded notation. Instead, use the idea of place value when adding the ones, the tens, the hundreds, and so on.

Oral For each open sentence below tell the sum. Which digit in 8765 also names this sum?

1.
$$\begin{array}{r} 3253 \\ +5512 \\ \hline 8765 \end{array}$$

$3+2=\square$ 5; 5

$50+10=\square$ 60; 6

$200+500=\square$ 700; 7

$3000+5000=\square$ 8000; 8

For each open sentence below tell the sum. Which digit in 7986 also names this sum?

2.
$$\begin{array}{r} 4362 \\ +3624 \\ \hline 7986 \end{array}$$

$2+4=\square$ 6; 6

$60+20=\square$ 80; 8

$300+600=\square$ 900; 9

$4000+3000=\square$ 7000; 7

For each open sentence below tell the sum. Which digit in 98,734 also names this sum?

3.
$$\begin{array}{r} 46310 \\ +52424 \\ \hline 98734 \end{array}$$

$0+4=\square$ 4; 4
 $10+20=\square$ 30; 3
 $300+400=\square$ 700; 7
 $6000+2000=\square$ 8000; 8
 $40000+50000=\square$ 90000; 9

For each open sentence below tell the sum. Which digit in 52,679 also names this sum?

4.
$$\begin{array}{r} 31219 \\ +21460 \\ \hline 52679 \end{array}$$

$9+0=\square$ 9; 9
 $10+60=\square$ 70; 7
 $200+400=\square$ 600; 6
 $1000+1000=\square$ 2000; 2
 $30000+20000=\square$ 50000; 5

Tell the steps you would take in finding each sum below.
See page T97.

5.
$$\begin{array}{r} a \quad 1248 \\ +2631 \\ \hline 3879 \end{array}$$

$$\begin{array}{r} b \quad 8417 \\ +1130 \\ \hline 9547 \end{array}$$

6.
$$\begin{array}{r} 43187 \\ +13602 \\ \hline 56789 \end{array}$$

$$\begin{array}{r} 82174 \\ +15315 \\ \hline 97489 \end{array}$$

Written Copy. Find each sum.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 1417 \\ +8342 \\ \hline 9759 \end{array}$	$\begin{array}{r} 1892 \\ +4106 \\ \hline 5998 \end{array}$	$\begin{array}{r} 8192 \\ +1806 \\ \hline 9998 \end{array}$
2.	$\begin{array}{r} 7411 \\ +2587 \\ \hline 9998 \end{array}$	$\begin{array}{r} 5286 \\ +4713 \\ \hline 9999 \end{array}$	$\begin{array}{r} 4192 \\ +1406 \\ \hline 5598 \end{array}$
3.	$\begin{array}{r} 1482 \\ +6312 \\ \hline 7794 \end{array}$	$\begin{array}{r} 3614 \\ +14281 \\ \hline 17895 \end{array}$	$\begin{array}{r} 26341 \\ +3558 \\ \hline 29899 \end{array}$
4.	$\begin{array}{r} 81427 \\ +3571 \\ \hline 84998 \end{array}$	$\begin{array}{r} 8122 \\ +51036 \\ \hline 59158 \end{array}$	$\begin{array}{r} 15034 \\ +44954 \\ \hline 59988 \end{array}$
5.	$\begin{array}{r} 50313 \\ +49656 \\ \hline 99969 \end{array}$	$\begin{array}{r} 84624 \\ +15212 \\ \hline 99836 \end{array}$	$\begin{array}{r} 21541 \\ +36247 \\ \hline 57788 \end{array}$

Write an open sentence for each problem. Solve the open sentence. Answer the problem.

6. Jeff's father drove 1282 miles in September. He drove 1407 miles in October. How many miles did he drive in all? $1282+1407=\square$; 2689 miles

7. On Saturday the attendance for a world series game was 52,496. On Sunday the attendance was 47,303. How many people attended these two baseball games? $52496+47303=\square$; 99,799 people

Can you do this? Replace each with the correct numeral.

<i>a</i>	<i>b</i>
$\begin{array}{r} 8 \ 4 \ 5 \ 3 \\ +1 \ 0 \ 1 \ 4 \\ \hline 9 \ 4 \ 6 \ 7 \end{array}$	$\begin{array}{r} 9 \ 2 \ 3 \ 4 \ 6 \\ +7 \ 4 \ 5 \ 0 \\ \hline 9 \ 9 \ 7 \ 9 \ 6 \end{array}$

Study the subtraction example below. Tell the simplest numeral for the \square in each sentence below.

3.
$$\begin{array}{r} 82496 \\ -42361 \\ \hline 40135 \end{array}$$

$6 - 1 = \square$
 $90 - 60 = \square$
 $400 - 300 = \square$
 $2000 - 2000 = \square$
 $80000 - 40000 = \square$

Study the subtraction example below. Tell the simplest numeral for the \square in each sentence below.

4.
$$\begin{array}{r} 94127 \\ -31102 \\ \hline 63025 \end{array}$$

$7 - 2 = \square$
 $20 - 0 = \square$
 $100 - 100 = \square$
 $4000 - 1000 = \square$
 $90000 - 30000 = \square$

Tell the steps you would take in finding each difference below. See page T99.

	a	b
5.	$\begin{array}{r} 9186 \\ -2171 \\ \hline 7015 \end{array}$	$\begin{array}{r} 6287 \\ -1246 \\ \hline 5041 \end{array}$
6.	$\begin{array}{r} 7764 \\ -1233 \\ \hline 6531 \end{array}$	$\begin{array}{r} 4875 \\ -2261 \\ \hline 2614 \end{array}$

Written Copy. Find each difference.

	a	b	c
1.	$\begin{array}{r} 7411 \\ -3210 \\ \hline 4201 \end{array}$	$\begin{array}{r} 6814 \\ -5812 \\ \hline 1002 \end{array}$	$\begin{array}{r} 8645 \\ -2615 \\ \hline 6030 \end{array}$
2.	$\begin{array}{r} 6827 \\ -6215 \\ \hline 612 \end{array}$	$\begin{array}{r} 9218 \\ -6118 \\ \hline 3100 \end{array}$	$\begin{array}{r} 2981 \\ -1681 \\ \hline 1300 \end{array}$
3.	$\begin{array}{r} 59214 \\ -28201 \\ \hline 31013 \end{array}$	$\begin{array}{r} 63219 \\ -62217 \\ \hline 1002 \end{array}$	$\begin{array}{r} 36291 \\ -34290 \\ \hline 2001 \end{array}$
4.	$\begin{array}{r} 64918 \\ -14902 \\ \hline 50016 \end{array}$	$\begin{array}{r} 49681 \\ -22380 \\ \hline 27301 \end{array}$	$\begin{array}{r} 68941 \\ -68841 \\ \hline 100 \end{array}$
5.	$\begin{array}{r} 49861 \\ -23431 \\ \hline 26430 \end{array}$	$\begin{array}{r} 94682 \\ -33211 \\ \hline 61471 \end{array}$	$\begin{array}{r} 89642 \\ -89341 \\ \hline 301 \end{array}$
6.	$\begin{array}{r} 84397 \\ -21364 \\ \hline 63033 \end{array}$	$\begin{array}{r} 49318 \\ -16216 \\ \hline 33102 \end{array}$	$\begin{array}{r} 39476 \\ -28175 \\ \hline 11301 \end{array}$

Write an open sentence for each problem. Solve the open sentence. Answer the problem.

7. In July and August Don drove his car 2838 miles. In August he drove 1432 miles. How many miles did he drive in July? $2838 - 1432 = \square$; 1406 miles

8. A loaded truck weighed 9452 pounds. The empty truck weighed 6351 pounds. What was the difference between the two weights? $9452 - 6351 = \square$; 3101 pounds

Tell why To find a difference you can think about expanded notations but just write place-value numerals. The place-value numerals indicate the place values shown in the expanded notation. 99

Addition with Renaming (Four- and Five-Digit Numerals)

A	B	C
	Th H T O	
2436	1 1 1	2436
2341	2 4 3 6	2341
+3855	2 3 4 1	+3855
	+3 8 5 5	
	8 6 3 2	8632

$6 + 1 + 5 = 12 = 1T + 2$
 $1T + 3T + 4T + 5T = 13T = 1H + 3T$
 $1H + 4H + 3H + 8H = 16H = 1Th + 6H$
 $1Th + 2Th + 2Th + 3Th = 8Th$

To find the sum in **A** you can think of place value as shown by the grid in **B**. You need to write only the place-value numerals as shown in **C**.

The greatest number you can name in the ones position in the numeral for the sum is 9. Why? ^{*1} Is 12 greater than 9? Think of 12 as $1T + 2$. Why? ^{*2} In **B**, where do you record the 1T of $1T + 2$? Where do you record the 2 of $1T + 2$? as a remainder numeral in the tens position; in the ones position

What is the greatest number you can name in the tens position in the numeral for the sum? Is $13T$ greater than $9T$? ^{Yes} Think of $13T$ as $1H + 3T$. Why? ^{*3} In **B**, where do you record the 1H of $1H + 3T$? Where do you record the 3T of $1H + 3T$? as a remainder numeral in the hundreds position; in the tens position

What is the greatest number you can name in the hundreds position in the numeral for the sum? ⁹⁰⁰ Is $16H$ greater than $9H$? Think of $16H$ as $1Th + 6H$. Why? ^{*4} In **B**, where do you record the 1Th of $1Th + 6H$? Where do you record the 6H of $1Th + 6H$? in the thousands position; in the hundreds position

What is the greatest number you can name in the thousands position in the numeral for the sum? Is $8Th$ greater than $9Th$? Do you need to rename the $8Th$ in order to record it? Where do you record the $8Th$? ⁹⁰⁰⁰ No; No; in the thousands position

Explain the steps you would take in doing the addition shown in **C** above. See thinking steps in **B** above.

- 100** *1 Only one digit can be recorded in a place-value position and the digit with the greatest value is 9.
- *2 12 cannot be recorded in the ones position.
- *3 $13T$ cannot be recorded in the tens position.
- *4 $16H$ cannot be recorded in the hundreds position.

Oral Study the following addition example. Use it to answer the questions below.

$$\begin{array}{r}
 \begin{array}{c} \text{Th} \mid \text{H} \mid \text{T} \mid \text{O} \\ 1 \mid 1 \mid 1 \mid \\ 5428 \\ 2694 \\ +1421 \\ \hline 9543 \end{array}
 \end{array}$$

1. $8+4+1=13=1\text{T}+3$

Why do you rename 13 as $1\text{T}+3$?
Where should you record the 1T of $1\text{T}+3$? Where should you record the 3 of $1\text{T}+3$? **See below.**

2. $1\text{T}+2\text{T}+9\text{T}+2\text{T}=14\text{T}=1\text{H}+4\text{T}$

Why do you rename 14T as $1\text{H}+4\text{T}$?
Where should you record 1H of $1\text{H}+4\text{T}$? Where should you record 4T of $1\text{H}+4\text{T}$? **See below.**

3. $1\text{H}+4\text{H}+6\text{H}+4\text{H}=15\text{H}=1\text{Th}+5\text{H}$

Why do you rename 15H as $1\text{Th}+5\text{H}$?
Where should you record 1Th of $1\text{Th}+5\text{H}$? Where should you record 5H of $1\text{Th}+5\text{H}$? **See below.**

4. $1\text{Th}+5\text{Th}+2\text{Th}+1\text{Th}=9\text{Th}$

Do you need to rename 9Th ? Where should you record 9Th ? **No; in the thousands position**

Written Copy. Find each sum.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 6872 \\ +1539 \\ \hline 8411 \end{array}$	$\begin{array}{r} 7762 \\ +1468 \\ \hline 9230 \end{array}$	$\begin{array}{r} 4683 \\ +2739 \\ \hline 7422 \end{array}$
2.	$\begin{array}{r} 6843 \\ +2499 \\ \hline 9342 \end{array}$	$\begin{array}{r} 69342 \\ +15789 \\ \hline 85131 \end{array}$	$\begin{array}{r} 36944 \\ +18458 \\ \hline 55402 \end{array}$

Oral

1. You cannot record 13 in the ones position; in the tens position
2. You cannot record 14T in the tens position; in the hundreds position
3. You cannot record 15H in the hundreds position; in the thousands position

3.	$\begin{array}{r} 1931 \\ 4862 \\ +2169 \\ \hline 8962 \end{array}$	$\begin{array}{r} 4076 \\ 2913 \\ +2945 \\ \hline 9934 \end{array}$	$\begin{array}{r} 3007 \\ 2967 \\ +3451 \\ \hline 9425 \end{array}$
4.	$\begin{array}{r} 34129 \\ 26981 \\ +36186 \\ \hline 97296 \end{array}$	$\begin{array}{r} 41390 \\ 19686 \\ +19426 \\ \hline 80502 \end{array}$	$\begin{array}{r} 63298 \\ 19448 \\ +7078 \\ \hline 89824 \end{array}$
5.	$\begin{array}{r} 72094 \\ 19418 \\ +9138 \\ \hline 100650 \end{array}$	$\begin{array}{r} 4869 \\ 34829 \\ +4016 \\ \hline 43714 \end{array}$	$\begin{array}{r} 85420 \\ 1639 \\ +74818 \\ \hline 161877 \end{array}$
6.	$\begin{array}{r} 63104 \\ 49937 \\ +62488 \\ \hline 175529 \end{array}$	$\begin{array}{r} 9101 \\ 36824 \\ +6359 \\ \hline 52284 \end{array}$	$\begin{array}{r} 82005 \\ 8490 \\ +61207 \\ \hline 151702 \end{array}$

Write an open sentence for each problem. Solve the open sentence. Answer the problem.

7. On Monday 17,384 people saw the Cubs play the Pirates. On Tuesday the attendance was 18,984. What was the attendance for both days?
 $17384+18984=\square$; **36,368 people**

8. On Tuesday 17,838 people saw the Indians play the Yankees. On Wednesday 15,494 people watched them play. On Thursday the attendance was 16,483. What was the attendance for these three days?
 $17838+15494+16483=\square$; **49815 people**

Can you do this? Replace each \square with the correct numeral.

$$\begin{array}{r}
 \begin{array}{c} a \\ 1 \mid 1 \mid \\ 4 \mid 3 \mid 1 \mid 9 \mid 2 \\ +6 \mid 9 \mid 3 \mid 8 \\ \hline 5 \mid 0 \mid 1 \mid 3 \mid 0 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{c} b \\ 1 \mid 1 \mid \\ 8 \mid 6 \mid 4 \mid 3 \mid 9 \\ +3 \mid 5 \mid 6 \mid 4 \\ \hline 9 \mid 0 \mid 0 \mid 0 \mid 3 \end{array}
 \end{array}$$

Subtraction with Renaming (Four- and Five-Digit Numerals)

$$\begin{array}{r} 74523 \\ -45328 \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{r} 70000+4000+500+20+3 \\ -(40000+5000+300+20+8) \\ \hline \end{array}$$

Can you subtract 8 from 3? ^{No} Since you cannot, rename 20+3 as 10+13. You now have:

$$\begin{array}{r} 70000+4000+500+10+13 \\ -(40000+5000+300+20+8) \\ \hline \end{array}$$

What is the difference of the ones? Can you subtract 20 from 10? ^{No} Since you cannot, rename 500+10 as 400+110. You now have:

$$\begin{array}{r} 70000+4000+400+110+13 \\ -(40000+5000+300+20+8) \\ \hline \end{array}$$

What is the difference of the tens? Can you subtract 300 from 400? ^{Yes} You now have:

$$\begin{array}{r} 70000+4000+400+110+13 \\ -(40000+5000+300+20+8) \\ \hline \end{array}$$

What is the difference of the hundreds? Can you subtract 5000 from 4000? ^{No} Since you cannot, how would you rename 70000+4000? You now have:

$$\begin{array}{r} 60000+14000+400+110+13 \\ -(40000+5000+300+20+8) \\ \hline \end{array}$$

What is the difference of the thousands? Can you subtract 40000 from 60000? ^{Yes} You now have:

$$\begin{array}{r} 60000+14000+400+110+13 \\ -(40000+5000+300+20+8) \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{r} \text{TTh} \mid \text{Th} \mid \text{H} \mid \text{T} \mid \text{O} \\ \begin{array}{r} 6 \quad 14 \quad 4 \quad 11 \quad 13 \\ \cancel{7} \quad \cancel{4} \quad \cancel{5} \quad \cancel{2} \quad \cancel{3} \\ -4 \quad 5 \quad 3 \quad 2 \quad 8 \\ \hline 2 \quad 9 \quad 1 \quad 9 \quad 5 \end{array} \end{array}$$

What is the difference of the ten thousands? What is the simplest numeral for 74523-45328? 20000; 29195

Oral What numeral should replace each \square in each of the following?

1.
$$\begin{array}{r} 4725 \\ -2816 \\ \hline \end{array}$$

Th	H	T	O
4	7	2	5
-2	8	1	6
1	9	0	9

2.
$$\begin{array}{r} 7452 \\ -2591 \\ \hline \end{array}$$

Th	H	T	O
7	4	5	2
-2	5	9	1
4	8	6	1

Tell how you would rename the minuend to subtract in each place-value position in each of the following.

3.
$$\begin{array}{r} 3692 \\ -1248 \\ \hline \end{array}$$

a b c

4.
$$\begin{array}{r} 9201 \\ -4180 \\ \hline \end{array}$$

a b c

5.
$$\begin{array}{r} 2847 \\ -1983 \\ \hline \end{array}$$

a b c

6.
$$\begin{array}{r} 43181 \\ -16290 \\ \hline \end{array}$$

a b c

7.
$$\begin{array}{r} 24062 \\ -11866 \\ \hline \end{array}$$

a b c

8.
$$\begin{array}{r} 94321 \\ -13982 \\ \hline \end{array}$$

a b c

Written Copy. Find each difference.

1.
$$\begin{array}{r} 4358 \\ -1239 \\ \hline \end{array}$$

a b c

2.
$$\begin{array}{r} 6812 \\ -1911 \\ \hline \end{array}$$

a b c

3.
$$\begin{array}{r} 7528 \\ -918 \\ \hline \end{array}$$

a b c

4.
$$\begin{array}{r} 5782 \\ -1893 \\ \hline \end{array}$$

a b c

5.
$$\begin{array}{r} 67183 \\ -48091 \\ \hline \end{array}$$

a b c

6.
$$\begin{array}{r} 76813 \\ -4923 \\ \hline \end{array}$$

a b c

7.
$$\begin{array}{r} 23194 \\ -14089 \\ \hline \end{array}$$

a b c

8.
$$\begin{array}{r} 49210 \\ -6873 \\ \hline \end{array}$$

a b c

Can you do this? Replace each \square with the correct numeral.

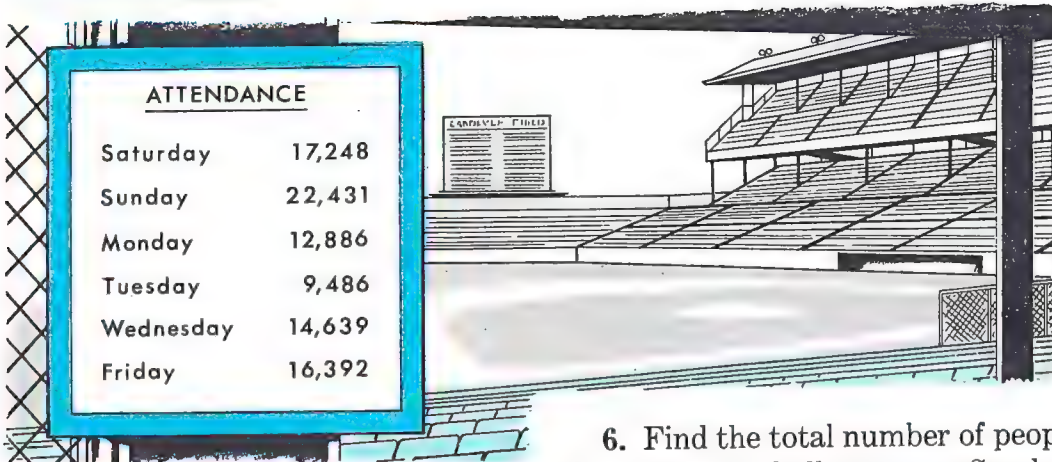
a b

$$\begin{array}{r} 5934 \\ -2899 \\ \hline 3035 \end{array}$$

$$\begin{array}{r} 23863 \\ -11237 \\ \hline 12626 \end{array}$$

Tell why Explain why you can only put one digit in each place-value position of a difference. The greatest number that can be recorded in a place-value position is 9 103 times that particular place value.

Solving Problems



Use the attendance figures shown above for problems 1–11. Write an open sentence for each problem. Solve the open sentence. Answer the problem.

1. How many people attended the ball game on Saturday and Sunday together? $17248+22431=\square$; 39679 people

2. Find the total number of people attending the ball game on Sunday and Monday. $22431+12886=\square$; 35317 people

3. How many more people attended the ball game on Sunday than on Monday? $22431-12886=\square$; 9545 people

4. How many more people attended the ball game on Sunday than on Saturday? $22431-17248=\square$; 5183 people

5. How many more people attended the ball game on Sunday than on Tuesday? $22431-9486=\square$; 12945 people

6. Find the total number of people attending the ball game on Sunday, Monday, and Tuesday. $22431+12886+9486=\square$; 44803 people

7. Find the total number of people attending the ball game on Tuesday, Wednesday, and Friday. $9486+14639+16392=\square$; 40517 people

8. Compare Wednesday's attendance with Friday's attendance. How many more people attended the ball game on Friday than on Wednesday? $16392-14639=\square$; 1753 people

9. Compare Sunday's attendance with Friday's attendance. How many more people attended the ball game on Sunday than on Friday? $22431-16392=\square$; 6039 people

10. Find the total number of people attending the ball game on Friday, Saturday, and Sunday. $16392+17248+22431=\square$; 56071 people

11. Compare Saturday's attendance with Tuesday's attendance. How many more people attended the ball game on Saturday than on Tuesday? $17248-9486=\square$; 7762 people

12. A ball park had a seating capacity of 64,382. The owners decided to increase the size of the ball park. Now the seating capacity is 72,468. How many new seats were added to the ball park? $72468 - 64382 = \square$; 8086 seats

13. The seating capacity of a stadium is 46,314. The attendance for a football game was 38,395. If everyone attending the ball game was occupying a seat, how many seats were empty? $46314 - 38395 = \square$; 7919 seats

14. Mr. Moore drove 1482 miles in August. He drove 1369 miles in September. He drove 1142 miles in October. How many miles did he drive in all? $1482 + 1369 + 1142 = \square$; 3993 miles

15. A service station sold 2461 gallons of gasoline on Saturday. It sold 1859 gallons of gasoline on Sunday. How many more gallons of gasoline did this service station sell on Saturday than on Sunday? $2461 - 1859 = \square$; 602 gallons

16. A farmer had 2248 chickens. He sent 1159 of these chickens to market. How many chickens did he keep on the farm? $2248 - 1159 = \square$; 1089 chickens

17. In the last two years, Mr. Ames drove his car 22,486 miles. Last year he drove 11,568 miles. How many miles did he drive this year? $22486 - 11568 = \square$; 10918 miles

18. An airplane was flying at an altitude of 11,348 feet. It was or-

dered to increase this altitude by 2865 feet. What is its new altitude? $11348 + 2865 = \square$; 14213 feet

19. There are 3186 pupils attending Washington High School. If 1842 of them are boys, how many girls attend Washington High School? $3186 - 1842 = \square$; 1344 girls

20. A truck weighing 7450 pounds was loaded with 1948 pounds of coal. How many pounds did the truck and its load weigh together? $7450 + 1948 = \square$; 9398 pounds

21. The enrollment of the three elementary schools in Westfield in a certain year was 478 pupils, 395 pupils, and 446 pupils. In all, how many pupils were enrolled in the elementary schools in Westfield that year? $478 + 395 + 446 = \square$; 1319 pupils

22. A record of milk delivered at one school was as follows: Monday, 1248; Tuesday, 1248; and Wednesday, 1150. How many cartons of milk were delivered in the 3 days? $1248 + 1248 + 1150 = \square$; 3646 cartons

Can you do this? Compose a story problem for each of the following.

Answers will vary.

1. $14000 + 18000 = \square$

2. $33000 - 16000 = \square$

Tell why Sally can find the sum of 12 eights as shown below.

$$\begin{array}{r} 48 \\ +48 \\ \hline 96 \end{array}$$

12 eights = 6 eights + 6 eights
 $= (6 \times 8) + (6 \times 8)$
 $= 48 + 48$ or 96

Practice in Addition and Subtraction

Part 1 Estimate and record the following sums and differences after rounding off each number to nearest 100, 1000, or 10,000. *only the exact answers shown*

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 389 \\ +412 \\ \hline 801 \end{array}$	$\begin{array}{r} 1628 \\ +3580 \\ \hline 5208 \end{array}$	$\begin{array}{r} 14280 \\ +1040 \\ \hline 15320 \end{array}$
2.	$\begin{array}{r} 839 \\ -461 \\ \hline 378 \end{array}$	$\begin{array}{r} 6128 \\ -2874 \\ \hline 3254 \end{array}$	$\begin{array}{r} 84182 \\ -62079 \\ \hline 22103 \end{array}$
3.	$\begin{array}{r} 296 \\ +408 \\ \hline 704 \end{array}$	$\begin{array}{r} 4084 \\ -1961 \\ \hline 2123 \end{array}$	$\begin{array}{r} 48621 \\ -11409 \\ \hline 37212 \end{array}$
4.	$\begin{array}{r} 611 \\ -292 \\ \hline 319 \end{array}$	$\begin{array}{r} 3120 \\ -1998 \\ \hline 1122 \end{array}$	$\begin{array}{r} 14631 \\ -8599 \\ \hline 6032 \end{array}$
5.	$\begin{array}{r} 649 \\ 302 \\ +60 \\ \hline 1011 \end{array}$	$\begin{array}{r} 8342 \\ 308 \\ +7091 \\ \hline 15741 \end{array}$	$\begin{array}{r} 60042 \\ 10904 \\ +7062 \\ \hline 78008 \end{array}$
6.	$\begin{array}{r} 843 \\ 216 \\ +348 \\ \hline 1407 \end{array}$	$\begin{array}{r} 9210 \\ 1706 \\ +4038 \\ \hline 14954 \end{array}$	$\begin{array}{r} 12456 \\ 1280 \\ +43809 \\ \hline 57545 \end{array}$

Part 2 Do the following.

1-6. Copy *Part 1*, 1-6. Find each sum or difference. Compare the exact answer to the estimated answer.
See above.

Part 3 Write an open sentence for each problem. Solve the open sentence. Answer the problem.

1. There are 23,382 people living in Centerville. There are 26,499 people living in Solon. How many people live in both towns together?
 $23382+26499=\square$; **49,881 people**

2. Mr. Jacobs earned \$14,280 last year. This year he earned \$16,940. In both years he earned a total of how many dollars?
 $14280+16940=\square$; **\$31,220**

3. Mr. Kline earned \$16,450 in two years. In one of those years he earned \$8485. How much did he earn the other year?
 $16450-8485=\square$; **\$7965**

4. Mr. Lester bought a new car for \$3862. He also had a garage built that cost him \$2468. How much money did he spend?
 $3862+2468=\square$; **\$6330**

5. Mr. Neal spent \$5492 for a new car and a garage. The garage cost him \$2139. How much did the new car cost?
 $5492-2139=\square$; **\$3353**

6. Jack is planning to buy a car for \$2840. How much less than \$3000 is this?
 $3000-2840=\square$; **\$160**

Can you do this? Replace each with the correct numeral.

<i>a</i>	<i>b</i>
$\begin{array}{r} 4\ 6\ 7\ 2 \\ 3\ 8\ 5\ 7 \\ +9\ 2\ 4\ 6 \\ \hline 1\ 7\ 7\ 7\ 5 \end{array}$	$\begin{array}{r} 7\ 2\ 4\ 6 \\ -5\ 2\ 8\ 7 \\ \hline 1\ 9\ 5\ 9 \end{array}$

Tell why The addition and the multiplication below both name the same number. Why?
 $4826+4826+4826+$
4826 means 4x4826.

$\begin{array}{r} 4826 \\ 4826 \\ 4826 \\ +4826 \\ \hline \end{array}$	$\begin{array}{r} 4826 \\ \times 4 \\ \hline \end{array}$
--	---

Checkup Time

The numerals in () tell the pages where you can turn for help.

Important Ideas

1. Simplest numerals are formed using ten digits and the idea of place value. (87)
2. Commas may be used to separate a numeral into periods of three digits each. (90)
3. To estimate a sum or difference, the numbers may be rounded off. (94)
4. Expanded notation helps you understand how to add or subtract. (96, 98)
5. When adding whole numbers, you add ones to ones, tens to tens, thousands to thousands, and ten-thousands to ten-thousands, and so on. (100)
6. You may rename the minuend to make subtraction possible in every place-value position. (102)

Words to Know

1. Place value, digits (87)
2. Number words, periods, millions, billions (90)
3. Rounding off (92)
4. Estimating, approximate answer (94)

Questions to Discuss

See page T107.

1. How do we use the idea of place value to construct simplest numerals for numbers? (87)
2. What is the value of each digit in 1,928,357? (88)
3. How does separating a numeral into sets of three digits each help you read the numeral? (90)
4. How would you rename the minuend in $4832 - 2983$ to make subtraction possible in every place-value position? (102)

Written Practice

1. Name the greatest possible number using each of these digits only once: 1, 3, 5, 0. (88) **5310**

2. Round off each number to the nearest 10, 100, 1000.

5280 **12900** **25680**
5283 **12898** **25682** (92)
5300, 5000 **12900, 13000** **25700, 26000**

Copy. Find each sum or difference below.

	<i>a</i>		<i>b</i>	
3.	$\begin{array}{r} 3409 \\ +5340 \\ \hline 8749 \end{array}$	(96)	$\begin{array}{r} 5340 \\ -2130 \\ \hline 3210 \end{array}$	(98)
4.	$\begin{array}{r} 4657 \\ +3784 \\ \hline 8441 \end{array}$	(100)	$\begin{array}{r} 4657 \\ -1948 \\ \hline 2709 \end{array}$	(102)

Self-Evaluation

Part 1 Write the simplest numeral for each of the following.

1. $2TTh + 3Th + 7H + 4T + 3$
23,743

2. $4Th + 3H + 0T + 0$
4300

3. $1HTh + 2TTh + 4Th + 2H + 0T + 3$
124,203

4. $5M + 6HTh + 5TTh + 0Th + 3H + 4T + 2$
5,650,342

Write the following numerals in expanded notation, like $1234 = (1 \times 1000) + (2 \times 100) + (3 \times 10) + 4$.

For column *a* see below. *b*

5. 29,648 $(2 \times 10000) + (9 \times 1000) + (6 \times 100) + (4 \times 10) + (8 \times 1)$

6. 10,048 $(1 \times 10000) + (0 \times 1000) + (0 \times 100) + (4 \times 10) + (8 \times 1)$

7. 99,639 $(9 \times 10000) + (9 \times 1000) + (6 \times 100) + (3 \times 10) + (9 \times 1)$

8. 33,333 $(3 \times 10000) + (3 \times 1000) + (3 \times 100) + (3 \times 10) + (3 \times 1)$

Part 2 Copy. Complete each row by naming the number that is 10 more, 100 less, and 1000 more than the number named below.

	10 more	100 less	1000 more
1. 6240	6250	6140	7240
2. 2640	2650	2540	3640
3. 2460	2470	2360	3460
4. 4620	4630	4520	5620

Part 3 Copy the numerals below and use commas to separate it into periods. Then replace each _ with the correct name of a period.

1. 968463241
million thousand
968_,463_,241

2. 1342046102
billion million thousand
1_,342_,046_,102

3. 14384210
million thousand
14_,384_,210

4. 23698764201
billion million thousand
23_,698_,764_,201

5. 16432406210
billion million thousand
16_,432_,406_,210

Part 4 Round off each of the following to the nearest hundred, then to the nearest thousand, and then to the nearest ten thousand.

	<i>a</i>	<i>b</i>
1.	23,469 23500; 23000; 20000	32,859 32900; 33000; 30000
2.	46,358 46400; 46000; 50000	36,450 36400; 36000; 40000
3.	36,584 36600; 37000; 40000	14,750 14800; 15000; 10000
4.	841,841 841800; 842000; 840000	169,507 169500; 170000; 170000
5.	39,421 39400; 39000; 40000	738,430 738400; 738000; 740000

- Part 1**
- 5b. $(1 \times 100000) + (4 \times 10000) + (8 \times 1000) + (6 \times 100) + (3 \times 10) + (4 \times 1)$
 6b. $(6 \times 100000) + (4 \times 10000) + (8 \times 1000) + (3 \times 100) + (7 \times 10) + (4 \times 1)$
 7b. $(2 \times 100000) + (3 \times 10000) + (3 \times 1000) + (0 \times 100) + (0 \times 10) + (6 \times 1)$
 8b. $(2 \times 100000) + (2 \times 10000) + (2 \times 1000) + (2 \times 100) + (2 \times 10) + (2 \times 1)$

Chapter 5

PATTERNS OF MULTIPLICATION AND DIVISION

Multiplication

Each of five children bought a box of crayons at a sale price of 32 cents a box. How much money did the five children spend for crayons?

Why is $5 \times 32 = \square$ a good open sentence to use for solving this problem?^{*1} Think of 5×32 as repeated addition. What open addition sentence can you think of for $5 \times 32 = \square$? This can be expressed as shown in **A** below.

$$32+32+32+32+32=\square$$

A	B
32	(30 + 2)
32	(30 + 2)
32	(30 + 2)
32	(30 + 2)
+32	+(30 + 2)
(5×32)	(5×30)+(5×2)

How is 32 named in **B**? Why? What is the simplest numeral for (5×30) ? For (5×2) ? For $(5 \times 30) + (5 \times 2)$? For (5×32) ? as 30+2; so the ones and tens may be added separately; 150; 10; 160; 160
How many cents did the five children spend? 160 cents

Oral What open addition sentence can you give for each open multiplication sentence below? See representative answers for 1 below.

a	b	c
1. $6 \times 18 = \square$ 108	4. $21 \times \square = \square$ 84	7. $42 \times \square = \square$ 294
2. $3 \times 39 = \square$ 117	5. $19 \times \square = \square$ 95	6. $24 \times \square = \square$ 144
3. $9 \times 31 = \square$ 279	7. $16 \times \square = \square$ 112	4. $29 \times \square = \square$ 116

Written Solve each open multiplication sentence in *Oral* 1–3 by adding as shown in **B** above. See Oral.

Tell why If you already know that $8 \times 16 = 128$, then you also know that $16 \times 8 = 128$. Why? Addition is commutative.

Tell how Tell how multiplication is related to addition. Repeated addition of whole numbers can be thought of as multiplication. 109

^{*1} 5 sets of 32 cents each are joined.

Oral 1a. $18+18+18+18+18$
b. $21+21+21+21$
c. $42+42+42+42+42$

The Distributive Property of Multiplication Over Addition

Each of four children bought a 10-cent pad of paper and a 3-cent pencil. How much money did the four children spend for pads of paper and pencils?

Tell how much *each* child spent as an addition numeral. $10+3$
 Tell how much *each* child spent as a simplest numeral.¹³ Why could you use either open sentence below to solve the problem above? **4 sets of $(10+3)$ or 13 members each are to be joined.**

$$4 \times (10+3) = \square \quad \text{or} \quad 4 \times 13 = \square$$

Do $4 \times (10+3)$ and 4×13 both name the same number? Which number? **Yes; 52**

Another way to think about this problem is that each child spent 10 cents for a pad of paper, so the four children spent (4×10) cents for pads of paper. Each child spent 3 cents for a pencil, so the four children spent (4×3) cents for pencils. Why could you use the open sentence below to solve the problem?
A set of (4×10) cents is joined to a set of (4×3) cents.

$$(4 \times 10) + (4 \times 3) = \square$$

52

What number does $(4 \times 10) + (4 \times 3)$ name? Is this the same number that is named by $4 \times (10+3)$ and 4×13 ? This can be expressed as shown below. **Yes**

$$\begin{array}{rcl}
 4 \times 13 & = & 4 \times (10+3) \\
 & = & (4 \times 10) + (4 \times 3) \\
 & = & 40 + 12 \\
 & = & 52
 \end{array}
 \qquad
 \begin{array}{r}
 10+3 \\
 \times 4 \\
 \hline
 40+12=52
 \end{array}$$

How was 13 renamed? Do the arrows show that multiplication is distributed over addition? **as $10+3$; Yes**

How many cents did the four children spend for pads of paper and pencils? **52 cents**

Multiplication can be distributed over addition.

Oral How would you rename each second factor below for easy multiplication?

- | a | b |
|--|------------------------------------|
| 1. $6 \times 17 = \square$ <u>102</u> | 8 $\times 21 = \square$ <u>168</u> |
| 2. $5 \times 19 = \square$ <u>95</u> | 9 $\times 38 = \square$ <u>342</u> |
| 3. $3 \times 14 = \square$ <u>42</u> | 7 $\times 58 = \square$ <u>406</u> |
| 4. $6 \times 35 = \square$ <u>210</u> | 3 $\times 41 = \square$ <u>123</u> |
| 5. $8 \times 14 = \square$ <u>112</u> | 6 $\times 24 = \square$ <u>144</u> |
| 6. $7 \times 25 = \square$ <u>175</u> | 4 $\times 31 = \square$ <u>124</u> |
| 7. $8 \times 39 = \square$ <u>312</u> | 9 $\times 92 = \square$ <u>828</u> |
| 8. $2 \times 18 = \square$ <u>36</u> | 5 $\times 71 = \square$ <u>355</u> |
| 9. $6 \times 22 = \square$ <u>132</u> | 9 $\times 66 = \square$ <u>594</u> |
| 10. $8 \times 52 = \square$ <u>416</u> | 3 $\times 29 = \square$ <u>87</u> |

Written Use the form shown below to find each product in Oral 1-10.
See beside each \square in Oral.

$$\begin{aligned} 6 \times 17 &= 6 \times (10 + 7) \\ &= (6 \times 10) + (6 \times 7) \\ &= 60 + 42 \\ &= 102 \end{aligned}$$

Copy. Replace each \square with the simplest numeral.

- | a | b |
|--|---|
| 11. $10 + 6$
$\times 7$
<u>70 + 42 = 112</u> | $10 + 4$
$\times 7$
<u>70 + 28 = 98</u> |

12. $60 + 7$ $\times 4$ <u>240 + 28 = 268</u>	$40 + 5$ $\times 6$ <u>240 + 30 = 270</u>
---	---

13. $50 + 4$ $\times 3$ <u>150 + 12 = 162</u>	$70 + 8$ $\times 4$ <u>280 + 32 = 312</u>
---	---

14. $50 + 3$ $\times 6$ <u>300 + 18 = 318</u>	$70 + 2$ $\times 7$ <u>490 + 14 = 504</u>
---	---

15. $30 + 7$ $\times 4$ <u>120 + 28 = 148</u>	$80 + 9$ $\times 6$ <u>480 + 54 = 534</u>
---	---

Another way Study the form shown below to find each product that follows.

$$\begin{aligned} 6 \times 19 &= 6 \times (20 - 1) \\ &= (6 \times 20) - (6 \times 1) \\ &= 120 - 6 \\ &= 114 \end{aligned}$$

- | a | b |
|---------------------------------------|------------------------------------|
| 1. $8 \times 18 = \square$ <u>144</u> | $7 \times 29 = \square$ <u>203</u> |
| 2. $6 \times 18 = \square$ <u>108</u> | $5 \times 19 = \square$ <u>95</u> |
| 3. $4 \times 28 = \square$ <u>112</u> | $3 \times 17 = \square$ <u>51</u> |
| 4. $2 \times 39 = \square$ <u>78</u> | $5 \times 49 = \square$ <u>245</u> |
| 5. $7 \times 17 = \square$ <u>119</u> | $4 \times 49 = \square$ <u>196</u> |

Tell why To solve $5 \times 17 = \square$, you can solve either $5 \times (10 + 7) = \square$ or $5 \times (20 - 3) = \square$. Why? 10 + 7 and 20 - 3 are both names for 17.

Multiplication (Two-Digit by One-Digit)

A

$$\begin{aligned}
 8 \times 64 &= 8 \times (60 + 4) \\
 &= (8 \times 60) + (8 \times 4) \\
 &= 480 + 32 \\
 &= 512
 \end{aligned}$$

B

$$\begin{array}{r}
 60 + 4 \\
 \times 8 \\
 \hline
 32 \text{ — } 8 \times 4 \\
 480 \text{ — } 8 \times 60 \\
 \hline
 512
 \end{array}$$

In **A**, the simplest numeral for 8×64 is found by renaming 64 as $60 + 4$ and distributing multiplication over addition. How do the arrows in **A** show that multiplication is distributed over addition? Each of the addends 60 and 4 is multiplied by 8.

Expanded notation in **B** shows another way to find the simplest numeral for 8×64 . How is 64 named in expanded notation? How is 32 obtained? How is 480 obtained? Is multiplication distributed over addition in **B**? Explain how. $60 + 4$; 8×4 ; 8×60 ; Yes; Each of the addends 60 and 4 is multiplied by 8.

The multiplication examples above can be expressed by using either the grid form as in **C** or place-value numerals as in **D**.

C

H	T	O
	6	4
	$\times 8$	
	3	2
4	8	0
5	1	2

D

	64
	$\times 8$
32	— 8×4 — 32
480	— 8×60 — 480
512	— $32 + 480$ — 512

In **A**, both 8 and 64 are called factors. When arranged as in **D**, 64 can be called the **multiplicand** (the number to be multiplied); 8 can be called the **multiplier** (the number by which you multiply); and 512 can be called the **product**.

To find the simplest numeral for a product, like 8×64 , name 64 in expanded notation and distribute multiplication over addition.

Oral Replace each with a single digit. Tell the steps you would take in finding the product.

$$\begin{array}{r} 13 \\ \times 5 \\ \hline 15 \\ 50 \\ \hline 65 \end{array}$$

5×3
 5×10
Add

$$\begin{array}{r} 14 \\ \times 7 \\ \hline 28 \\ 70 \\ \hline 98 \end{array}$$

7×4
 7×10
Add

a	b
$\begin{array}{r} 15 \\ \times 5 \\ \hline 25 \\ 50 \\ \hline 75 \end{array}$	$\begin{array}{r} 17 \\ \times 8 \\ \hline 56 \\ 80 \\ \hline 136 \end{array}$

$\begin{array}{r} 16 \\ \times 7 \\ \hline 42 \\ 70 \\ \hline 112 \end{array}$	$\begin{array}{r} 13 \\ \times 8 \\ \hline 24 \\ 80 \\ \hline 104 \end{array}$
--	--

Explain each example below.

See below.

a	b	c
$\begin{array}{r} 16 \\ \times 8 \\ \hline 48 \\ 80 \\ \hline 128 \end{array}$	$\begin{array}{r} 17 \\ \times 6 \\ \hline 42 \\ 60 \\ \hline 102 \end{array}$	$\begin{array}{r} 18 \\ \times 8 \\ \hline 64 \\ 80 \\ \hline 144 \end{array}$

Written Copy. Find each product.

a	b	c
$\begin{array}{r} 14 \\ \times 8 \\ \hline 112 \end{array}$	$\begin{array}{r} 15 \\ \times 4 \\ \hline 60 \end{array}$	$\begin{array}{r} 13 \\ \times 9 \\ \hline 117 \end{array}$
$\begin{array}{r} 17 \\ \times 9 \\ \hline 153 \end{array}$	$\begin{array}{r} 18 \\ \times 6 \\ \hline 108 \end{array}$	$\begin{array}{r} 17 \\ \times 5 \\ \hline 85 \end{array}$
$\begin{array}{r} 48 \\ \times 3 \\ \hline 144 \end{array}$	$\begin{array}{r} 18 \\ \times 7 \\ \hline 126 \end{array}$	$\begin{array}{r} 74 \\ \times 2 \\ \hline 148 \end{array}$
$\begin{array}{r} 46 \\ \times 9 \\ \hline 414 \end{array}$	$\begin{array}{r} 19 \\ \times 8 \\ \hline 152 \end{array}$	$\begin{array}{r} 14 \\ \times 7 \\ \hline 98 \end{array}$
$\begin{array}{r} 39 \\ \times 4 \\ \hline 156 \end{array}$	$\begin{array}{r} 24 \\ \times 6 \\ \hline 144 \end{array}$	$\begin{array}{r} 87 \\ \times 4 \\ \hline 348 \end{array}$
$\begin{array}{r} 37 \\ \times 8 \\ \hline 296 \end{array}$	$\begin{array}{r} 26 \\ \times 4 \\ \hline 104 \end{array}$	$\begin{array}{r} 55 \\ \times 6 \\ \hline 330 \end{array}$

Write an open sentence for each problem below. Solve the open sentence. Answer the problem.

7. How far can a car go if it travels 23 miles per gallon and has 8 gallons of gasoline in its tank?

$8 \times 23 = \square$; 184 miles

8. One yard is equivalent to 36 inches. How many inches are equivalent to 6 yards? $6 \times 36 = \square$; 216 inches

Can you do this? To find 6×19 as a simplest numeral you can add 60 and 54. How are 60 and 54 obtained? What is the product? $60 = 6 \times 10$, $54 = 6 \times 9$; 114

Tell why The first nine multiples of eleven are easy to remember. Why? In the simplest numeral for each of these multiples, both digits are the same as that for the multiplier. 113

Oral

5a. $8 \times 6 = 48$, $8 \times 10 = 80$, $48 + 80 = 128$
 b. $6 \times 7 = 42$, $6 \times 10 = 60$, $42 + 60 = 102$
 c. $8 \times 8 = 64$, $8 \times 10 = 80$, $64 + 80 = 144$

Multiplication (Three-Digit by One-Digit)

Multiplying a 3-digit number by a 1-digit number is like multiplying a 2-digit number by a 1-digit number. Study the examples shown below.

A

$$\begin{aligned}
 3 \times 423 &= 3 \times (400 + 20 + 3) \\
 &= (3 \times 400) + (3 \times 20) + (3 \times 3) \\
 &= 1200 + 60 + 9 \\
 &= 1269
 \end{aligned}$$

$400+20+3$

B

$$\begin{array}{r}
 423 \\
 \times 3 \\
 \hline
 9 \\
 60 \\
 1200 \\
 \hline
 1269
 \end{array}$$

$9 \text{ --- } 3 \times 3$
 $60 \text{ --- } 3 \times 20$
 $1200 \text{ --- } 3 \times 400$

In **A**, how is 423 renamed? How do you obtain (3×400) ? (3×20) ? (3×3) ? Is multiplication distributed over addition? **Yes**. What is the simplest numeral for (3×400) ? For (3×20) ? For (3×3) ? How do you obtain the 1269? **1200; 60; 9; Add 1200, 60, and 9.**

Does the multiplication example in **B** illustrate how the multiplication example in **A** can be written by using place-value numerals? **Yes**

In **B**, how do you obtain the 9? The 60? The 1200? Tell how multiplication is distributed over addition in **B**. How is the 1269 obtained? **Add 9, 60, and 1200.**

Another way to perform multiplication is shown below. Study the example and then explain each step.

$$\begin{array}{r}
 1 \\
 483 \\
 \times 5 \\
 \hline
 5
 \end{array}$$

$$\begin{array}{r}
 41 \\
 483 \\
 \times 5 \\
 \hline
 15
 \end{array}$$

$$\begin{array}{r}
 41 \\
 483 \\
 \times 5 \\
 \hline
 2415
 \end{array}$$

$$5 \times 3 = 10 + 5$$

$$(5 \times 80) + 10 = 400 + 10$$

$$(5 \times 400) + 400 = 2000 + 400$$

To find the simplest numeral for a product like 3×423 , name 423 in expanded notation and distribute multiplication over addition.

Oral Replace each with a single digit. Then tell the steps you would take in finding each product.

1.
$$\begin{array}{r} 1\ 3\ 4 \\ \times 4 \\ \hline \end{array}$$

1 6 ——— 4×4
1 2 0 ——— 4×30
4 0 0 ——— 4×100

$$\begin{array}{r} 5\ 3\ 6 \end{array}$$

2.
$$\begin{array}{r} 4\ 1\ 5 \\ \times 3 \\ \hline \end{array}$$

1 5 ——— 3×5
3 0 ——— 3×10
1 2 0 0 ——— 3×400

$$\begin{array}{r} 1\ 2\ 4\ 5 \end{array}$$

Tell the steps you would take in finding each product below.
See page T115.

<i>a</i>	<i>b</i>	<i>c</i>
3. $\begin{array}{r} 154 \\ \times 2 \\ \hline 8 \\ 100 \\ 200 \\ \hline 308 \end{array}$	$\begin{array}{r} 143 \\ \times 5 \\ \hline 15 \\ 200 \\ 500 \\ \hline 715 \end{array}$	$\begin{array}{r} 162 \\ \times 7 \\ \hline 14 \\ 420 \\ 700 \\ \hline 1134 \end{array}$

Tell the steps you would take in finding each product below.
See page T115.

<i>a</i>	<i>b</i>	<i>c</i>
4. $\begin{array}{r} 472 \\ \times 3 \\ \hline 1416 \end{array}$	$\begin{array}{r} 384 \\ \times 6 \\ \hline 2304 \end{array}$	$\begin{array}{r} 209 \\ \times 4 \\ \hline 836 \end{array}$
5. $\begin{array}{r} 149 \\ \times 7 \\ \hline 1043 \end{array}$	$\begin{array}{r} 488 \\ \times 3 \\ \hline 1464 \end{array}$	$\begin{array}{r} 834 \\ \times 4 \\ \hline 3336 \end{array}$

Written Copy. Find each product.

<i>a</i>	<i>b</i>	<i>c</i>
1. $\begin{array}{r} 124 \\ \times 3 \\ \hline 372 \end{array}$	$\begin{array}{r} 142 \\ \times 4 \\ \hline 568 \end{array}$	$\begin{array}{r} 124 \\ \times 5 \\ \hline 620 \end{array}$
2. $\begin{array}{r} 135 \\ \times 4 \\ \hline 540 \end{array}$	$\begin{array}{r} 148 \\ \times 6 \\ \hline 888 \end{array}$	$\begin{array}{r} 105 \\ \times 7 \\ \hline 735 \end{array}$
3. $\begin{array}{r} 153 \\ \times 4 \\ \hline 612 \end{array}$	$\begin{array}{r} 184 \\ \times 7 \\ \hline 1288 \end{array}$	$\begin{array}{r} 156 \\ \times 6 \\ \hline 936 \end{array}$
4. $\begin{array}{r} 138 \\ \times 5 \\ \hline 690 \end{array}$	$\begin{array}{r} 102 \\ \times 8 \\ \hline 816 \end{array}$	$\begin{array}{r} 129 \\ \times 5 \\ \hline 645 \end{array}$
5. $\begin{array}{r} 145 \\ \times 5 \\ \hline 725 \end{array}$	$\begin{array}{r} 154 \\ \times 6 \\ \hline 924 \end{array}$	$\begin{array}{r} 737 \\ \times 7 \\ \hline 5159 \end{array}$
6. $\begin{array}{r} 873 \\ \times 8 \\ \hline 6984 \end{array}$	$\begin{array}{r} 304 \\ \times 5 \\ \hline 1520 \end{array}$	$\begin{array}{r} 639 \\ \times 4 \\ \hline 2556 \end{array}$
7. $\begin{array}{r} 707 \\ \times 5 \\ \hline 3535 \end{array}$	$\begin{array}{r} 643 \\ \times 3 \\ \hline 1929 \end{array}$	$\begin{array}{r} 734 \\ \times 4 \\ \hline 2936 \end{array}$

Write an open sentence for each problem below. Solve the open sentence. Answer the problem.

8. There are 144 items in one gross. How many items are there in 8 gross? $8 \times 144 = \square$; 1152 items

9. A milkman delivered 120 cartons of milk to a store on each of 6 days. How many cartons of milk did he deliver to the store in all?

$6 \times 120 = \square$; 720 cartons

Tell why. You can find the simplest numeral for 8×56 by adding (8×50) and (8×6) . Why? $56 = 50 + 6$

then distribute multiplication over addition

Multiplication (Four-Digit by One-Digit)

The examples given below show how you can solve the open sentence $5 \times 4321 = \square$. Study these examples.

A	B	C
$\begin{array}{r} 4321 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 4000 + 300 + 20 + 1 \\ \times 5 \\ \hline 20000 + 1500 + 100 + 5 \end{array}$	$\begin{array}{r} 4321 \\ \times 5 \\ \hline 5 \\ 100 \\ 1500 \\ 20000 \\ \hline 21605 \end{array}$
		<div style="display: flex; justify-content: flex-end; margin-right: 10px;"> <div style="margin-right: 10px;">5</div> <div>5 × 1</div> </div> <div style="display: flex; justify-content: flex-end; margin-right: 10px;"> <div style="margin-right: 10px;">100</div> <div>5 × 20</div> </div> <div style="display: flex; justify-content: flex-end; margin-right: 10px;"> <div style="margin-right: 10px;">1500</div> <div>5 × 300</div> </div> <div style="display: flex; justify-content: flex-end; margin-right: 10px;"> <div style="margin-right: 10px;">20000</div> <div>5 × 4000</div> </div>

Do 4321 and $4000 + 300 + 20 + 1$ name the same number? How do you know? *Yes; $4000 + 300 + 20 + 1$ is the expanded notation for 4321.*

In $20000 + 1500 + 100 + 5$ in **B**, how do you obtain the 5? The 100? The 1500? The 20000? Is multiplication distributed over addition? How is multiplication distributed over addition shown in **B**? What is the simplest numeral for $20000 + 1500 + 100 + 5$? *5×1 ; 5×20 ; 5×300 ; 5×4000 ; Yes; see above; 21605*

In **C**, how do you obtain the 5? The 100? The 1500? The 20000? Is multiplication distributed over addition? How is multiplication distributed over addition shown in **C**? How do you obtain the 21605? Do $20000 + 1500 + 100 + 5$ and 21605 both name the same number? *5×1 ; 5×20 ; 5×300 ; 5×4000 ; Yes; see above; add 5, 100, 1500, and 20000; Yes*

A more convenient way to perform this multiplication is shown below. Study the example that follows and then explain each step.

$$\begin{array}{r} 4321 \\ \times 5 \\ \hline 21605 \end{array}$$

To find the simplest numeral for a product like 5×4321 , name 4321 in expanded notation and distribute multiplication over addition.

Oral Replace each with a single digit. Then tell the steps you would take in finding each product.

1.
$$\begin{array}{r} 1342 \\ \times 3 \\ \hline \end{array}$$

6 — 3×2
120 — 3×40
900 — 3×300
3000 — 3×1000

$$\hline 4026$$

2.
$$\begin{array}{r} 5631 \\ \times 5 \\ \hline \end{array}$$

5 — 5×1
150 — 5×30
3000 — 5×600
25000 — 5×5000

$$\hline 28155$$

Tell the steps you would take in finding each product below.

See page T117.

a	b	c
3. $\begin{array}{r} 3014 \\ \times 8 \\ \hline 32 \\ 80 \\ 000 \\ \hline 24000 \\ 24112 \end{array}$	$\begin{array}{r} 6103 \\ \times 5 \\ \hline 15 \\ 00 \\ 500 \\ \hline 30000 \\ 30515 \end{array}$	$\begin{array}{r} 4120 \\ \times 6 \\ \hline 0 \\ 120 \\ 600 \\ \hline 24000 \\ 24720 \end{array}$

Tell the steps you would take in finding each product below.

See page T117.

a	b	c
4. $\begin{array}{r} 741 \\ 4962 \\ \times 8 \\ \hline 39696 \end{array}$	$\begin{array}{r} 1 \\ 6021 \\ \times 7 \\ \hline 42147 \end{array}$	$\begin{array}{r} 11 \\ 3202 \\ \times 6 \\ \hline 19212 \end{array}$

Written Copy. Find each product.

a	b	c
1. $\begin{array}{r} 1352 \\ \times 4 \\ \hline 5408 \end{array}$	$\begin{array}{r} 1253 \\ \times 3 \\ \hline 3759 \end{array}$	$\begin{array}{r} 2315 \\ \times 2 \\ \hline 4630 \end{array}$
2. $\begin{array}{r} 1652 \\ \times 5 \\ \hline 8260 \end{array}$	$\begin{array}{r} 1492 \\ \times 3 \\ \hline 4476 \end{array}$	$\begin{array}{r} 6942 \\ \times 7 \\ \hline 48594 \end{array}$
3. $\begin{array}{r} 2906 \\ \times 7 \\ \hline 20342 \end{array}$	$\begin{array}{r} 1482 \\ \times 8 \\ \hline 11856 \end{array}$	$\begin{array}{r} 7436 \\ \times 9 \\ \hline 66924 \end{array}$
4. $\begin{array}{r} 1132 \\ \times 9 \\ \hline 10188 \end{array}$	$\begin{array}{r} 3102 \\ \times 9 \\ \hline 27918 \end{array}$	$\begin{array}{r} 2014 \\ \times 6 \\ \hline 12084 \end{array}$
5. $\begin{array}{r} 7604 \\ \times 4 \\ \hline 30416 \end{array}$	$\begin{array}{r} 5268 \\ \times 9 \\ \hline 47412 \end{array}$	$\begin{array}{r} 6522 \\ \times 6 \\ \hline 39132 \end{array}$
6. $\begin{array}{r} 2101 \\ \times 6 \\ \hline 12606 \end{array}$	$\begin{array}{r} 3120 \\ \times 4 \\ \hline 12480 \end{array}$	$\begin{array}{r} 4821 \\ \times 5 \\ \hline 24105 \end{array}$
7. $\begin{array}{r} 3010 \\ \times 7 \\ \hline 21070 \end{array}$	$\begin{array}{r} 3121 \\ \times 2 \\ \hline 6242 \end{array}$	$\begin{array}{r} 4002 \\ \times 8 \\ \hline 32016 \end{array}$

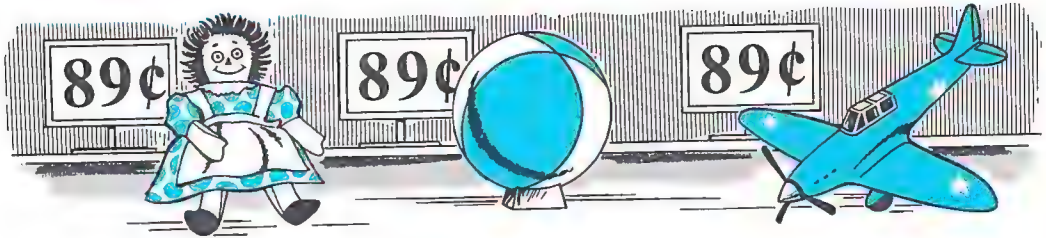
Write an open sentence for each problem below. Solve the open sentence. Answer the problem.

8. One mile is equivalent to 1760 yards. How many yards are equivalent to 3 miles? $3 \times 1760 = \square$; 5280 yards

9. One mile is equivalent to 5280 feet. How many feet are equivalent to 3 miles? $3 \times 5280 = \square$; 15,840 feet

Tell why You can find the simplest numeral for 4×124 by adding 400, 80, and 16. Why? $124 = 100 + 20 + 4$; Distribute multiplication over addition. The addends will be 400, 80, and 16.

Estimating Products



John's father is going to buy the toys shown in the picture above. He would like to know approximately how much the toys will cost him.

For many problems of this kind, you are able to give an approximate answer without doing any written computation.

When you give an approximate answer without doing any written computation, you are **estimating**.

An open sentence for the problem is shown below.

$$3 \times 89 = \square$$

You can estimate the answer mentally by rounding off 89 to 90. Why would you round off 89 to 90? Then use the open sentence below to find the estimated product.

$$3 \times 90 = \square$$

What is the simplest numeral for 3×90 ? Approximately how much money is John's father going to spend? 270; 270 cents or \$2.70

You estimate a product mentally by rounding the numbers given in the problem to the nearest 10, 100, or 1000, whichever is most convenient.

To estimate the product of 3 and 89, rounding 89 to 100 would allow you to use what open sentence? What is the simplest numeral for 3×100 ? $3 \times 100 = \square$; 300

Find the simplest numeral for 3×89 . Which estimated answer, 270 cents or 300 cents, is nearer to the exact answer? Which estimate would you prefer? Why? 270 cents; 270 cents; It is nearer to the exact answer.

Oral Round off each of the following numbers to the nearest 10. Then round off each of the following numbers to the nearest 100. Compare the two results.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	90; 100 89	70; 100 68	50; 100 52
2.	100; 100 98	60; 100 57	90; 100 93
3.	140; 100 142	190; 200 189	200; 200 202
4.	210; 200 212	300; 300 304	300; 300 298
5.	390; 400 391	210; 200 209	620; 600 621

Estimate each product below. In doing so, round off each multiplicand to the nearest 10 or 100, whichever is more convenient. *Estimated results are given to the right of each exercise.*

	<i>a</i>	<i>b</i>	<i>c</i>
6.	89 540; $\times 6$ 600 <hr/> 534	68 490; $\times 7$ 700 <hr/> 476	52 250; $\times 5$ 500 <hr/> 260
7.	98 800; $\times 8$ 800 <hr/> 784	97 600; $\times 6$ 600 <hr/> 582	93 360; $\times 4$ 400 <hr/> 372
8.	142 280; $\times 2$ 200 <hr/> 284	189 380; $\times 2$ 400 <hr/> 378	202 800; $\times 4$ 800 <hr/> 808
9.	212 630; $\times 3$ 600 <hr/> 636	304 2100; $\times 7$ 2100 <hr/> 2128	298 600; $\times 2$ 600 <hr/> 596
10.	432 3010; $\times 7$ 2800 <hr/> 3024	268 810; $\times 3$ 900 <hr/> 804	325 2880; $\times 9$ 2700 <hr/> 2925
11.	498 3500; $\times 7$ 3500 <hr/> 3486	678 2040; $\times 3$ 2100 <hr/> 2034	721 3600; $\times 5$ 3500 <hr/> 3605

Written Answer the following.

1-6. Copy *Oral* 6-11. Write an estimate for each product. Find each product. Compare the exact answer with the estimated answer.

See Oral 6-11.

Write an open sentence for each problem below. Estimate and record the product. Find the exact product. Answer the problem.

7. Jim bought 3 pairs of socks that cost 79 cents a pair. What was the total cost? $3 \times 79 = \square$; *estimate 240; 237 cents or \$2.37*

8. A cup holds 8 ounces. How many ounces are there in 19 cups? $19 \times 8 = \square$; *estimate 160; 152 ounces*

9. One yard is equivalent to 36 inches. How many inches are equivalent to 9 yards? $9 \times 36 = \square$; *estimate 360; 324 inches*

10. One foot is equivalent to 12 inches. How many inches are equivalent to 7 feet? $7 \times 12 = \square$; *estimate 70; 84 inches*

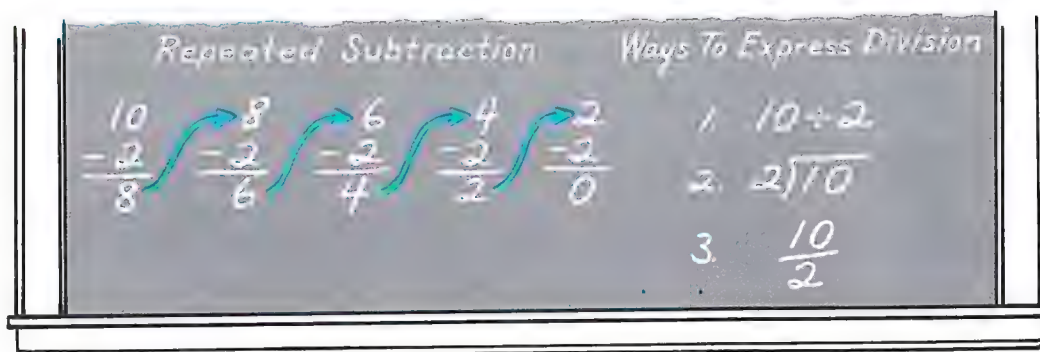
11. One yard is equivalent to 3 feet. How many feet are equivalent to 29 yards? $29 \times 3 = \square$; *estimate 90; 87 feet*

12. One pound is equivalent to 16 ounces. How many ounces are equivalent to 8 pounds? $8 \times 16 = \square$; *estimate 160; 128 ounces*

13. One year is equivalent to 12 months. How many months are equivalent to 5 years? $5 \times 12 = \square$; *estimate 50; 60 months*

14. Each of 5 trucks hauled 14 tons of gravel. Altogether, how many tons of gravel were hauled? $5 \times 14 = \square$; *estimate 50; 70 tons*

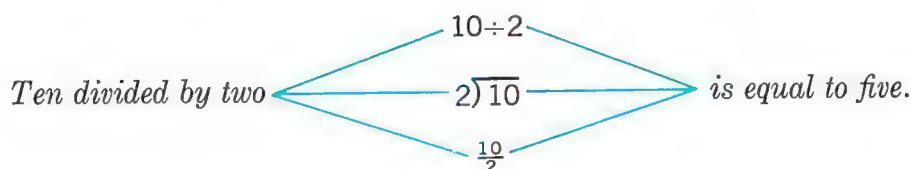
Naming Quotients



The picture above will help you understand what a quotient is. Study the repeated subtraction above. What number do you start with? ¹⁰ How many times is two subtracted until you reach zero? ⁵ How many times was the number 2 contained in the number 10? ⁵ What closed division sentence can you write to express the number of times that the number 2 is contained in the number 10? In $10 \div 2 = 5$, you call 5 the *quotient*.
 $10 \div 2 = 5$

A quotient is the number of times that one number is contained in another number.

A quotient may be expressed in the ways shown below.



Since you already know many products, such as $9 \times 2 = 18$, you also already know many quotients, such as $18 \div 2 = 9$. Why? What is the simplest numeral for $12 \div 3$? What is the simplest numeral for $\frac{18}{6}$? What is the simplest numeral for $4 \overline{)20}$?

Multiplication and division are inverse operations; 4; 3; 5

When numbers are greater, as in $144 \div 9$ or $4 \overline{)64}$, you will want to discover an easy way for finding the simplest numeral for the quotient.

Oral Match each expression in column *a* with the expression in column *b* that names the same number.

<i>a</i>	<i>b</i>
h 1. $\frac{45}{5}$	a. $\frac{48}{8}$
i 2. 8	b. $8 \div 8$
f 3. $28 \div 4$	c. $14 \div 7$
a 4. $8 \overline{)48}$	d. $9 \overline{)45}$
g 5. $3 \overline{)12}$	e. $5 \overline{)15}$
e 6. $15 \div 5$	f. 7
b 7. $\frac{8}{8}$	g. $\frac{12}{3}$
c 8. 2	h. 9
d 9. $45 \div 9$	i. $48 \div 6$

Answer the following.

10. Do each of the following expressions name the same number? Why or why not? **No; $\frac{18}{3}$ names 6, all other expressions name 3.**
 $\frac{18}{3}$, $18 \div 6$, $6 \overline{)18}$, 3

11. Do each of the following expressions name the same number? Why or why not? **No; All expressions except 7 name 6.**
 $7 \overline{)42}$, 7, 6, $\frac{42}{7}$

12. Do each of the following expressions name the same number? Why or why not? **No; All expressions except 8 name 7.**
 7, $\frac{56}{8}$, 8, $56 \div 8$

Written Study exercise 1 below. Then copy and complete each row so that the symbols in each row name the same number.

1. $32 \div 8$	$\frac{32}{8}$	$8 \overline{)32}$	$\frac{4}{1}$
2. $18 \div 9$	$\frac{18}{9}$	$9 \overline{)18}$	$\frac{2}{1}$
3. $63 \div 7$	$\frac{63}{7}$	$7 \overline{)63}$	$\frac{9}{1}$
4. $54 \div 6$	$\frac{54}{6}$	$6 \overline{)54}$	$\frac{9}{1}$
5. $9 \div 9$	$\frac{9}{9}$	$9 \overline{)9}$	$\frac{1}{1}$
6. $16 \div 2$	$\frac{16}{2}$	$2 \overline{)16}$	$\frac{8}{1}$
7. $6 \div 3$	$\frac{6}{3}$	$3 \overline{)6}$	$\frac{2}{1}$
8. $6 \div 6$	$\frac{6}{6}$	$6 \overline{)6}$	$\frac{1}{1}$
9. $28 \div 7$	$\frac{28}{7}$	$7 \overline{)28}$	$\frac{4}{1}$
10. $40 \div 5$	$\frac{40}{5}$	$5 \overline{)40}$	$\frac{8}{1}$

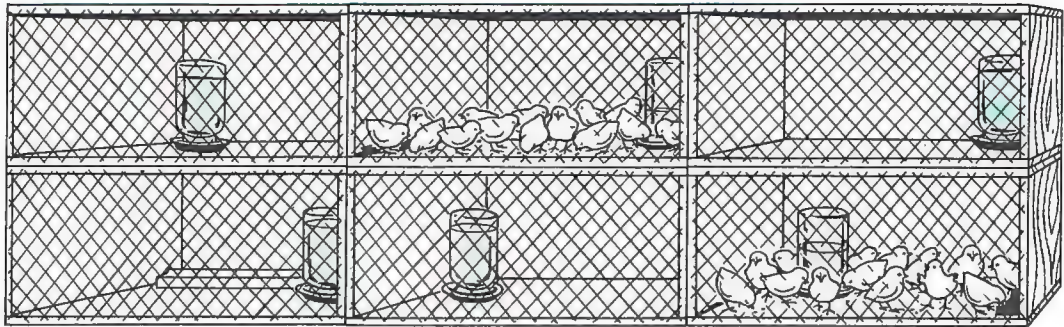
Copy. Replace each \square with the simplest numeral.

- If $7 \times 5 = 35$, then $35 \div 5 = \square$.
- If $6 \times 8 = 48$, then $48 \div 8 = \square$.
- If $9 \times 4 = 36$, then $36 \div 4 = \square$.
- If $4 \times 3 = 12$, then $12 \div 3 = \square$.
- If $5 \times 7 = 35$, then $35 \div 7 = \square$.
- If $3 \times 9 = 27$, then $27 \div 9 = \square$.
- If $3 \times 7 = 21$, then $21 \div 7 = \square$.
- If $7 \times 7 = 49$, then $49 \div 7 = \square$.

Tell why If you know $6 \times 7 = 42$, then you also know $42 \div 7 = 6$ and $42 \div 6 = 7$. Why?

Mult. is commutative. \downarrow Division undoes mult.
 $6 \times 7 = 42 \longrightarrow 42 \div 7 = 6$
 $7 \times 6 = 42 \longrightarrow 42 \div 6 = 7$

The Distributive Property of Division Over Addition



Tom has 84 baby chicks. He plans to put them in the pens shown above. If Tom puts the same number of chicks in each pen, how many chicks will there be in each pen?

Why is $84 \div 6 = \square$ a suitable open sentence for this problem? A set of 84 members is to be separated into 6 equivalent subsets. You can think about solving $84 \div 6 = \square$ as shown below.

$$\begin{aligned}
 84 \div 6 &= (60 + 24) \div 6 \\
 &= (60 \div 6) + (24 \div 6) \\
 &= 10 + 4 \\
 &= 14
 \end{aligned}$$

as $60 + 24$

84 members is to be separated into 6 equivalent subsets.

How is 84 renamed? Why is 84 renamed as $60 + 24$ instead of $80 + 4$ or some other numeral? What is the simplest numeral for $60 \div 6$? For $24 \div 6$? Do $84 \div 6$ and 14 name the same number? Do the arrows above indicate that division is distributed over addition? Is the dividend or the divisor of $84 \div 6$ named as a sum? How many chicks will there be in each pen? 10; 4; Yes; Yes; the dividend; 14 chicks

Distributing division over addition makes it easy to find the simplest numeral for a quotient. In renaming the dividend, it is most convenient to rename the dividend so that both addends are divisible by the divisor.

Division can be distributed over addition when the dividend is named as a sum.

Oral Answer the following.

1. Do you rename the dividend or the divisor as a sum when distributing division over addition? **dividend**

2. Tell why it is best to rename 72 as $40+32$ in finding the simplest numeral for $72 \div 4$. **40 is the greatest multiple of 2 and 10 that is less than 72.**

3. Tell why it is best to rename 36 as $20+16$ in finding the simplest numeral for $36 \div 2$. **20 is the greatest multiple of 2 and 10 that is less than 36.**

Tell the simplest numeral that you would use to replace each \square below that makes each sentence true.

$$\begin{aligned} 4. \quad 42 \div 3 &= (\underline{30} + 12) \div 3 \\ &= (\underline{30} \div 3) + (\underline{12} \div 3) \\ &= 10 + 4 \end{aligned}$$

$$\begin{aligned} 5. \quad 60 \div 4 &= (\underline{40} + 20) \div 4 \\ &= (\underline{40} \div 4) + (\underline{20} \div 4) \\ &= 10 + 5 \end{aligned}$$

$$\begin{aligned} 6. \quad 72 \div 6 &= (\underline{60} + 12) \div 6 \\ &= (\underline{60} \div 6) + (\underline{12} \div 6) \\ &= 10 + 2 \end{aligned}$$

$$\begin{aligned} 7. \quad 96 \div 8 &= (\underline{80} + 16) \div 8 \\ &= (\underline{80} \div 8) + (\underline{16} \div 8) \\ &= 10 + 2 \end{aligned}$$

$$\begin{aligned} 8. \quad 84 \div 3 &= (60 + \underline{24}) \div 3 \\ &= (\underline{60} \div 3) + (\underline{24} \div 3) \\ &= 20 + 8 \end{aligned}$$

Tell how you would rename each dividend to find each quotient.

	a	b	c
9.	$\frac{30+6}{36 \div 3}$	$\frac{40+8}{48 \div 4}$	$\frac{50+10}{60 \div 5}$
10.	$\frac{60+18}{78 \div 6}$	$\frac{70+14}{84 \div 7}$	$\frac{20+16}{36 \div 2}$
11.	$\frac{40+12}{52 \div 4}$	$\frac{50+20}{70 \div 5}$	$\frac{30+9}{39 \div 3}$
12.	$\frac{40+6}{46 \div 2}$	$\frac{80+16}{96 \div 8}$	$\frac{90+9}{99 \div 9}$

Written Copy. Find each quotient by renaming the dividend as a sum of 2 addends so that each addend is divisible by the divisor.

	a	b	c
1.	$87 \div 3 = \square$ ²⁹	$78 \div 6 = \square$ ¹³	$64 \div 4 = \square$ ¹⁶
2.	$48 \div 3 = \square$ ¹⁶	$70 \div 5 = \square$ ¹⁴	$72 \div 6 = \square$ ¹²
3.	$84 \div 7 = \square$ ¹²	$48 \div 4 = \square$ ¹²	$60 \div 5 = \square$ ¹²
4.	$99 \div 9 = \square$ ¹¹	$52 \div 4 = \square$ ¹³	$96 \div 8 = \square$ ¹²
5.	$56 \div 4 = \square$ ¹⁴	$75 \div 5 = \square$ ¹⁵	$65 \div 5 = \square$ ¹³
6.	$60 \div 4 = \square$ ¹⁵	$42 \div 3 = \square$ ¹⁴	$42 \div 2 = \square$ ²¹
7.	$72 \div 3 = \square$ ²⁴	$90 \div 5 = \square$ ¹⁸	$85 \div 5 = \square$ ¹⁷

Write an open division sentence for each multiplication sentence below. Then find each quotient.

	a	b	c
8.	$\square \times 3 = 63$ $63 \div 3 = \square ; 21$	$\square \times 7 = 91$ $91 \div 7 = \square ; 13$	$\square \times 6 = 84$ $84 \div 6 = \square ; 14$
9.	$\square \times 7 = 98$ $98 \div 7 = \square ; 14$	$\square \times 4 = 52$ $52 \div 4 = \square ; 13$	$\square \times 5 = 80$ $80 \div 5 = \square ; 16$

Division (Two-Digit by One-Digit)

The examples given below show how you can solve the open sentence $52 \div 4 = \square$. Study these examples.

A

$$\begin{aligned}
 52 \div 4 &= (40 + 12) \div 4 \\
 &= (40 \div 4) + (12 \div 4) \\
 &= 10 + 3 \\
 &= 13
 \end{aligned}$$

as 40+12

B

$$\begin{array}{r}
 10 + 3 = 13 \\
 4 \overline{)40+12} \\
 \underline{40} 4 \times 10 \\
 12 \\
 \underline{12} 4 \times 3 \\
 0
 \end{array}$$

Yes

How is 52 renamed in **A**? Is 52 renamed the same way in **B**? Why can $(40+12) \div 4$ be written as $(40 \div 4) + (12 \div 4)$? What is the simplest numeral for $40 \div 4$? Where is this shown in **A**? In **B**? What is the tens digit of the quotient numeral? *Division can be distributed over addition; 10; below $40 \div 4$; above the 40; 1* What is the simplest numeral for $12 \div 4$? Where is this shown in **A**? In **B**? What is the ones digit of the quotient numeral? *3; below $12 \div 4$; above the 12; 3*

To find the simplest numeral for a quotient like $52 \div 4$, name 52 in expanded notation and distribute division over addition.

Oral Tell how you would rename each dividend below so that you could divide as shown in **B** above.

Written Copy. Rename each dividend as a sum. Find the simplest numeral for each quotient.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>		<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
1.	$7 \overline{)98}$ <i>70+28</i>	$8 \overline{)88}$ <i>80+8</i>	$3 \overline{)45}$ <i>30+15</i>	$5 \overline{)75}$ <i>50+25</i>	1.	$4 \overline{)60}$ <i>15</i>	$3 \overline{)54}$ <i>18</i>	$2 \overline{)34}$ <i>17</i>	$7 \overline{)91}$ <i>13</i>
2.	$6 \overline{)78}$ <i>60+18</i>	$4 \overline{)48}$ <i>40+8</i>	$6 \overline{)84}$ <i>60+24</i>	$3 \overline{)48}$ <i>30+18</i>	2.	$5 \overline{)60}$ <i>12</i>	$4 \overline{)68}$ <i>17</i>	$7 \overline{)84}$ <i>12</i>	$8 \overline{)88}$ <i>11</i>
3.	$4 \overline{)56}$ <i>40+16</i>	$3 \overline{)54}$ <i>30+24</i>	$5 \overline{)90}$ <i>50+40</i>	$2 \overline{)32}$ <i>20+12</i>	3.	$3 \overline{)48}$ <i>16</i>	$5 \overline{)85}$ <i>17</i>	$6 \overline{)84}$ <i>14</i>	$4 \overline{)52}$ <i>13</i>
4.	$7 \overline{)84}$ <i>70+14</i>	$8 \overline{)96}$ <i>80+16</i>	$5 \overline{)80}$ <i>50+30</i>	$3 \overline{)42}$ <i>30+12</i>	4.	$6 \overline{)90}$ <i>15</i>	$3 \overline{)51}$ <i>17</i>	$3 \overline{)57}$ <i>19</i>	$5 \overline{)95}$ <i>19</i>
5.	$4 \overline{)64}$ <i>40+24</i>	$3 \overline{)57}$ <i>30+27</i>	$6 \overline{)72}$ <i>60+12</i>	$7 \overline{)77}$ <i>70+7</i>	5.	$5 \overline{)55}$ <i>11</i>	$2 \overline{)28}$ <i>14</i>	$5 \overline{)75}$ <i>15</i>	$6 \overline{)96}$ <i>16</i>

Division (Two-Digit by One-Digit)

The example below also shows how you can solve the open sentence $52 \div 4 = \square$. Study this example.

C

$$\begin{array}{r} \text{T} \text{ } | \text{ } \text{O} \\ 1 \\ 4 \overline{) 52} \\ \underline{40} 4 \times 10 \\ 12 52 - 40 \end{array}$$

Since $4 \times 10 < 52$ and $4 \times 20 > 52$, the quotient is between 10 and 20. Why? Write 1 in the tens place of the numeral for the quotient.

D

$$\begin{array}{r} \text{T} \text{ } | \text{ } \text{O} \\ 13 \\ 4 \overline{) 52} \\ \underline{40} \\ 12 \\ \underline{12} 4 \times 3 \\ 0 12 - 12 \end{array}$$

Since $4 \times 3 = 12$, write 3 in the ones place of the numeral for the quotient.

E

$$\begin{array}{r} 13 \\ 4 \overline{) 52} \\ \underline{40} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

In C, how do you obtain the 40? How do you record the 10 in the numeral for the quotient? How do you obtain the 12? 4×10 ; Write 1 in the tens position of the quotient numeral; $52 - 40$

In D, do 4×3 and 12 name the same number? Where do you record the 3 in the numeral for the quotient? Yes; in the ones position of the quotient numeral

If $52 \div 4 = 13$, then $13 \times 4 = 52$. Why? How can you check division results? Multiplication is the inverse of division; The product of the quotient and the divisor should be the dividend.

To check a division result, multiply the quotient by the divisor. This product should be the dividend.

Oral Explain each example below.

See examples C and D above.

a	b	c	d
$\begin{array}{r} 13 \\ 3 \overline{) 39} \\ \underline{30} \\ 9 \\ \underline{9} \\ 0 \end{array}$	$\begin{array}{r} 21 \\ 4 \overline{) 84} \\ \underline{80} \\ 4 \\ \underline{4} \\ 0 \end{array}$	$\begin{array}{r} 17 \\ 5 \overline{) 85} \\ \underline{50} \\ 35 \\ \underline{35} \\ 0 \end{array}$	$\begin{array}{r} 13 \\ 7 \overline{) 91} \\ \underline{70} \\ 21 \\ \underline{21} \\ 0 \end{array}$

Written Copy. Find each quotient.

	a	b	c	d
1.	$\begin{array}{r} 13 \\ 78 \div 6 \end{array}$	$\begin{array}{r} 15 \\ 90 \div 6 \end{array}$	$\begin{array}{r} 12 \\ 84 \div 7 \end{array}$	$\begin{array}{r} 28 \\ 56 \div 2 \end{array}$
2.	$\begin{array}{r} 17 \\ 51 \div 3 \end{array}$	$\begin{array}{r} 16 \\ 48 \div 3 \end{array}$	$\begin{array}{r} 14 \\ 98 \div 7 \end{array}$	$\begin{array}{r} 23 \\ 92 \div 4 \end{array}$
3.	$\begin{array}{r} 14 \\ 70 \div 5 \end{array}$	$\begin{array}{r} 12 \\ 48 \div 4 \end{array}$	$\begin{array}{r} 38 \\ 76 \div 2 \end{array}$	$\begin{array}{r} 19 \\ 95 \div 5 \end{array}$
4.	$\begin{array}{r} 12 \\ 96 \div 8 \end{array}$	$\begin{array}{r} 17 \\ 85 \div 5 \end{array}$	$\begin{array}{r} 19 \\ 57 \div 3 \end{array}$	$\begin{array}{r} 11 \\ 88 \div 8 \end{array}$

Division (Three-Digit by One-Digit)

The example below shows how to solve the open sentence $144 \div 8 = \square$. Is the quotient for $144 \div 8$ less than 100? Why? $8 \times 100 > 144$
 Study this example. Yes

A	B												
<div style="text-align: center; margin-bottom: 10px;"> <table border="1" style="margin: auto;"> <tr><td>H</td><td>T</td><td>O</td></tr> <tr><td>1</td><td></td><td></td></tr> </table> </div> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> $\begin{array}{r} 8 \overline{) 144} \\ \underline{80} \\ 64 \end{array}$ </div> <div> 8×10 $144 - 80$ </div> </div>	H	T	O	1			<div style="text-align: center; margin-bottom: 10px;"> <table border="1" style="margin: auto;"> <tr><td>H</td><td>T</td><td>O</td></tr> <tr><td>1</td><td>8</td><td></td></tr> </table> </div> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> $\begin{array}{r} 8 \overline{) 144} \\ \underline{80} \\ 64 \\ \underline{64} \\ 0 \end{array}$ </div> <div> 8×8 $64 - 64$ </div> </div>	H	T	O	1	8	
H	T	O											
1													
H	T	O											
1	8												
<p>Since $8 \times 10 < 144$ and $8 \times 20 > 144$, the quotient is between 10 and 20. Write 1 in the tens place of the numeral for the quotient.</p>	<p>Since $8 \times 8 = 64$, write 8 in the ones place of the numeral for the quotient.</p>												

In **A**, how do you know the quotient is between 10 and 20? How do you obtain the 80? The 64? How do you record the 10 in the numeral for the quotient? $8 \times 10 < 144$ and $8 \times 20 > 144$; 8×10 ; $144 - 80$; Write 1 in the tens position of the quotient numeral.

In **B**, do 8×8 and 64 name the same number? Where do you record the 8 in the numeral for the quotient? How do you obtain 0? Yes; in the ones position; $64 - 64 = 0$

How would you check the division result in **C**? Find the product of 8 and 18. The result should be 144.

Oral Explain each step in the examples below. See examples A, B, and C above.

Written Copy. Find each quotient. Check each result.

a	b	c	a	b	c	d
$\begin{array}{r} 24 \\ 6 \overline{) 144} \\ \underline{120} \\ 24 \\ \underline{24} \\ 0 \end{array}$	$\begin{array}{r} 25 \\ 5 \overline{) 125} \\ \underline{100} \\ 25 \\ \underline{25} \\ 0 \end{array}$	$\begin{array}{r} 64 \\ 3 \overline{) 192} \\ \underline{180} \\ 12 \\ \underline{12} \\ 0 \end{array}$	1. $\begin{array}{r} 25 \\ 6 \overline{) 150} \\ \underline{120} \\ 30 \\ \underline{30} \\ 0 \end{array}$	$\begin{array}{r} 23 \\ 7 \overline{) 161} \\ \underline{140} \\ 21 \\ \underline{21} \\ 0 \end{array}$	$\begin{array}{r} 22 \\ 8 \overline{) 176} \\ \underline{160} \\ 16 \\ \underline{16} \\ 0 \end{array}$	$\begin{array}{r} 24 \\ 5 \overline{) 120} \\ \underline{100} \\ 20 \\ \underline{20} \\ 0 \end{array}$
			2. $\begin{array}{r} 37 \\ 3 \overline{) 111} \\ \underline{111} \\ 0 \end{array}$	$\begin{array}{r} 49 \\ 4 \overline{) 196} \\ \underline{196} \\ 0 \end{array}$	$\begin{array}{r} 86 \\ 2 \overline{) 172} \\ \underline{172} \\ 0 \end{array}$	$\begin{array}{r} 82 \\ 9 \overline{) 738} \\ \underline{738} \\ 0 \end{array}$
			3. $\begin{array}{r} 83 \\ 8 \overline{) 664} \\ \underline{640} \\ 24 \\ \underline{24} \\ 0 \end{array}$	$\begin{array}{r} 52 \\ 7 \overline{) 364} \\ \underline{350} \\ 14 \\ \underline{14} \\ 0 \end{array}$	$\begin{array}{r} 65 \\ 3 \overline{) 195} \\ \underline{180} \\ 15 \\ \underline{15} \\ 0 \end{array}$	$\begin{array}{r} 47 \\ 5 \overline{) 235} \\ \underline{235} \\ 0 \end{array}$

Division (Three-Digit by One-Digit)

The example below shows how to solve the open division sentence $992 \div 8 = \square$. Study this example.

A				B			
	H	T	O				
	1	2	4				
8)	9	9	2				
	8	0	0				
	1	9	2				
	1	6	0				
		3	2				
		3	2				
			0				

In A, tell the thinking steps you would take to find the digit 1 of the numeral for the quotient. Tell the thinking steps you would take to find the digit 2 of the numeral for the quotient. Tell the thinking steps you would take to find the digit 4 of the numeral for the quotient. What is the simplest numeral for $992 \div 8$? **124**

Oral Tell the steps you would take to find the hundreds digit of the numeral for each quotient. See example above for style.

- | | a | b | c | d |
|----|---------------------|---------------------|---------------------|---------------------|
| 1. | $6 \overline{)612}$ | $5 \overline{)820}$ | $7 \overline{)847}$ | $8 \overline{)968}$ |
| 2. | $3 \overline{)801}$ | $2 \overline{)932}$ | $4 \overline{)932}$ | $5 \overline{)735}$ |
| 3. | $6 \overline{)846}$ | $3 \overline{)702}$ | $5 \overline{)930}$ | $3 \overline{)522}$ |
| 4. | $4 \overline{)620}$ | $8 \overline{)992}$ | $7 \overline{)966}$ | $8 \overline{)928}$ |
| 5. | $7 \overline{)833}$ | $4 \overline{)732}$ | $5 \overline{)875}$ | $2 \overline{)732}$ |

Written Copy. Find each quotient.

- | | a | b | c | d |
|----|---------------------|---------------------|---------------------|---------------------|
| 1. | $6 \overline{)618}$ | $5 \overline{)825}$ | $7 \overline{)854}$ | $8 \overline{)976}$ |
| 2. | $3 \overline{)804}$ | $2 \overline{)834}$ | $4 \overline{)936}$ | $5 \overline{)740}$ |
| 3. | $3 \overline{)705}$ | $6 \overline{)852}$ | $5 \overline{)955}$ | $3 \overline{)528}$ |
| 4. | $8 \overline{)896}$ | $3 \overline{)816}$ | $2 \overline{)870}$ | $6 \overline{)864}$ |
| 5. | $4 \overline{)620}$ | $8 \overline{)928}$ | $6 \overline{)912}$ | $7 \overline{)938}$ |

Tell how How can you tell when a three-digit number is divisible by 5? If the ones digit of the decimal numeral for that number is **127** 0 or 5.

Division (Four-Digit by One-Digit)

The example below shows how you can solve which open division sentence? Is $9 \times 1000 > 2862$ a true sentence? Should you write a digit in the thousands place of the numeral for the quotient? Why or why not? $2862 \div 9 = \square$; Yes; No; The quotient is less than 1000.

Thinking Steps

$$\begin{array}{r}
 318 \\
 9 \overline{) 2862} \quad \text{—————} \quad 9 \times 300 < 2862 \text{ and } 9 \times 400 > 2862 \\
 \underline{2700} \quad \text{—————} \quad 9 \times 300 = 2700 \\
 162 \quad \text{—————} \quad 9 \times 10 < 162 \text{ and } 9 \times 20 > 162 \\
 \underline{90} \quad \text{—————} \quad 9 \times 10 = 90 \\
 \underline{72} \quad \text{—————} \quad 9 \times 8 = 72 \\
 \underline{72} \\
 0
 \end{array}$$

Tell the thinking steps you would take to find the digit 3 of the numeral for the quotient. Tell the thinking steps you would take to find the digit 1 of the numeral for the quotient. Tell the thinking steps you would take to find the digit 8 of the numeral for the quotient. What is the simplest numeral for $2862 \div 9$? 318

Oral Tell the steps you would take to find the hundreds digit of the numeral for each quotient.

See page T128.

- | | <i>a</i> | <i>b</i> | <i>c</i> |
|--------------------------|-----------------------|-----------------------|----------|
| 1. $9 \overline{) 7317}$ | $5 \overline{) 4690}$ | $8 \overline{) 6536}$ | |
| 2. $6 \overline{) 3732}$ | $4 \overline{) 2932}$ | $3 \overline{) 2298}$ | |
| 3. $7 \overline{) 4837}$ | $6 \overline{) 4032}$ | $8 \overline{) 5704}$ | |
| 4. $5 \overline{) 4865}$ | $4 \overline{) 3856}$ | $3 \overline{) 2928}$ | |
| 5. $9 \overline{) 7578}$ | $5 \overline{) 4875}$ | $7 \overline{) 5124}$ | |
| 6. $2 \overline{) 1972}$ | $8 \overline{) 6336}$ | $4 \overline{) 3772}$ | |

Written Copy. Find each quotient.

- | | <i>a</i> | <i>b</i> | <i>c</i> |
|--------------------------|-----------------------|-----------------------|----------|
| 1. $7 \overline{) 6524}$ | $4 \overline{) 3856}$ | $3 \overline{) 2928}$ | |
| 2. $8 \overline{) 7488}$ | $7 \overline{) 4907}$ | $2 \overline{) 1588}$ | |
| 3. $3 \overline{) 1665}$ | $5 \overline{) 4220}$ | $9 \overline{) 5238}$ | |
| 4. $6 \overline{) 4152}$ | $7 \overline{) 4865}$ | $8 \overline{) 6664}$ | |
| 5. $4 \overline{) 3112}$ | $3 \overline{) 1965}$ | $9 \overline{) 8847}$ | |
| 6. $6 \overline{) 4572}$ | $5 \overline{) 4320}$ | $2 \overline{) 1312}$ | |
| 7. $9 \overline{) 8487}$ | $7 \overline{) 5754}$ | $4 \overline{) 3064}$ | |

Division (Four-Digit by One-Digit)

The example below shows how you can solve which open division sentence? Study this example. $9344 \div 7 = \square$

Thinking Steps

$$\begin{array}{r}
 1342 \\
 7 \overline{)9344} \\
 \underline{7000} \\
 2344 \\
 \underline{2100} \\
 294 \\
 \underline{280} \\
 14 \\
 \underline{14} \\
 0
 \end{array}$$

$7 \times 1000 < 9344$ and $7 \times 2000 > 9344$
 $7 \times 1000 = 7000$
 $7 \times 300 < 2344$ and $7 \times 400 > 2344$
 $7 \times 300 = 2100$
 $7 \times 40 < 294$ and $7 \times 50 > 294$
 $7 \times 40 = 280$
 $7 \times 2 = 14$

Tell the thinking steps you would take to find the digit 1 of the numeral for the quotient; the digit 3 of the numeral for the quotient; the digit 4 of the numeral for the quotient; the digit 2 of the numeral for the quotient. What is the simplest numeral for $9344 \div 7$? **1342**

Oral Tell the steps you would take to find the thousands digit of the numeral for each quotient.

See page T129.

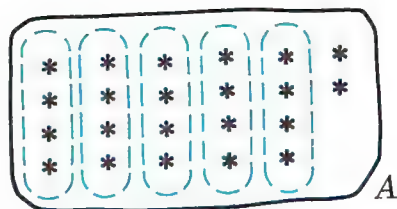
- | | a | b | c |
|----|----------------------|----------------------|----------------------|
| 1. | $9 \overline{)9189}$ | $6 \overline{)7254}$ | $5 \overline{)6115}$ |
| 2. | $3 \overline{)9843}$ | $2 \overline{)9642}$ | $4 \overline{)9328}$ |
| 3. | $5 \overline{)9860}$ | $3 \overline{)8439}$ | $2 \overline{)7788}$ |
| 4. | $4 \overline{)4252}$ | $6 \overline{)7392}$ | $5 \overline{)6315}$ |
| 5. | $8 \overline{)9768}$ | $4 \overline{)9132}$ | $3 \overline{)9621}$ |
| 6. | $7 \overline{)9044}$ | $8 \overline{)9856}$ | $7 \overline{)8085}$ |

Written Copy. Find each quotient.

- | | a | b | c |
|----|----------------------|----------------------|----------------------|
| 1. | $3 \overline{)7653}$ | $2 \overline{)5848}$ | $4 \overline{)7352}$ |
| 2. | $5 \overline{)7210}$ | $6 \overline{)8388}$ | $3 \overline{)9444}$ |
| 3. | $3 \overline{)6243}$ | $9 \overline{)9198}$ | $7 \overline{)9247}$ |
| 4. | $3 \overline{)5736}$ | $2 \overline{)7492}$ | $4 \overline{)6448}$ |
| 5. | $2 \overline{)9544}$ | $5 \overline{)8635}$ | $4 \overline{)7356}$ |
| 6. | $4 \overline{)5776}$ | $3 \overline{)8562}$ | $5 \overline{)8055}$ |
| 7. | $8 \overline{)9072}$ | $6 \overline{)7056}$ | $7 \overline{)8001}$ |

Remainders in Division

Can set A be separated into subsets of 4 stars each? Why or why not? If you remove the subsets of 4, how many single stars will remain? **No; 2 stars are left over; 2**

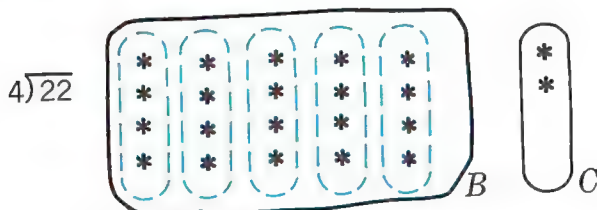


Some of the numerals are circled on the number line below. These numerals name multiples of 4. Are they all divisible by 4? **Yes**



Is 22 a multiple of 4? Is 22 divisible by 4? What is the greatest number less than 22 that is divisible by 4? **No; No; 20**

You can rearrange the stars in set A and think about $22 \div 4$ as shown below.



$4 \overline{)20+2}$
 Is 20 divisible by 4? **Yes**
 Is 2 less than 4? **Yes**

Can set B be separated into subsets of 4 each? Is 20 divisible by 4? Can Set C be separated into subsets of 4 each? Will the addend 2 in $20+2$ remain undivided? **Yes; Yes; No; Yes**

In division, the number that remains undivided is called the **remainder**.

Study how the quotient and the remainder are recorded below.

$$\begin{array}{r}
 5 \text{ ————— quotient ————— } 5 \text{ r}2 \\
 4 \overline{)20+2} \qquad \qquad \qquad 4 \overline{)22} \\
 \underline{20} \qquad \qquad \qquad \underline{20} \\
 2 \text{ ————— remainder ————— } 2
 \end{array}$$

Oral Tell the number less than, but nearest to, the dividend that is divisible by the divisor.

Written Copy. Find each quotient and remainder.

a	b	c	d	a	b	c
1. $4\overline{)14}$ 12	5 $\overline{)18}$ 15	6 $\overline{)23}$ 18	9 $\overline{)35}$ 27	1. $6\overline{)25}$ 4r1	3 $\overline{)138}$ 46r0	7 $\overline{)1461}$ 208r5
2. $8\overline{)68}$ 64	2 $\overline{)17}$ 16	8 $\overline{)82}$ 80	5 $\overline{)13}$ 10	2. $8\overline{)75}$ 9r3	2 $\overline{)317}$ 158r1	5 $\overline{)3407}$ 681r2
3. $4\overline{)11}$ 8	7 $\overline{)48}$ 42	6 $\overline{)62}$ 60	5 $\overline{)54}$ 50	3. $3\overline{)35}$ 11r2	9 $\overline{)845}$ 93r8	4 $\overline{)6211}$ 1552r3
4. $8\overline{)79}$ 72	4 $\overline{)42}$ 40	5 $\overline{)21}$ 20	9 $\overline{)95}$ 90	4. $2\overline{)27}$ 13r1	7 $\overline{)642}$ 91r5	8 $\overline{)9001}$ 1125r1
5. $3\overline{)29}$ 27	6 $\overline{)41}$ 36	2 $\overline{)21}$ 20	3 $\overline{)61}$ 60	5. $9\overline{)91}$ 10r1	5 $\overline{)249}$ 49r4	6 $\overline{)7041}$ 1173r3
				6. $7\overline{)85}$ 12r1	4 $\overline{)338}$ 84r2	9 $\overline{)9010}$ 1001r1
				7. $5\overline{)62}$ 12r2	3 $\overline{)761}$ 253r2	2 $\overline{)2011}$ 1005r1
				8. $4\overline{)51}$ 12r3	8 $\overline{)362}$ 45r2	3 $\overline{)6472}$ 2157r1
				9. $3\overline{)53}$ 17r2	6 $\overline{)526}$ 87r4	5 $\overline{)7819}$ 1563r4
				10. $8\overline{)65}$ 8r1	7 $\overline{)446}$ 63r5	4 $\overline{)3806}$ 951r2

Tell the quotient and the remainder for each division below.

a	b	c	d	a	b	c
6. $4\overline{)28}$ 7r0	5 $\overline{)14}$ 2r4	6 $\overline{)24}$ 4r0	9 $\overline{)37}$ 4r1	1. $6\overline{)25}$ 4r1	3 $\overline{)138}$ 46r0	7 $\overline{)1461}$ 208r5
7. $8\overline{)69}$ 8r5	2 $\overline{)19}$ 9r1	8 $\overline{)87}$ 10r7	5 $\overline{)12}$ 2r2	2. $8\overline{)75}$ 9r3	2 $\overline{)317}$ 158r1	5 $\overline{)3407}$ 681r2
8. $4\overline{)17}$ 4r1	7 $\overline{)50}$ 7r1	6 $\overline{)64}$ 10r4	5 $\overline{)49}$ 9r4	3. $3\overline{)35}$ 11r2	9 $\overline{)845}$ 93r8	4 $\overline{)6211}$ 1552r3
9. $8\overline{)75}$ 9r3	4 $\overline{)41}$ 10r1	5 $\overline{)19}$ 3r4	9 $\overline{)84}$ 9r3	4. $2\overline{)27}$ 13r1	7 $\overline{)642}$ 91r5	8 $\overline{)9001}$ 1125r1
10. $3\overline{)25}$ 8r1	6 $\overline{)45}$ 7r3	2 $\overline{)15}$ 7r1	3 $\overline{)31}$ 10r1	5. $9\overline{)91}$ 10r1	5 $\overline{)249}$ 49r4	6 $\overline{)7041}$ 1173r3

Answer the following questions.

- If the divisor is 4, what are all the possible remainders? 0, 1, 2, and 3
- If the divisor is 5, what are all the possible remainders? 0, 1, 2, 3, and 4
- If the divisor is 9, what are all the possible remainders? 0, 1, 2, 3, 4, 5, 6, 7, and 8

Can you do this? Draw set pictures to represent each division shown below.

a	b	c
$5\overline{)22}$	$3\overline{)19}$	$6\overline{)35}$
$\frac{20}{2}$	$\frac{18}{1}$	$\frac{30}{5}$

Tell why You should keep dividing until the remainder is less than the divisor. Why? The remainder should always be less than the divisor.

Tell how If a remainder is 6, how do you know that the divisor is greater than 6? If the division is complete, the divisor is greater than the remainder.

Solving Problems

Write an open sentence for each problem. Solve the open sentence. Answer the problem.

1. A marching band contained 8 rows with the same number of members in each row. If there were 96 members in the band, how many were in each row? $96 \div 8 = \square$;
12 members

2. A marching band had 112 members. It had 8 members in each row. How many rows of members did the band have? $112 \div 8 = \square$; 14 rows

3. Six buses from one school brought 456 pupils to see the football game. If all six buses carried the same number of pupils, how many pupils rode on each bus?
 $456 \div 6 = \square$; 76 pupils

4. Stanton is 112 miles from Newberry. A bus can travel 7 miles on each gallon of gasoline. How many gallons of gasoline are needed to drive from Stanton to Newberry?
 $112 \div 7 = \square$; 16 gallons

5. Mr. Johnson drove 80 miles. In doing so, he used 5 gallons of gasoline. How many miles did he travel on each gallon of gasoline?
 $80 \div 5 = \square$; 16 miles

6. A milkman delivers 8244 quarts of milk in 6 days. If the milkman delivered an equal amount of milk each day, how many quarts of milk were delivered each day?

$$8244 \div 6 = \square ; 1374 \text{ quarts}$$

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7. One mile is equivalent to 5280 feet. One yard is equivalent to 3 feet. How many yards are equivalent to 5280 feet or 1 mile?

$$5280 \div 3 = \square ; 1760 \text{ yards}$$

8. There are 144 items packed in a box in 4 equal layers. How many items are contained in each layer?

$$144 \div 4 = \square ; 36 \text{ items}$$

9. Mr. Buber drove 207 miles on 9 gallons of gasoline. How far did he drive on 1 gallon of gasoline?

$$207 \div 9 = \square ; 23 \text{ miles}$$

10. Mr. Hauser borrowed \$1248. He is to pay it back in 8 equal monthly payments. How much money is he to pay back each month?

$$1248 \div 8 = \square ; \$156$$

11. A company packs 4 gallons of paint in a box. A store ordered 532 gallons of paint. How many boxes of paint did the store receive?

$$532 \div 4 = \square ; 133 \text{ boxes}$$

12. Mr. Clark is to pay \$504 in 6 equal monthly payments. How much money will Mr. Clark pay in one monthly payment?

$$504 \div 6 = \square ;$$

\$84

13. An airplane flew 1960 miles in 7 hours. If the airplane flew at the same rate each of those 7 hours, how many miles per hour was the airplane flying?

$$1960 \div 7 = \square ; 280 \text{ miles per hour}$$

Can you do this? Replace \square with a numeral to make the sentence below true.

$$146 \div 8 = (8 \times 18) + \square$$

Practice in Division

Part 1 Copy. Find each quotient.

a	b	c
1. $4 \overline{)44}$ <u>18</u>	$6 \overline{)72}$ <u>24</u>	$8 \overline{)96}$ <u>17</u>
2. $4 \overline{)72}$ <u>29</u>	$3 \overline{)72}$ <u>28</u>	$5 \overline{)85}$ <u>24</u>
3. $2 \overline{)58}$ <u>11</u>	$3 \overline{)84}$ <u>12</u>	$4 \overline{)96}$ <u>18</u>
4. $9 \overline{)99}$ <u>15</u>	$7 \overline{)84}$ <u>16</u>	$5 \overline{)90}$ <u>13</u>
5. $5 \overline{)75}$	$6 \overline{)96}$	$7 \overline{)91}$

Part 2 Copy. Find each quotient.

a	b	c
1. $3 \overline{)108}$ <u>54</u>	$6 \overline{)132}$ <u>63</u>	$4 \overline{)192}$ <u>25</u>
2. $7 \overline{)378}$ <u>234</u>	$8 \overline{)504}$ <u>175</u>	$5 \overline{)125}$ <u>146</u>
3. $4 \overline{)936}$ <u>106</u>	$3 \overline{)525}$ <u>132</u>	$6 \overline{)876}$ <u>499</u>
4. $6 \overline{)636}$ <u>94</u>	$7 \overline{)924}$ <u>224</u>	$2 \overline{)998}$ <u>26</u>
5. $9 \overline{)846}$	$4 \overline{)896}$	$5 \overline{)130}$

Part 3 Copy. Find each quotient.

a	b	c
1. $9 \overline{)7398}$ <u>660</u>	$8 \overline{)7392}$ <u>799</u>	$4 \overline{)3124}$ <u>966</u>
2. $3 \overline{)1980}$ <u>2284</u>	$2 \overline{)1598}$ <u>1222</u>	$4 \overline{)3864}$ <u>1223</u>
3. $4 \overline{)9136}$ <u>2535</u>	$8 \overline{)9776}$ <u>1613</u>	$6 \overline{)7338}$ <u>1612</u>
4. $3 \overline{)7605}$ <u>1358</u>	$4 \overline{)6452}$ <u>942</u>	$5 \overline{)8060}$ <u>1652</u>
5. $7 \overline{)9506}$	$9 \overline{)8478}$	$2 \overline{)3304}$

Part 4 Copy. Find each quotient and remainder.

a	b	c
1. $3 \overline{)44}$ <u>14r2</u>	$8 \overline{)109}$ <u>13r5</u>	$5 \overline{)8321}$ <u>1664r1</u>
2. $4 \overline{)95}$ <u>23r3</u>	$6 \overline{)904}$ <u>150r4</u>	$3 \overline{)7004}$ <u>2334r2</u>
3. $7 \overline{)68}$ <u>9r5</u>	$7 \overline{)807}$ <u>115r2</u>	$6 \overline{)9076}$ <u>1512r4</u>
4. $5 \overline{)81}$ <u>16r1</u>	$4 \overline{)937}$ <u>234r1</u>	$8 \overline{)9001}$ <u>1125r1</u>
5. $6 \overline{)85}$ <u>14r1</u>	$9 \overline{)858}$ <u>95r3</u>	$7 \overline{)9686}$ <u>1383r5</u>

Part 5 Write an open sentence for each problem. Solve the open sentence. Answer the problem.

1. Sally has 65 cents. What is the greatest number of 5-cent candy bars she can buy with this amount of money? $65 \div 5 = \square$; 13 candy bars

2. A newsboy delivers 270 newspapers in 5 days. If he delivers the same number of newspapers each day, how many newspapers does he deliver each day? $270 \div 5 = \square$; 54 newspapers

3. What is the greatest number of 5-cent stamps that Len could buy with 85 cents? $85 \div 5 = \square$; 17 stamps

Can you do this? What number is named by $(8 \div 4) \div 2$? What number is named by $8 \div (4 \div 2)$? Is division associative? Why or why not? 1; 4; No; $(8 \div 4) \div 2$ and $8 \div (4 \div 2)$ name different numbers.

Solving Problems

Write an open sentence for each problem. Solve the open sentence. Answer the problem.

1. The Boy Scouts had a paper drive. Troop 13 collected 4312 pounds of paper. Troop 17 collected 5687 pounds, and Troop 20 collected 4769 pounds. In all, how many pounds of paper did the three troops collect?

$$4312+5687+4769=\square; 14,768 \text{ pounds}$$

2. The population of Grafton is 37,462. It is expected that the next census will show an increase to 49,000. What is the expected increase? $49000-37462=\square; 11,538$ people

3. A truck hauled coal to a heating plant. On the first trip the truck hauled 16,240 pounds. On the second trip it hauled 15,842 pounds. On the third trip it hauled 16,152 pounds. In all, how many pounds of coal were hauled? $16240+15842+16152=\square; 48,234$ pounds

4. Quart cans of paint are shipped 6 in a box. A paint store ordered 744 quarts of paint. How many boxes of paint should the paint store receive? $744\div 6=\square; 124$ boxes

5. In 9 basketball games Howard scored 162 points. Assuming he scored the same number of points in each game, how many points did he score in each game? $162\div 9=\square; 18$ points

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6. On three successive Sundays, Jack sold 324, 291, and 311 newspapers. In all, how many Sunday newspapers did Jack sell? $324+291+311=\square; 926$ newspapers

7. Mr. Baker earns \$8245 a year. Mr. Everett earns \$9430 a year. Find the difference in their annual salaries. $9430-8245=\square; \$1185$

8. Larry delivered 312 newspapers in 6 days. If he delivered the same number of newspapers each day, how many newspapers did he deliver each day? $312\div 6=\square; 52$ newspapers

9. John purchased 4 packs of paper with 125 sheets of paper in each pack. How many sheets of paper does he have in all? $4\times 125=\square; 500$ sheets

10. Suppose a guided missile travels at a steady rate of 1250 miles an hour. How far will it travel in 6 hours? $6\times 1250=\square; 7500$ miles

11. Mary has a ribbon 72 inches long. She cut it into pieces 6 inches long. How many pieces 6 inches long does she now have? $72\div 6=\square; 12$ pieces

Can you do this? Compose a story problem for each of the following.

Answers will vary.

a

b

1. $5\times 5280=\square$ $72\div 3=\square$

2. $3\times 144=\square$ $5280\div 3=\square$

Checkup Time

The numerals in () tell the pages where you can turn for help.

Important Ideas

1. Multiplication can be distributed over addition. (110)
2. To find a product such as 8×24 , rename 24 in expanded notation and distribute multiplication over addition. (112)
3. In multiplication you can estimate a product by rounding off one or both factors. (118)
4. Division can be distributed over addition when the dividend is named as a sum. (122)
5. In division, the number that remains undivided is called the remainder. (130)

Words to Know

1. Distributive property of multiplication over addition (110)
2. Multiplicand, multiplier, product (112)
3. Estimate (118)
4. Quotient (120)
5. Distributive property of division over addition (122)
6. Remainder (130)

Questions to Discuss

See page T135.

1. How does the distributive property of multiplication over addition make finding the simplest numeral for 8×14 easy? (112)
2. Explain how you would estimate the product for $3 \times 69 = \square$. (118)
3. How does the distributive property of division over addition make finding the simplest numeral for $96 \div 6$ easy? (122)
4. Explain how you find the tens digit of the quotient numeral for $6 \overline{)576}$. (126)

Written Practice

Copy. Find each product or quotient.

	<i>a</i>	<i>b</i>
1.	$\begin{array}{r} 15 \\ \times 8 \\ \hline 120 \end{array}$	$\begin{array}{r} 49 \\ \times 3 \\ \hline 147 \end{array}$
2.	$\begin{array}{r} 146 \\ \times 4 \\ \hline 584 \end{array}$	$\begin{array}{r} 1422 \\ \times 3 \\ \hline 4266 \end{array}$
3.	$\begin{array}{r} 7 \\ 9 \overline{)63} \\ 19 \end{array}$	$\begin{array}{r} 29 \\ 3 \overline{)87} \\ 17 \end{array}$
4.	$\begin{array}{r} 57 \\ 3 \overline{)57} \\ 57 \end{array}$	$\begin{array}{r} 176 \\ 5 \overline{)85} \\ 176 \end{array}$
5.	$\begin{array}{r} 57 \\ 5 \overline{)285} \\ 285 \end{array}$	$\begin{array}{r} 176 \\ 3 \overline{)528} \\ 528 \end{array}$

Self-Evaluation

Part 1 Copy. Find each product.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 18 \\ \times 9 \\ \hline 162 \end{array}$	$\begin{array}{r} 162 \\ \times 4 \\ \hline 648 \end{array}$	$\begin{array}{r} 1428 \\ \times 4 \\ \hline 5712 \end{array}$
2.	$\begin{array}{r} 16 \\ \times 8 \\ \hline 128 \end{array}$	$\begin{array}{r} 149 \\ \times 6 \\ \hline 894 \end{array}$	$\begin{array}{r} 1760 \\ \times 6 \\ \hline 10560 \end{array}$
3.	$\begin{array}{r} 35 \\ \times 5 \\ \hline 175 \end{array}$	$\begin{array}{r} 133 \\ \times 9 \\ \hline 1197 \end{array}$	$\begin{array}{r} 1338 \\ \times 5 \\ \hline 6690 \end{array}$
4.	$\begin{array}{r} 72 \\ \times 8 \\ \hline 576 \end{array}$	$\begin{array}{r} 127 \\ \times 8 \\ \hline 1016 \end{array}$	$\begin{array}{r} 1428 \\ \times 3 \\ \hline 4284 \end{array}$

Part 2 Copy. Find each quotient.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 12 \\ 7 \overline{)84} \end{array}$	$\begin{array}{r} 71 \\ 9 \overline{)639} \end{array}$	$\begin{array}{r} 4070 \\ 2 \overline{)8140} \end{array}$
2.	$\begin{array}{r} 25 \\ 3 \overline{)75} \end{array}$	$\begin{array}{r} 82 \\ 6 \overline{)492} \end{array}$	$\begin{array}{r} 1081 \\ 5 \overline{)5405} \end{array}$
3.	$\begin{array}{r} 16 \\ 6 \overline{)96} \end{array}$	$\begin{array}{r} 78 \\ 7 \overline{)546} \end{array}$	$\begin{array}{r} 1031 \\ 4 \overline{)4124} \end{array}$
4.	$\begin{array}{r} 28 \\ 3 \overline{)84} \end{array}$	$\begin{array}{r} 86 \\ 8 \overline{)688} \end{array}$	$\begin{array}{r} 1121 \\ 6 \overline{)6726} \end{array}$

Part 3 Estimate each of the following products. Then find the exact product. **Only exact answers shown.**

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 69 \\ \times 3 \\ \hline 207 \end{array}$	$\begin{array}{r} 996 \\ \times 4 \\ \hline 3984 \end{array}$	$\begin{array}{r} 1989 \\ \times 6 \\ \hline 11934 \end{array}$
2.	$\begin{array}{r} 95 \\ \times 7 \\ \hline 665 \end{array}$	$\begin{array}{r} 893 \\ \times 8 \\ \hline 7144 \end{array}$	$\begin{array}{r} 1296 \\ \times 3 \\ \hline 3888 \end{array}$

3.	$\begin{array}{r} 71 \\ \times 8 \\ \hline 568 \end{array}$	$\begin{array}{r} 902 \\ \times 6 \\ \hline 5412 \end{array}$	$\begin{array}{r} 1004 \\ \times 5 \\ \hline 5020 \end{array}$
4.	$\begin{array}{r} 82 \\ \times 6 \\ \hline 492 \end{array}$	$\begin{array}{r} 291 \\ \times 6 \\ \hline 1746 \end{array}$	$\begin{array}{r} 1096 \\ \times 3 \\ \hline 3288 \end{array}$

Part 4 Copy. Find each product or quotient and remainder.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 17 \\ \times 6 \\ \hline 102 \\ 13 \end{array}$	$\begin{array}{r} 143 \\ \times 4 \\ \hline 572 \\ 72 \end{array}$	$\begin{array}{r} 1432 \\ \times 5 \\ \hline 7160 \\ 4071 \end{array}$
2.	$7 \overline{)91}$	$9 \overline{)648}$	$2 \overline{)8142}$
3.	$\begin{array}{r} 28 \\ \times 5 \\ \hline 140 \end{array}$	$\begin{array}{r} 912 \\ \times 3 \\ \hline 2736 \end{array}$	$\begin{array}{r} 1048 \\ \times 2 \\ \hline 2096 \end{array}$
4.	$\begin{array}{r} 31 \\ 3 \overline{)93} \end{array}$	$\begin{array}{r} 105r6 \\ 8 \overline{)846} \end{array}$	$\begin{array}{r} 1123r4 \\ 6 \overline{)6742} \end{array}$
5.	$\begin{array}{r} 45 \\ \times 4 \\ \hline 180 \end{array}$	$\begin{array}{r} 873 \\ \times 5 \\ \hline 4365 \end{array}$	$\begin{array}{r} 1391 \\ \times 3 \\ \hline 4173 \end{array}$

Part 5 Copy. Find each quotient and remainder.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 26 \\ 3 \overline{)78} \end{array}$	$\begin{array}{r} 232r3 \\ 4 \overline{)931} \end{array}$	$\begin{array}{r} 1367r1 \\ 6 \overline{)8203} \end{array}$
2.	$\begin{array}{r} 29r1 \\ 2 \overline{)59} \end{array}$	$\begin{array}{r} 122r3 \\ 6 \overline{)735} \end{array}$	$\begin{array}{r} 2533r2 \\ 3 \overline{)7601} \end{array}$
3.	$\begin{array}{r} 22r3 \\ 4 \overline{)91} \end{array}$	$\begin{array}{r} 121r2 \\ 5 \overline{)607} \end{array}$	$\begin{array}{r} 1321r3 \\ 7 \overline{)9250} \end{array}$
4.	$\begin{array}{r} 16r4 \\ 5 \overline{)84} \end{array}$	$\begin{array}{r} 97r2 \\ 9 \overline{)875} \end{array}$	$\begin{array}{r} 932r1 \\ 8 \overline{)7457} \end{array}$
5.	$\begin{array}{r} 16r1 \\ 6 \overline{)97} \end{array}$	$\begin{array}{r} 66r3 \\ 7 \overline{)465} \end{array}$	$\begin{array}{r} 1870 \\ 3 \overline{)5610} \end{array}$

Chapter 6

PRACTICE IN MULTIPLICATION AND DIVISION

Multiples of Ten as Factors

A

$$\begin{aligned} 36 \times 20 &= 36 \times (2 \times 10) \\ &= (36 \times 2) \times 10 \\ &= 72 \times 10 \\ &= 720 \end{aligned}$$

B

$$\begin{aligned} 50 \times 30 &= (5 \times 10) \times (3 \times 10) \\ &= (5 \times 3) \times (10 \times 10) \\ &= 15 \times 100 \\ &= 1500 \end{aligned}$$

A way to find the simplest numeral for 36×20 is shown in **A**. How is 20 renamed? Then what property of multiplication is used? What is the simplest numeral for 36×2 ? What is the simplest numeral for 36×20 ? **as 2×10 ; associative; 72; 720**

How many 0's are there in 720? In 36? In 20? How many 0's are there in both 36 and 20 together? Is this the same number of 0's as in 720? If $36 \times 2 = 72$, then $36 \times 20 = 720$. **one; none; one; one; Yes**

A way to find the simplest numeral for 50×30 is shown in **B**. How is 50 renamed? How is 30 renamed? What two properties are used to change $(5 \times 10) \times (3 \times 10)$ to $(5 \times 3) \times (10 \times 10)$? What is the simplest numeral for 5×3 ? For 10×10 ? For 50×30 ? **as 5×10 ; as 3×10 ; associative and commutative; 15; 100; 1500**

How many 0's are there in 1500? In 50? In 30? How many 0's are there in 50 and 30 together? Is this the same number of 0's as in 1500? If $3 \times 5 = 15$, then $30 \times 50 = 1500$. **2; 1; 1; 2; Yes**

Oral Tell the simplest numeral that should replace the \square in each of the following.

- If $4 \times 8 = 32$, then $4 \times 80 = \square$. **320**
- If $6 \times 5 = 30$, then $6 \times 50 = \square$. **300**
- If $8 \times 7 = 56$, then $80 \times 70 = \square$. **5600**

Written Find each product. Use the method shown in **A** or **B** above.

- $8 \times 90 = \square$ **a 720**
- $4 \times 70 = \square$ **280**
- $6 \times 80 = \square$ **480**

- $20 \times 50 = \square$ **b 1000**
- $30 \times 60 = \square$ **1800**
- $60 \times 70 = \square$ **4200**

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Multiplication (Two-Digit by Two-Digit)

Bill delivers 38 newspapers each morning. How many newspapers does he deliver in two weeks, including Sundays?

You can think of 14 equivalent sets of 38 newspapers each being joined so you multiply 38 by 14. Their product can be expressed as follows.

$$\begin{array}{r} 38 \\ \times 14 \\ \hline \end{array}$$

To find the simplest numeral for the product you can think about expanded notation and write place-value numerals.

	Think		Write
A	B	C	D
$\begin{array}{r} 38 \\ \times 14 \\ \hline \end{array}$	$\begin{array}{r} 30+8 \\ \times (10+4) \\ \hline 32 \text{ --- } 4 \times 8 \\ 120 \text{ --- } 4 \times 30 \\ 80 \text{ --- } 10 \times 8 \\ 300 \text{ --- } 10 \times 30 \\ \hline 532 \end{array}$	<div style="text-align: center; font-size: small; margin-bottom: 5px;">H T O</div> $\begin{array}{r} 38 \\ \times 14 \\ \hline 32 \\ 120 \\ 80 \\ 300 \\ \hline 532 \end{array}$	$\begin{array}{r} 38 \\ \times 14 \\ \hline 152 \text{ --- } 4 \times 38 \\ 380 \text{ --- } 10 \times 38 \\ \hline 532 \end{array}$

In **B**, how is 38 named? How is 14 named? How do you obtain the 32? The 120? The 80? The 300? The 532? See thinking steps above.

Explain how the method in **C** is like the method in **B**. How are the two methods different? **B** shows expanded notation and **C** shows the place-value grid.

In **D**, how do you obtain the 152? The 380? The 532? The steps taken in **D** are shown below. Explain each step. 4×38 ; 10×38 ; $152 + 380$

$\begin{array}{r} 38 \\ \times 14 \\ \hline \end{array}$	$\begin{array}{r} 38 \\ \times 14 \\ \hline 152 \end{array}$	$\begin{array}{r} 38 \\ \times 14 \\ \hline 152 \\ 380 \end{array}$	$\begin{array}{r} 38 \\ \times 14 \\ \hline 152 \\ 380 \\ \hline 532 \end{array}$	$\begin{array}{r} 38 \\ \times 14 \\ \hline 152 \\ 380 \\ \hline 532 \end{array}$
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$$4 \times 38 = 152 \quad 10 \times 38 = 380 \quad 152 + 380 = 532$$

Oral Use the following multiplication example to answer questions 1–3.

$$\begin{array}{r} 43 \\ \times 12 \\ \hline 86 \text{ — } 2 \times 43 \\ 430 \text{ — } 10 \times 43 \\ \hline 516 \end{array}$$

- How is the 86 obtained? 2×43
- How is the 430 obtained? 10×43
- How is the 516 obtained? $86 + 430$

Use the following multiplication example to answer questions 4–6.

$$\begin{array}{r} 84 \\ \times 36 \\ \hline 504 \text{ — } 6 \times 84 \\ 2520 \text{ — } 30 \times 84 \\ \hline 3024 \end{array}$$

- How is the 504 obtained? 6×84
- How is the 2520 obtained? 30×84
- How is the 3024 obtained?

$$504 + 2520$$

Explain the steps you would take in finding each of the following products.

a	b
7. $\begin{array}{r} 37 \\ \times 12 \\ \hline \end{array}$	$\begin{array}{r} 92 \\ \times 34 \\ \hline \end{array}$
2×37 74	4×92 368
10×37 370	30×92 2760
$74 + 370$ 444	$368 + 2760$ 3128

Written Copy. Find each product.

	a	b	c
1.	$\begin{array}{r} 23 \\ \times 34 \\ \hline \end{array}$	$\begin{array}{r} 18 \\ \times 31 \\ \hline \end{array}$	$\begin{array}{r} 14 \\ \times 25 \\ \hline \end{array}$
	782	558	350
2.	$\begin{array}{r} 36 \\ \times 21 \\ \hline \end{array}$	$\begin{array}{r} 18 \\ \times 30 \\ \hline \end{array}$	$\begin{array}{r} 38 \\ \times 22 \\ \hline \end{array}$
	756	540	836
3.	$\begin{array}{r} 14 \\ \times 31 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ \times 27 \\ \hline \end{array}$	$\begin{array}{r} 16 \\ \times 41 \\ \hline \end{array}$
	434	405	656
4.	$\begin{array}{r} 86 \\ \times 14 \\ \hline \end{array}$	$\begin{array}{r} 28 \\ \times 32 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ \times 18 \\ \hline \end{array}$
	1204	896	216
5.	$\begin{array}{r} 17 \\ \times 42 \\ \hline \end{array}$	$\begin{array}{r} 29 \\ \times 31 \\ \hline \end{array}$	$\begin{array}{r} 14 \\ \times 40 \\ \hline \end{array}$
	714	899	560
6.	$\begin{array}{r} 15 \\ \times 20 \\ \hline \end{array}$	$\begin{array}{r} 28 \\ \times 16 \\ \hline \end{array}$	$\begin{array}{r} 17 \\ \times 23 \\ \hline \end{array}$
	300	448	391
7.	$\begin{array}{r} 16 \\ \times 24 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ \times 32 \\ \hline \end{array}$	$\begin{array}{r} 33 \\ \times 27 \\ \hline \end{array}$
	384	480	891

Write an open sentence for each problem below. Solve the open sentence. Answer the problem.

8. Sally solved 22 arithmetic problems each day for 28 days. How many arithmetic problems did she solve altogether? $28 \times 22 = \square$; 616 problems

9. John put 32 apples in each of 14 baskets. How many apples did John put in all the baskets? $14 \times 32 = \square$; 448 apples

Tell why Think of any two numbers, each named by a 2-digit numeral. The numeral for their product contains at least three digits. Why? The least number named by 2 digits is 10. Since $10 \times 10 = 100$, 139 and 100 has 3 digits, every such product numeral has at least 3 digits.

Multiplication (Three-Digit by Two-Digit)

Jim wants to know the simplest numeral for 37×144 . This product can be expressed as follows.

$$\begin{array}{r} 144 \\ \times 37 \\ \hline \end{array}$$

To find the simplest numeral for the product you can think about expanded notation and write place-value numerals.

Think

A

Th	H	T	O
1	4	4	
\times	3	7	
<hr/>			
	2	8	
	2	8	0
	7	0	0
	1	2	0
1	2	0	0
3	0	0	0
<hr/>			
5	3	2	8

$$\left. \begin{array}{l} 28 \text{ — } 7 \times 4 \\ 280 \text{ — } 7 \times 40 \\ 700 \text{ — } 7 \times 100 \end{array} \right\} \text{ — } 1008 \text{ — } (7 \times 4) + (7 \times 40) + (7 \times 100)$$

$$\left. \begin{array}{l} 120 \text{ — } 30 \times 4 \\ 1200 \text{ — } 30 \times 40 \\ 3000 \text{ — } 30 \times 100 \end{array} \right\} \text{ — } 4320 \text{ — } (30 \times 4) + (30 \times 40) + (30 \times 100)$$

Write

B

$$\begin{array}{r} 144 \\ \times 37 \\ \hline \end{array}$$

$$\begin{array}{r} 5328 \end{array}$$

In **A**, how do you obtain the 28? The 280? The 700? The 120? The 1200? The 3000? How do you obtain the 5328?

See thinking steps above.

How is the method used in **B** like the method used in **A**? How are the two methods different? The methods are the same except for the place-value grid and combining some steps in **B**.

In **B**, how do you obtain the 1008? The 4320? How do you obtain the 5328? The steps taken in **B** are shown below. Explain each step. 7×144 ; 30×144 ; $1008 + 4320$

$$\begin{array}{r} ^3 ^2 \\ 144 \\ \times 37 \\ \hline 1008 \end{array}$$

$$\begin{array}{r} ^1 ^1 \\ 144 \\ \times 37 \\ \hline 1008 \\ 4320 \end{array}$$

$$\begin{array}{r} 144 \\ \times 37 \\ \hline 1008 \\ 4320 \\ \hline 5328 \end{array}$$

$$\begin{array}{r} 144 \\ \times 37 \\ \hline 1008 \\ 4320 \\ \hline 5328 \end{array}$$

$$7 \times 144 = 1008$$

$$30 \times 144 = 4320$$

$$1008 + 4320 = 5328$$

Oral Use the following multiplication example to answer questions 1–3.

$$\begin{array}{r}
 123 \\
 \times 23 \\
 \hline
 9 \text{ ————— } 3 \times 3 \\
 60 \text{ ————— } 3 \times 20 \\
 300 \text{ ————— } 3 \times 100 \\
 60 \text{ ————— } 20 \times 3 \\
 400 \text{ ————— } 20 \times 20 \\
 2000 \text{ ————— } 20 \times 100 \\
 \hline
 2829
 \end{array}$$

1. How do you obtain the 9? The 60? The 300? **See the blue type in the example.**

2. How do you obtain the 60? The 400? The 2000? **See the blue type in the example.**

3. How is the 2829 obtained?
 $9+60+300+60+400+2000$

Use the following multiplication example to answer questions 4–6.

$$\begin{array}{r}
 123 \\
 \times 23 \\
 \hline
 369 \text{ ————— } 3 \times 123 \\
 2460 \text{ ————— } 20 \times 123 \\
 \hline
 2829
 \end{array}$$

4. How is the 369 obtained?

See the blue type in the example. 192 books

5. How is the 2460 obtained?

See the blue type in the example.

6. How is the 2829 obtained?

$369+2460$

Answer the following.

7. Explain the steps you would take in solving the open sentence $15 \times 621 = \square$.

$$\begin{array}{r}
 621 \\
 \times 15 \\
 \hline
 3105 \\
 6210 \\
 \hline
 9315
 \end{array}
 \quad
 \begin{array}{l}
 5 \times 621 \\
 10 \times 621 \\
 3105 + 6210
 \end{array}$$

Written Copy. Find each product.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$ \begin{array}{r} 150 \\ \times 13 \\ \hline 1950 \end{array} $	$ \begin{array}{r} 224 \\ \times 12 \\ \hline 2688 \end{array} $	$ \begin{array}{r} 111 \\ \times 15 \\ \hline 1665 \end{array} $
2.	$ \begin{array}{r} 123 \\ \times 31 \\ \hline 3813 \end{array} $	$ \begin{array}{r} 237 \\ \times 22 \\ \hline 5214 \end{array} $	$ \begin{array}{r} 345 \\ \times 21 \\ \hline 7245 \end{array} $
3.	$ \begin{array}{r} 409 \\ \times 22 \\ \hline 8998 \end{array} $	$ \begin{array}{r} 413 \\ \times 54 \\ \hline 22302 \end{array} $	$ \begin{array}{r} 731 \\ \times 90 \\ \hline 65790 \end{array} $
4.	$ \begin{array}{r} 258 \\ \times 20 \\ \hline 5160 \end{array} $	$ \begin{array}{r} 814 \\ \times 67 \\ \hline 54538 \end{array} $	$ \begin{array}{r} 390 \\ \times 67 \\ \hline 26130 \end{array} $
5.	$ \begin{array}{r} 218 \\ \times 32 \\ \hline 6976 \end{array} $	$ \begin{array}{r} 165 \\ \times 43 \\ \hline 7095 \end{array} $	$ \begin{array}{r} 491 \\ \times 73 \\ \hline 35843 \end{array} $
6.	$ \begin{array}{r} 619 \\ \times 48 \\ \hline 29712 \end{array} $	$ \begin{array}{r} 304 \\ \times 20 \\ \hline 6080 \end{array} $	$ \begin{array}{r} 705 \\ \times 89 \\ \hline 62745 \end{array} $
7.	$ \begin{array}{r} 905 \\ \times 69 \\ \hline 62445 \end{array} $	$ \begin{array}{r} 541 \\ \times 51 \\ \hline 27591 \end{array} $	$ \begin{array}{r} 397 \\ \times 64 \\ \hline 25408 \end{array} $

Write an open sentence for each problem below. Solve the open sentence. Answer the problem.

8. Books were being moved from one room to another. If each of 16 boys carried 12 books, how many books were moved? **$16 \times 12 = \square$; 192 books**

9. Each of 25 pupils drew 14 pictures one month. Altogether that was how many pictures? **$25 \times 14 = \square$; 350 pictures**

10. The 24 girl scouts in a troop hope to sell 24 boxes of cookies each. How many boxes of cookies will that be in all? **$24 \times 24 = \square$; 576 boxes**

Multiplication (Three-Digit by Three-Digit)

Paula wants to know the simplest numeral for 145×462 . This product can be expressed as follows.

$$\begin{array}{r} 462 \\ \times 145 \\ \hline \end{array}$$

To find the simplest numeral for the product you can think about expanded notation and write place-value numerals. The multiplication example shown below helps you see how to find the simplest numeral for 145×462 .

$$\begin{array}{r} 462 \\ \times 145 \\ \hline 2310 \text{ ————— } 5 \times 462 \\ 18480 \text{ ————— } 40 \times 462 \\ 46200 \text{ ————— } 100 \times 462 \\ \hline 66990 \end{array}$$

How is multiplying by a 3-digit number like multiplying by a 2-digit number? How is it different? *The pattern is the same except for the hundreds digit.*

In the multiplication example above, you are multiplying by what number? The digit 5 in the ones place of 145 means 5 ones or 5. What does the digit 4 in the tens place of 145 mean? What does the digit 1 in the hundreds place of 145 mean? Do you multiply 462 by 5, then by 40, and then by 100? *145; 4 tens; 1 hundred; Yes*

How do you obtain the 2310? The 18480? The 46200? The 66990? *5×462 ; 40×462 ; 100×462 ; $2310 + 18480 + 46200$*

The steps used in the method illustrated above are shown below. Explain each step.

$$\begin{array}{r} \overset{3}{4} \overset{1}{6} 2 \\ \times 145 \\ \hline 2310 \end{array}$$

$$\begin{array}{r} \overset{2}{4} 62 \\ \times 145 \\ \hline 2310 \\ 18480 \end{array}$$

$$\begin{array}{r} 462 \\ \times 145 \\ \hline 2310 \\ 18480 \\ 46200 \end{array}$$

$$\begin{array}{r} 462 \\ \times 145 \\ \hline 2310 \\ 18480 \\ 46200 \\ \hline 66990 \end{array}$$

$$5 \times 462 = 2310 \quad 40 \times 462 = 18480 \quad 100 \times 462 = 46200$$

Oral Use the following multiplication example to answer the questions below.

$$\begin{array}{r} 123 \\ \times 213 \\ \hline 369 \text{ --- } 3 \times 123 \\ 1230 \text{ --- } 10 \times 123 \\ 24600 \text{ --- } 200 \times 123 \\ \hline 26199 \end{array}$$

- How is 369 obtained?
See example above.
- How is 1230 obtained?
See example above.
- How is 24600 obtained?
See example above.
- How is 26199 obtained?
 $369 + 1230 + 24600$

Tell the steps you would take in doing each example below. Typical answer for 5a and 6a only.

	a	b
5.	$\begin{array}{r} 421 \\ \times 312 \\ \hline 842 \quad 2 \times 421 \\ 4210 \quad 10 \times 421 \\ 126300 \quad 300 \times 421 \\ \hline 131352 \quad \text{Add.} \end{array}$	$\begin{array}{r} 123 \\ \times 123 \\ \hline 369 \\ 2460 \\ 12300 \\ \hline 15129 \end{array}$
6.	$\begin{array}{r} 311 \\ \times 213 \\ \hline 933 \quad 3 \times 311 \\ 3110 \quad 10 \times 311 \\ 62200 \quad 200 \times 311 \\ \hline 66243 \quad \text{Add.} \end{array}$	$\begin{array}{r} 113 \\ \times 213 \\ \hline 339 \\ 1130 \\ 22600 \\ \hline 24069 \end{array}$

Answer the following.

7. Explain how you would solve $453 \times 248 = \square$. 248

$$\begin{array}{r} \times 453 \\ 744 \quad 3 \times 248 \\ 12400 \quad 50 \times 248 \\ 99200 \quad 400 \times 248 \\ \hline 112344 \quad \text{Add.} \end{array}$$

Written Copy. Find each product.

	a	b	c
1.	$\begin{array}{r} 123 \\ \times 212 \\ \hline 26076 \end{array}$	$\begin{array}{r} 212 \\ \times 113 \\ \hline 23956 \end{array}$	$\begin{array}{r} 122 \\ \times 311 \\ \hline 37942 \end{array}$
2.	$\begin{array}{r} 321 \\ \times 122 \\ \hline 39162 \end{array}$	$\begin{array}{r} 402 \\ \times 312 \\ \hline 125424 \end{array}$	$\begin{array}{r} 204 \\ \times 123 \\ \hline 25092 \end{array}$
3.	$\begin{array}{r} 724 \\ \times 713 \\ \hline 516212 \end{array}$	$\begin{array}{r} 124 \\ \times 172 \\ \hline 21328 \end{array}$	$\begin{array}{r} 344 \\ \times 234 \\ \hline 80496 \end{array}$
4.	$\begin{array}{r} 174 \\ \times 417 \\ \hline 72558 \end{array}$	$\begin{array}{r} 661 \\ \times 906 \\ \hline 598866 \end{array}$	$\begin{array}{r} 339 \\ \times 358 \\ \hline 121362 \end{array}$
5.	$\begin{array}{r} 845 \\ \times 285 \\ \hline 240825 \end{array}$	$\begin{array}{r} 297 \\ \times 317 \\ \hline 94149 \end{array}$	$\begin{array}{r} 258 \\ \times 260 \\ \hline 67080 \end{array}$
6.	$\begin{array}{r} 309 \\ \times 305 \\ \hline 94245 \end{array}$	$\begin{array}{r} 274 \\ \times 137 \\ \hline 37538 \end{array}$	$\begin{array}{r} 714 \\ \times 107 \\ \hline 76398 \end{array}$
7.	$\begin{array}{r} 217 \\ \times 127 \\ \hline 27559 \end{array}$	$\begin{array}{r} 335 \\ \times 533 \\ \hline 178555 \end{array}$	$\begin{array}{r} 370 \\ \times 207 \\ \hline 76590 \end{array}$

Can you do this? Find the missing numerals below.

	a	b
1.	$\begin{array}{r} 1 \square 7 \\ \times 312 \\ \hline \square 5 \square \\ 3 \square \square 00 \\ 3 \square 6 \square 4 \end{array}$	$\begin{array}{r} 13\square \\ \times 213 \\ \hline 4\square 2 \\ \square \square \square 0 \\ 26\square 00 \\ \square 8\square 4 \square \end{array}$
2.	$\begin{array}{r} \square 24 \\ \times 3\square 1 \\ \hline \square 2 \square \\ \square \square 40 \\ 3 \square \square 00 \\ 38\square \square 4 \end{array}$	$\begin{array}{r} 12\square \\ \times \square 15 \\ \hline \square 25 \\ \square \square 50 \\ 2\square 000 \\ \square 68\square 5 \end{array}$

Estimating Products

Each day for 298 days a punch press operator in a factory worked on making spark plug wrenches. He was expected to work on 602 spark plug wrenches each day. How many spark plug wrenches did he work on in the 298 days?

You can think of 298 equivalent sets of 602 wrenches each being joined so you multiply 602 by 298. Their product can be expressed as follows.

$$\begin{array}{r} 602 \\ \times 298 \\ \hline \end{array}$$

For problems like the one above, you can easily find an *approximate answer*. When you do this, you **estimate** the answer. The *estimate* for a product is another product, which is either greater or less than the exact product but much easier to find. Study the example shown below.

$$\begin{array}{r} 602 \text{ — rounded to the nearest 100 — } 600 \\ \times 298 \text{ — rounded to the nearest 100 — } \times 300 \\ \hline 600 \end{array}$$

Is 602 nearer to 600 or to 700? Is 298 nearer to 200 or to 300?^{*1} To estimate the product 298×602 multiply 600 by 300. What is the simplest numeral for 300×600 ? How does knowing $3 \times 6 = 18$ help you find the simplest numeral for 300×600 ? 180000; If $3 \times 6 = 18$, then $3H \times 6H = 18TT$.

Find the exact product by multiplying 602 by 298. Is the exact product greater or less than 180,000? Does 180,000 tell you *about* how many spark plug wrenches the operator worked on? less; Yes

Which of the two products, the estimated product or the exact product, is easier to find? Why? estimated; only multiples of 100 needed

To estimate a product such as 298×602 , you round off the multiplier and the multiplicand to the nearest ten, hundred, or thousand, whichever is most convenient. Then you multiply the rounded off numbers.

Oral Answer the following questions.

1. Is 204 nearer to 200 or 210?

2. Is 198 nearer to 190 or 200?

3. Is 185 nearer to 100 or 200?

4. Is 365 nearer to 300 or 400?

5. Is 428 nearer to 400 or 500?

Think of estimating the product of 398 and 188. Answer the following questions.

6. Would you round off 398 to 300 or to 400? Why? **400; nearer to 400**

7. Would you round off 188 to 100 or to 200? Why? **200; nearer to 200**

8. What is your estimate for the product? **80,000**

Think of estimating the product of 425 and 375. Answer the following questions.

9. Would you round off 425 to 400 or to 500? Why? **400; nearer to 400**

10. Would you round off 375 to 300 or to 400? Why? **400; nearer to 400**

11. What is your estimate for the product? **160,000**

12. Could you estimate 375×425 by rounding to the nearest 10? Would this be easier than rounding to the nearest 100? Why? **Yes; No; more computation required**

Written Copy. Estimate each of the following products. Record your estimate. Find the exact product. Compare the estimated product with the exact product. **Estimated products on page T145.**

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 903 \\ \times 601 \\ \hline 542703 \end{array}$	$\begin{array}{r} 895 \\ \times 205 \\ \hline 183475 \end{array}$	$\begin{array}{r} 709 \\ \times 198 \\ \hline 140382 \end{array}$
2.	$\begin{array}{r} 688 \\ \times 393 \\ \hline 270384 \end{array}$	$\begin{array}{r} 782 \\ \times 321 \\ \hline 251022 \end{array}$	$\begin{array}{r} 891 \\ \times 192 \\ \hline 171072 \end{array}$
3.	$\begin{array}{r} 683 \\ \times 701 \\ \hline 478783 \end{array}$	$\begin{array}{r} 579 \\ \times 602 \\ \hline 348558 \end{array}$	$\begin{array}{r} 504 \\ \times 201 \\ \hline 101304 \end{array}$
4.	$\begin{array}{r} 283 \\ \times 608 \\ \hline 172064 \end{array}$	$\begin{array}{r} 614 \\ \times 302 \\ \hline 185428 \end{array}$	$\begin{array}{r} 709 \\ \times 500 \\ \hline 354500 \end{array}$
5.	$\begin{array}{r} 231 \\ \times 198 \\ \hline 45738 \end{array}$	$\begin{array}{r} 196 \\ \times 192 \\ \hline 37632 \end{array}$	$\begin{array}{r} 321 \\ \times 194 \\ \hline 62274 \end{array}$
6.	$\begin{array}{r} 188 \\ \times 189 \\ \hline 35532 \end{array}$	$\begin{array}{r} 195 \\ \times 204 \\ \hline 39780 \end{array}$	$\begin{array}{r} 298 \\ \times 382 \\ \hline 113836 \end{array}$

Write an open sentence for each problem below. Estimate the product. Find the exact product. Compare the estimated product with the exact product. Answer the problem.

7. John delivers 108 newspapers each day. How many newspapers does he deliver in 52 days?

$52 \times 108 = \square$; 5000; 5616 papers

8. Mr. Hauser plans to buy a new car. His monthly payment will be \$114 each month. He must make the payments for 36 months. How much will he pay in 36 months?

$36 \times 114 = \square$; 4000; \$4104

Division (Two-Digit by Two-Digit)

You can think of $65 \div 13$ as repeated subtraction.

$$\begin{array}{r}
 65 \\
 -13 \\
 \hline
 52
 \end{array}
 \rightarrow
 \begin{array}{r}
 52 \\
 -13 \\
 \hline
 39
 \end{array}
 \rightarrow
 \begin{array}{r}
 39 \\
 -13 \\
 \hline
 26
 \end{array}
 \rightarrow
 \begin{array}{r}
 26 \\
 -13 \\
 \hline
 13
 \end{array}
 \rightarrow
 \begin{array}{r}
 13 \\
 -13 \\
 \hline
 0
 \end{array}$$

Starting with 65, how many times can you subtract 13 before reaching zero? What is the simplest numeral for the quotient of 65 divided by 13? **5; 5**

Instead of subtracting the thirteens one at a time, you could subtract all five of the thirteens at the same time.

Think

$$\begin{array}{r}
 5 \\
 13 \overline{) 65} \\
 \underline{65} \\
 0
 \end{array}
 \quad
 \begin{array}{l}
 17 \times _ < 67 \\
 5 \times 13
 \end{array}$$

Round off 13, the divisor, to 10 or 17. Round off 65, the dividend, to 60 or 67. To solve $65 \div 13 = \square$, think of $13 \times \square = 65$ and use $17 \times _ < 67$ to estimate a solution.

What is the greatest whole number whose numeral can replace the $_$ to make $17 \times _ < 67$ a true sentence? Where do you record the 5 in the numeral for the quotient? Why? **5; in ones place; ones digit**

When 5×13 or 65 is subtracted from 65, the difference is less than the divisor. You have subtracted the greatest possible multiple of the divisor.

If you use too small a multiple of the divisor, this difference is greater than the divisor. Try the next greater number as the quotient.

If you use too great a multiple of the divisor, you cannot subtract it from the dividend. Try the next lesser number as the quotient.

Oral In $65 \div 13 = \square$, you rounded off 65 and 13 and then used $1T \times \square < 6T$ to estimate the ones digit of the numeral for the quotient. In each of the following, tell the sentence you would use to estimate the ones digit of the numeral for the quotient.

- | <i>a</i> | <i>b</i> |
|--|---|
| 1. $84 \div 12 = \boxed{7}$
$1T \times \boxed{8}T$ | $55 \div 11 = \boxed{5}$
$1T \times \boxed{6}T$ |
| 2. $84 \div 14 = \boxed{6}$
$1T \times \boxed{8}T$ | $70 \div 14 = \boxed{5}$
$1T \times \boxed{7}T$ |
| 3. $88 \div 11 = \boxed{8}$
$1T \times \boxed{9}T$ | $81 \div 27 = \boxed{3}$
$3T \times \boxed{8}T$ |
| 4. $54 \div 18 = \boxed{3}$
$2T \times \boxed{5}T$ | $76 \div 19 = \boxed{4}$
$2T \times \boxed{8}T$ |
| 5. $48 \div 24 = \boxed{2}$
$2T \times \boxed{5}T$ | $96 \div 32 = \boxed{3}$
$3T \times \boxed{10}T$ |
| 6. $78 \div 13 = \boxed{6}$
$1T \times \boxed{8}T$ | $62 \div 31 = \boxed{2}$
$3T \times \boxed{6}T$ |
| 7. $96 \div 16 = \boxed{6}$
$2T \times \boxed{10}T$ | $86 \div 43 = \boxed{2}$
$4T \times \boxed{9}T$ |
| 8. $84 \div 21 = \boxed{4}$
$2T \times \boxed{8}T$ | $96 \div 12 = \boxed{8}$
$1T \times \boxed{10}T$ |

Explain each of the following division examples. Typical answers for 9a, 10a, and 11a only.

- | <i>a</i> | <i>b</i> |
|--|--|
| 9. $\begin{array}{r} 9 \\ 11 \overline{)99} \\ \underline{99} \\ 0 \end{array}$
$1T \times \boxed{10}T$
9×11
$99 - 99$ | $\begin{array}{r} 7 \\ 14 \overline{)98} \\ \underline{98} \\ 0 \end{array}$ |
| 10. $\begin{array}{r} 6 \\ 14 \overline{)84} \\ \underline{84} \\ 0 \end{array}$
$1T \times \boxed{8}T$
6×14
$84 - 84$ | $\begin{array}{r} 7 \\ 13 \overline{)91} \\ \underline{91} \\ 0 \end{array}$ |
| 11. $\begin{array}{r} 3 \\ 16 \overline{)48} \\ \underline{48} \\ 0 \end{array}$
$2T \times \boxed{5}T$
3×16
$48 - 48$ | $\begin{array}{r} 3 \\ 14 \overline{)42} \\ \underline{42} \\ 0 \end{array}$ |

Written Copy. Change each open sentence below to the $\overline{\hspace{1cm}}$ form. Find each quotient.

- | <i>a</i> | <i>b</i> |
|-----------------------------|--------------------------|
| 1. $48 \div 16 = \boxed{3}$ | $96 \div 16 = \boxed{6}$ |
| 2. $76 \div 19 = \boxed{4}$ | $96 \div 24 = \boxed{4}$ |
| 3. $75 \div 25 = \boxed{3}$ | $72 \div 18 = \boxed{4}$ |
| 4. $88 \div 22 = \boxed{4}$ | $57 \div 19 = \boxed{3}$ |
| 5. $72 \div 24 = \boxed{3}$ | $81 \div 27 = \boxed{3}$ |
| 6. $96 \div 32 = \boxed{3}$ | $80 \div 16 = \boxed{5}$ |
| 7. $85 \div 17 = \boxed{5}$ | $70 \div 14 = \boxed{5}$ |

Write an open sentence for each problem below. Solve the open sentence. Answer the problem.

8. Mary Ann's mother bought 12 yards of ribbon for 96 cents. How much money did she pay for one yard of ribbon? $96 \div 12 = \square$; 8 cents

9. George took 90 pictures on his vacation. If each roll contained 15 pictures, how many rolls of film did he use? $90 \div 15 = \square$; 6 rolls

Can you do this? Find the missing digits in these division examples.

- | <i>a</i> | <i>b</i> | <i>c</i> |
|--|--|--|
| $\begin{array}{r} \boxed{4} \\ 19 \overline{)76} \\ \underline{76} \\ 0 \end{array}$ | $\begin{array}{r} \boxed{6} \\ 16 \overline{)96} \\ \underline{96} \\ 0 \end{array}$ | $\begin{array}{r} \boxed{3} \\ 27 \overline{)81} \\ \underline{81} \\ 0 \end{array}$ |

Division (Three-Digit by Two-Digit)

Helen's father drove his car 768 miles. If he can drive 24 miles on one gallon of gasoline, how many gallons of gasoline did he use?

Why is $768 \div 24 = \square$ a suitable open sentence for this problem? You can think of this division as shown below.

$$24 \overline{)768}$$

Suppose you start with 768 and repeatedly subtract 24. Is that a convenient way to find the simplest numeral for $768 \div 24$? Why? Instead, think of $768 \div 24 = \square$ as $24 \times \square = 768$. Since $24 \times 10 < 768$ and $24 \times 100 > 768$, the solution is between 10 and 100. To find the tens digit of the quotient numeral, use $2T \times _ T < 8H$ as shown below.

A	B	C
$ \begin{array}{r} \text{H} \mid \text{T} \mid \text{O} \\ 24 \overline{)768} \\ \underline{720} \\ 48 \end{array} $	$ \begin{array}{r} \text{H} \mid \text{T} \mid \text{O} \\ 24 \overline{)768} \\ \underline{720} \\ 48 \\ \underline{48} \\ 0 \end{array} $	$ \begin{array}{r} 32 \\ 24 \overline{)768} \\ \underline{720} \\ 48 \\ \underline{48} \\ 0 \end{array} $
$2T \times _ T < 8H$	$2T \times _ < 5T$	

In A, 24 is rounded off to 20 or 2T and 768 is rounded off to 800 or 8H. What is the greatest whole number whose numeral can replace the $_$ to make $2T \times _ T < 8H$ a true sentence? Why is 3 recorded in the tens place of the quotient numeral? 3 means 3 tens.

In B, how do you obtain $2T \times _ < 5T$? How does $2T \times _ < 5T$ help you decide to record 2 in the ones place of the quotient numeral? Round 24 to 2T and round 48 to 5T; 2 is the greatest solution of the open sentence.

How many gallons of gasoline did Helen's father use? 32

To solve an open sentence like $768 \div 24 = \square$, think of rounding off the dividend and the divisor to estimate each digit of the quotient numeral.

Oral In $768 \div 24 = \square$, you rounded off 768 and 24 and then used $2T \times \square T < 8H$ to estimate the tens digit of the numeral for the quotient. In each of the following, tell the sentence you would use to estimate the tens digit of the numeral for the quotient.

- | <i>a</i> | <i>b</i> |
|--|---|
| 1. $391 \div 23 = \square$
$2T \times \underline{T} < 4H$ | $391 \div 17 = \square$
$2T \times \underline{T} < 4H$ |
| 2. $396 \div 18 = \square$
$2T \times \underline{T} < 4H$ | $384 \div 24 = \square$
$2T \times \underline{T} < 4H$ |
| 3. $884 \div 26 = \square$
$3T \times \underline{T} < 9H$ | $875 \div 25 = \square$
$2T \times \underline{T} < 9H$ |
| 4. $891 \div 33 = \square$
$3T \times \underline{T} < 9H$ | $396 \div 22 = \square$
$2T \times \underline{T} < 4H$ |
| 5. $875 \div 35 = \square$
$4T \times \underline{T} < 9H$ | $884 \div 34 = \square$
$3T \times \underline{T} < 9H$ |
| 6. $836 \div 19 = \square$
$2T \times \underline{T} < 8H$ | $714 \div 17 = \square$
$2T \times \underline{T} < 7H$ |
| 7. $798 \div 42 = \square$
$4T \times \underline{T} < 8H$ | $891 \div 27 = \square$
$3T \times \underline{T} < 9H$ |
| 8. $384 \div 16 = \square$
$2T \times \underline{T} < 4H$ | $352 \div 16 = \square$
$2T \times \underline{T} < 4H$ |
| 9. $414 \div 18 = \square$
$2T \times \underline{T} < 4H$ | $552 \div 23 = \square$
$2T \times \underline{T} < 6H$ |

Explain each of the following division examples. Typical answers for 10a and 11a only.

- | <i>a</i> | <i>b</i> |
|--|---|
| 10. $\begin{array}{r} 8 \\ 51 \overline{)408} \\ \underline{408} \\ 0 \end{array}$
$5T \times \underline{T} < 4H$
8×51
$408 - 408$ | $\begin{array}{r} 8 \\ 47 \overline{)376} \\ \underline{376} \\ 0 \end{array}$ |
| 11. $\begin{array}{r} 23 \\ 41 \overline{)943} \\ \underline{820} \\ 123 \\ \underline{123} \\ 0 \end{array}$
$4T \times \underline{T} < 9H$
20×41
$943 - 820$
3×41
$123 - 123$ | $\begin{array}{r} 41 \\ 12 \overline{)492} \\ \underline{480} \\ 12 \\ \underline{12} \\ 0 \end{array}$ |

Written Copy. Find each quotient.

- | <i>a</i> | <i>b</i> |
|--|---|
| 1. $387 \div 43 = \square$
9 | $259 \div 37 = \square$
7 |
| 2. $156 \div 52 = \square$
3 | $576 \div 24 = \square$
24 |
| 3. $286 \div 13 = \square$
22 | $528 \div 22 = \square$
24 |
| 4. $384 \div 24 = \square$
16 | $884 \div 34 = \square$
26 |
| 5. $900 \div 25 = \square$
36 | $391 \div 23 = \square$
17 |
| 6. $875 \div 35 = \square$
25 | $391 \div 17 = \square$
23 |
| 7. $884 \div 26 = \square$
34 | $396 \div 22 = \square$
18 |

Write an open sentence for each problem below. Solve the open sentence. Answer the problem.

8. There are 144 items in a gross. There are 12 items in a dozen. How many dozen are in a gross?
 $144 \div 12 = \square$; 12 items

9. Don drove 396 miles on 22 gallons of gasoline. How many miles did he drive on one gallon of gasoline?
 $396 \div 22 = \square$; 18 miles

Can you do this? Find the missing digits in these division examples.

- | <i>a</i> | <i>b</i> |
|--|--|
| $\begin{array}{r} 25 \\ 15 \overline{)375} \\ \underline{300} \\ 75 \\ \underline{75} \\ 0 \end{array}$ | $\begin{array}{r} 26 \\ 14 \overline{)364} \\ \underline{280} \\ 84 \\ \underline{84} \\ 0 \end{array}$ |

Division (Four-Digit by Two-Digit)

Mr. Noyes owns a piece of property that is 5270 feet long. He wants to separate this property into lots, each 85 feet wide. How many lots can he make?

Why is $5270 \div 85 = \square$ a suitable open sentence for this problem? You can think of this division as shown below.

$$85 \overline{)5270}$$

You can round off the divisor and the dividend to estimate each digit of the quotient numeral. Study how this is done to find the simplest numeral for $5270 \div 85$.

A	B	C	D
$\begin{array}{r} \text{Th} \mid \text{H} \mid \text{T} \mid \text{O} \\ 85 \overline{)5270} \end{array}$	$\begin{array}{r} \text{Th} \mid \text{H} \mid \text{T} \mid \text{O} \\ 85 \overline{)5270} \\ \underline{5100} \\ 170 \end{array}$	$\begin{array}{r} \text{Th} \mid \text{H} \mid \text{T} \mid \text{O} \\ 85 \overline{)5270} \\ \underline{5100} \\ 170 \\ \underline{170} \\ 0 \end{array}$	$\begin{array}{r} 62 \\ 85 \overline{)5270} \\ \underline{5100} \\ 170 \\ \underline{170} \\ 0 \end{array}$
$8T \times _ H < 5Th$	$8T \times _ T < 53H$	$8T \times _ < 17T$	

In A, the divisor is rounded off to 80 or 8T. The dividend is rounded off to 5000 or 5Th. Estimate the quotient by thinking of $5Th \div 8T = \square$ as $8T \times \square = 5Th$. Since $8T \times 1H > 5Th$, what do you know about the quotient? Should a digit be recorded in hundreds place of the quotient numeral? Why? *less than 100; No; quotient is less than 100*

In B, how do you obtain $8T \times _ T < 53H$? How does $8T \times _ T < 53H$ help you decide to record 6 in the tens place of the quotient numeral? *Round 85 to 8T and round 5270 to 53H; 6 is the greatest solution of the open sentence.*

In C, how do you obtain $8T \times _ < 17T$? How does $8T \times _ < 17T$ help you decide to record 2 in the ones place? *Round 85 to 8T and round 170 to 17T; 2 is greatest solution of the open sentence.*

You need to write only the place-value numerals as shown in D. How many lots can Mr. Noyes make? *62*

Oral Use the example below to answer the following questions.

$$\begin{array}{r} 34 \\ 52 \overline{)1768} \\ \underline{1560} \\ 208 \\ \underline{208} \\ 0 \end{array} \quad \left\{ \begin{array}{l} 5T \times _ H < 2Th \\ 5T \times _ T < 18H \\ 5T \times _ _ < 21T \end{array} \right.$$

1. How is the thinking step $5T \times _ H < 2Th$ obtained? **Round 52 to 5T and round 1768 to 2Th.**

2. How does the thinking step $5T \times _ H < 2Th$ help you decide that there is no hundreds digit in the quotient numeral? **0 is the greatest solution of the open sentence.**

3. How is the thinking step $5T \times _ T < 18H$ obtained? **Round 52 to 5T and round 1768 to 18T.**

4. How does $5T \times _ T < 18H$ help you decide to record 3 in the tens place of the quotient numeral? **See below.**

5. How is the thinking step $5T \times _ _ < 21T$ obtained? **Round 52 to 5T and round 208 to 21T.**

6. How does $5T \times _ _ < 21T$ help you decide to record 4 in the ones place of the quotient numeral? **See below.**

7. If $1768 \div 52 = 34$ is a true sentence, is $34 \times 52 = 1768$ a true sentence? Why? **Yes. Multiplication is the inverse of division.**

8. Does *Oral 7* help you decide what to do to check a division example? How can you check the division example above? **Yes. Multiply the quotient by the divisor.**

Written Copy. Find each quotient.

	a	b	c
1.	$43 \overline{)1548} \quad \underline{36}$	$54 \overline{)1242} \quad \underline{23}$	$63 \overline{)2772} \quad \underline{44}$
2.	$52 \overline{)3328} \quad \underline{64}$	$76 \overline{)3420} \quad \underline{45}$	$84 \overline{)5292} \quad \underline{63}$
3.	$37 \overline{)1184} \quad \underline{32}$	$50 \overline{)2300} \quad \underline{46}$	$38 \overline{)1976} \quad \underline{52}$
4.	$87 \overline{)3306} \quad \underline{38}$	$73 \overline{)3066} \quad \underline{42}$	$27 \overline{)2025} \quad \underline{75}$
5.	$37 \overline{)2997} \quad \underline{81}$	$31 \overline{)2604} \quad \underline{84}$	$61 \overline{)3660} \quad \underline{60}$
6.	$44 \overline{)2332} \quad \underline{53}$	$87 \overline{)3567} \quad \underline{41}$	$45 \overline{)2295} \quad \underline{51}$
7.	$38 \overline{)1596} \quad \underline{42}$	$53 \overline{)2491} \quad \underline{47}$	$41 \overline{)1599} \quad \underline{39}$

Write an open sentence for each problem below. Solve the open sentence. Answer the problem.

8. Mr. Barnes earns \$7428 a year. How much does he earn in one month? $7428 \div 12 = \square$; \$619

9. Mr. Adams earns \$5424 a year. How much does he earn in one month? $5424 \div 12 = \square$; \$452

Can you do this? Find the missing digits in each division example.

a	b
$34 \overline{)9996} \quad \underline{294}$	$37 \overline{)851} \quad \underline{23}$
$\quad \underline{6800}$	$\quad \underline{740}$
$\quad \quad \underline{3196}$	$\quad \quad \underline{111}$
$\quad \quad \underline{3060}$	$\quad \quad \underline{111}$
$\quad \quad \quad \underline{136}$	$\quad \quad \quad 0$
$\quad \quad \quad \underline{136}$	
$\quad \quad \quad \quad 0$	

Oral 4. 3 is the greatest solution of the open sentence.
6. 4 is the greatest solution of the open sentence.

Solving Problems



A boy scout troop from Gary, Indiana, is going to a boy scout jamboree in Irwin, Pennsylvania. How far will they travel in going to the jamboree?

The procedure given below can be helpful in solving story problems like the one above.

Read the problem carefully.	The boy scouts will travel 121 miles in Indiana. They will travel 249 miles across Ohio. They will travel 62 miles in Pennsylvania.	
Draw a diagram, if needed.		
Decide which operation to use.	Think: The diagram suggests that you are joining non-equivalent sets. The joining of such sets suggests which operation? addition	
Write an open sentence:	$121 + 249 + 62 = \square$	What does the \square stand for? total number of miles
Solve the open sentence.	$\begin{array}{r} 121 \\ 249 \\ +62 \\ \hline \end{array}$	What is the sum? 432
Answer the problem.	432 miles	Why do you write the word <i>miles</i> after the numeral 432?

Use the following suggestions to solve each problem below.

- Read the problem carefully.
- Decide which operation to use.
- Write an open sentence.
- Solve the open sentence.
- Answer the problem.

1. John delivered 610 newspapers. Stan delivered 582 newspapers. Jim delivered 649 newspapers. Altogether, the three boys delivered how many newspapers? $610+582+649=\square$; **1841 newspapers**

2. A truck loaded with furniture weighs 12,575 pounds. The empty truck weighs 8800 pounds. What is the weight of the furniture? $12575-8800=\square$; **3775 pounds**

3. A farmer delivered 14 loads of potatoes to the market. Each load had a weight of 850 pounds. What was the total weight of the potatoes in pounds? $14\times 850=\square$; **11,900 pounds**

4. A newsboy delivered 682 newspapers in the 31 days in October. He delivered the same number of newspapers each day, including Sundays. How many newspapers did he deliver each day? $682\div 31=\square$; **22 newspapers**

5. A library had 228 science books, 138 mathematics books, and 160 history books. How many books on all three subjects does the library have? $228+138+160=\square$; **526 books**

6. The children of a school went on a picnic. Each of 12 buses carried 48 children. How many children were at the picnic? $12\times 48=\square$; **576 children**

7. One tank held 1750 gallons of gasoline. Another tank held 1575 gallons. How many more gallons of gasoline were in the larger tank than in the smaller tank? $1750-1575=\square$; **175 gallons**

8. A milkman delivers 1248 quarts of milk on Tuesday, 1096 quarts of milk on Thursday, and 982 quarts of milk on Saturday. How many quarts of milk does the milkman deliver on the three days? $1248+1096+982=\square$; **3326 quarts**

9. A book contains 1024 pages. Betsy said she would read 32 pages a day. How many days will it take her to read the book? $1024\div 32=\square$; **32 days**

10. A teacher had 648 sheets of paper. She gave the same number of sheets of paper to each of her 24 pupils. How many sheets of paper did each pupil get? $648\div 24=\square$; **27 sheets**

11. Don earns \$120 a week. How much money does he earn in 4 weeks? $120\times 4=\square$; **\$480**

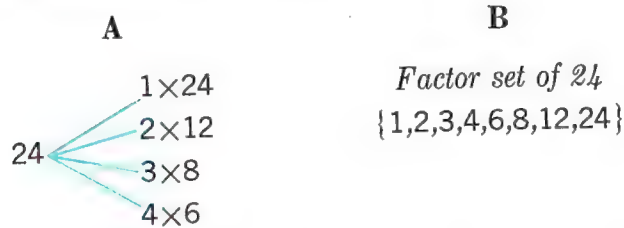
Can you do this? Compose a story problem for each open sentence below.
Answers will vary.

<i>a</i>	<i>b</i>
1. $430+380=\square$	$25\times 6=\square$

2. $305-162=\square$	$192\div 48=\square$
----------------------	----------------------

Greatest Common Factor

You can name 24 as the product of two whole numbers in several ways as shown in **A** below.



Is 2 a factor of 24? How do you know? Is 12 a factor of 24? What are some other factors of 24? Are all of these factors of 24 named in **B** above? Are all of these factors listed in their natural order beginning with one? *Yes; $2 \times 12 = 24$; Yes; See above; Yes; Yes*

How do you find the **factor set** of a number? **1*

Is $\{1, 2, 3, 6, 9, 18\}$ the factor set of 18? *Yes See *1.* How do you know?

Study the factor set of 18 and the factor set of 24 that are shown below.

$$\text{Factor set of } 24 = \{1, 2, 3, 4, 6, 8, 12, 24\}$$

$$\text{Factor set of } 18 = \{1, 2, 3, 6, 9, 18\}$$

Is one a factor of both 24 and 18? How do you know? Is 2 a factor of both 24 and 18? How do you know? Is 8 a factor of both 24 and 18? How do you know? Which numbers are factors of both 24 and 18? You call 1, 2, 3, and 6 the **common factors** of 24 and 18. Why? *Yes; in both factor sets; Yes; in both factor sets; No; not in both factor sets; 1, 2, 3, 6; common to both*

Which of the **common factors** of 24 and 18 is the greatest factor number? For this reason you call 6 the **greatest common factor** of 24 and 18. *sets*

To find the *greatest common factor* of 24 and 18 you think of the factor set of each number. Then you find the common factors of the two numbers. Then you choose the greatest of the common factors.

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**1 Find all the products of two whole numbers that are equal to the number.*

**2 6*

Oral Tell the factor set of each of the following numbers. See page T155.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	8	10	12
2.	14	16	18
3.	20	21	24
4.	25	27	30
5.	32	36	40

Tell the common factors of each pair of numbers below. Then tell the greatest common factor of the two numbers. **Greatest common factor is underlined.**

- Factor set of 12
{1,2,3,4,6,12} 1, 2, 3, 6
Factor set of 18
{1,2,3,6,9,18}
- Factor set of 9
{1,3,9} 1, 3
Factor set of 12
{1,2,3,4,6,12}
- Factor set of 6
{1,2,3,6} 1, 2, 3, 6
Factor set of 12
{1,2,3,4,6,12}
- Factor set of 21
{1,3,7,21} 1, 7
Factor set of 28
{1,2,4,7,14,28}

Tell how you would find the greatest common factor of each pair of numbers listed below. See below.

	<i>a</i>	<i>b</i>
10.	6 and 9 3	12 and 28 4
11.	9 and 18 9	9 and 21 3
12.	6 and 18 6	12 and 21 3

Written Copy. Find the greatest common factor of each of the following pairs of numbers.

	<i>a</i>	<i>b</i>
1.	6 and 15 3	8 and 12 4
2.	18 and 24 6	20 and 24 4
3.	12 and 20 4	20 and 36 4
4.	18 and 20 2	24 and 36 12
5.	36 and 18 18	16 and 12 4
6.	12 and 14 2	20 and 30 10
7.	21 and 14 7	6 and 24 6
8.	8 and 32 8	14 and 42 14

Can you do this? Find the greatest common factor of 18, 20, and 36. ²
Of 18, 20, and 24. 2

Tell why The greatest common factor of 3 and 5 or of 7 and 11 is one. Why? **Greatest common factor is 1.**

Oral 10-12

Find the factor set of each number. Choose the greatest common factor.

Least Common Multiple

	1×4	2×4	3×4	4×4	5×4	6×4	7×4	8×4	9×4
Multiples of 4	4	8	12	16	20	24	28	32	36
	1×6	2×6	3×6	4×6	5×6	6×6	7×6	8×6	9×6
Multiples of 6	6	12	18	24	30	36	42	48	54

Which multiples of 4 shown above are also multiples of 6? You call 12, 24, and 36 **common multiples** of 4 and 6. Why? Do 4 and 6 have more common multiples than those shown? What are the next two common multiples of 4 and 6? 12, 24, 36; members of both factor sets; Yes; 48 and 60

Which of the common multiples of 4 and 6 is the least? You call 12 the **least common multiple** of 4 and 6. Why? 12; It is the least of the common multiples.

A more convenient way to find the least common multiple of two natural numbers is described below.

Is the product 4×6 or 24 a common multiple of 4 and 6? Is it their least common multiple? Yes; No

What number should replace the \square in the open sentence below? 2

$$24 \div \square = 12$$

Product of
4 and 6
Least common multiple
of 4 and 6.

To answer this question, think of the factor sets of 4 and 6.

Factor set of 4 = { 1, 2, 4 }

Factor set of 6 = { 1, 2, 3, 6 }

What is the greatest common factor of 4 and 6? Is this the same number that should be named in the \square in $24 \div \square = 12$? 2; Yes

To find the least common multiple of two natural numbers, divide their product by their greatest common factor.

Oral Tell the first six multiples of each of the following numbers.
See page T157.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
1.	2	3	4	5
2.	6	7	8	9
3.	10	11	12	13

What common multiples of each pair of numbers are shown below? What is the least common multiple of each pair of numbers? **Least common multiple is underlined.**

4. Multiples of 8:
{8,16,24,32,40,48,56,64}
Multiples of 12: 24, 48
{12,24,36,48,60,72,84,96}
5. Multiples of 4:
{4,8,12,16,20,24,28,32}
Multiples of 10: 20
{10,20,30,40,50,60,70,80}
6. Multiples of 6:
{6,12,18,24,30,36,42,48}
Multiples of 9: 18, 36
{9,18,27,36,45,54,63,72}

Tell how you would find the least common multiple of each pair of numbers below. See below.

	<i>a</i>	<i>b</i>
7.	6 and 15 <u>30</u>	4 and 8 <u>8</u>
8.	8 and 14 <u>56</u>	4 and 5 <u>20</u>

Written Find the greatest common factor of each pair of numbers listed below. Then find their least common multiple.

	<i>a</i>	<i>b</i>
1.	6 and 8 <u>2</u> ; <u>24</u>	8 and 12 <u>4</u> ; <u>24</u>
2.	9 and 12 <u>3</u> ; <u>36</u>	6 and 12 <u>6</u> ; <u>12</u>
3.	12 and 20 <u>4</u> ; <u>60</u>	6 and 9 <u>3</u> ; <u>18</u>
4.	6 and 20 <u>2</u> ; <u>60</u>	8 and 20 <u>4</u> ; <u>40</u>
5.	9 and 24 <u>3</u> ; <u>72</u>	8 and 10 <u>2</u> ; <u>40</u>
6.	6 and 15 <u>3</u> ; <u>30</u>	4 and 9 <u>1</u> ; <u>36</u>
7.	3 and 8 <u>1</u> ; <u>24</u>	4 and 10 <u>2</u> ; <u>20</u>
8.	9 and 15 <u>3</u> ; <u>45</u>	6 and 10 <u>2</u> ; <u>30</u>
9.	6 and 14 <u>2</u> ; <u>42</u>	3 and 12 <u>3</u> ; <u>12</u>

Can you do this? Find the least common multiple of each set of three numbers given below. To help you do this, list the first six multiples of each number as shown at the top of page 156.

	<i>a</i>	<i>b</i>
1.	2, 4, 8 <u>8</u>	3, 6, 9 <u>18</u>
2.	2, 3, 4 <u>12</u>	2, 3, 6 <u>6</u>

Tell why The least common multiple of two numbers like 3 and 5 is their product. Why? **Their greatest common factor is 1.**

Oral 7-8: Divide their product by their greatest common factor.

Solving Problems

Write an open sentence for each problem. Solve the open sentence. Answer the problem.

1. Shaffer Elementary School has 14 classes of pupils. There are 35 pupils in each class. How many pupils are there in the Shaffer Elementary School? $14 \times 35 = \square$; 490 pupils

2. The Smiths were on a touring vacation. They traveled for 21 days. Each day they traveled 225 miles. In all, how many miles did they travel? $21 \times 225 = \square$; 4725 miles

3. Each month the Neals buy 38 quarts of milk. How many quarts of milk do they buy in one year? $12 \times 38 = \square$; 456 quarts

4. An airplane was flying at a height of 12 miles above sea level. How many feet is that? (1 mile is equivalent to 5280 feet.) $12 \times 5280 = \square$; 63,360 feet

5. A truck hauled 96 tons of sand to a building site. The truck hauled 12 tons on each trip. How many trips did the truck make? $96 \div 12 = \square$; 8 trips

6. Gene delivers 48 newspapers each day. In 21 days Gene delivers how many newspapers? $21 \times 48 = \square$; 1008 newspapers

7. Mr. Watson teaches 175 pupils a day. If each class has 35 pupils, how many different classes does he teach each day? $175 \div 35 = \square$; 5 classes

8. A farmer had 2724 eggs to pack in cartons holding a dozen eggs each. How many cartons did he need to pack all of the eggs? $2724 \div 12 = \square$; 227 cartons

9. Margie made 36 cookies each week for an entire year. How many cookies did she make that year? $52 \times 36 = \square$; 1872 cookies

10. An auditorium had every one of its 1008 seats filled for a school play. If 36 people sat in each row, how many rows of seats were in the auditorium? $1008 \div 36 = \square$; 28 rows

11. A storekeeper sold 16 color television sets at \$545 each. How much money did he get for the television sets? $16 \times 545 = \square$; \$8720

12. Fourteen school buses carried children to the school picnic. There were 602 children at the picnic. If each bus carried the same number of children, how many children rode in each bus? $602 \div 14 = \square$; 43 children

Can you do this? Compose a story problem that can be solved by multiplication. Compose a story problem that can be solved by division. Answers will vary.

Tell why The least common multiple of two consecutive numbers like 5 and 6 is the product of the two numbers. Why? See page T158.

Checkup Time

The numerals in () tell the pages where you can turn for help.

Important Ideas

1. To find the simplest numeral for a product like 32×14 , think about expanded notation and write only place-value numerals. (138)

2. You can estimate a product by rounding off the factors. (144)

3. To find the simplest numeral for a quotient like $768 \div 24$, think of rounding off the dividend and the divisor to find each digit of the quotient numeral. (148)

4. To solve a story problem it is best to:

- Read the problem carefully.
- Draw a diagram, if needed.
- Decide which operation to use.
- Write an open sentence.
- Solve the open sentence.
- Answer the problem. (152)

Words to Know

- Estimate (144)
- Factor set, common factors, greatest common factor (154)
- Common multiple, least common multiple (156)

Questions to Discuss

See page T159.

1. Why do you multiply when you think of equivalent sets being joined? (138)

2. Why can you think about expanded notation and write only place-value numerals in finding the simplest numeral for 27×146 ? (140)

3. How do you find the greatest common factor of 8 and 10? (154)

4. How do you find the least common multiple of 8 and 10? (156)

Written Practice

Copy. Find the simplest numeral for each product or quotient.

<i>a</i>		<i>b</i>	
1.	$\begin{array}{r} 84 \\ \times 23 \\ \hline 1932 \end{array}$ (138)		$\begin{array}{r} 482 \\ \times 24 \\ \hline 11568 \end{array}$ (140)
2.	$\begin{array}{r} 248 \\ \times 226 \\ \hline 56048 \end{array}$ (142)		$\begin{array}{r} 631 \\ \times 321 \\ \hline 202551 \end{array}$ (142)
3.	$\begin{array}{r} 6 \\ 14 \overline{)84} \end{array}$ (146)		$\begin{array}{r} 6 \\ 17 \overline{)102} \end{array}$ (148)
4.	$\begin{array}{r} 15 \\ 26 \overline{)390} \end{array}$ (148)		$\begin{array}{r} 28 \\ 13 \overline{)364} \end{array}$ (148)
5.	$\begin{array}{r} 43 \\ 87 \overline{)3741} \end{array}$ (150)		$\begin{array}{r} 76 \\ 27 \overline{)2052} \end{array}$ (150)
6.	$\begin{array}{r} 95 \\ 85 \overline{)8075} \end{array}$ (150)		$\begin{array}{r} 72 \\ 68 \overline{)4896} \end{array}$ (150)

Self-Evaluation

Part 1 Copy. Find each product.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 13 \\ \times 54 \\ \hline 702 \\ \hline \end{array}$	$\begin{array}{r} 27 \\ \times 24 \\ \hline 648 \\ \hline \end{array}$	$\begin{array}{r} 63 \\ \times 14 \\ \hline 882 \\ \hline \end{array}$
2.	$\begin{array}{r} 74 \\ \times 23 \\ \hline 1702 \\ \hline \end{array}$	$\begin{array}{r} 52 \\ \times 61 \\ \hline 3172 \\ \hline \end{array}$	$\begin{array}{r} 129 \\ \times 34 \\ \hline 4386 \\ \hline \end{array}$
3.	$\begin{array}{r} 134 \\ \times 35 \\ \hline 4690 \\ \hline \end{array}$	$\begin{array}{r} 128 \\ \times 41 \\ \hline 5248 \\ \hline \end{array}$	$\begin{array}{r} 241 \\ \times 36 \\ \hline 8676 \\ \hline \end{array}$
4.	$\begin{array}{r} 413 \\ \times 234 \\ \hline 96542 \\ \hline \end{array}$	$\begin{array}{r} 821 \\ \times 314 \\ \hline 257794 \\ \hline \end{array}$	$\begin{array}{r} 461 \\ \times 312 \\ \hline 143832 \\ \hline \end{array}$
5.	$\begin{array}{r} 863 \\ \times 302 \\ \hline 260626 \\ \hline \end{array}$	$\begin{array}{r} 605 \\ \times 204 \\ \hline 123420 \\ \hline \end{array}$	$\begin{array}{r} 513 \\ \times 105 \\ \hline 53865 \\ \hline \end{array}$

Part 2 Copy. Find each quotient.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 4 \\ 16 \overline{)64} \\ \hline 6 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ 18 \overline{)90} \\ \hline 4 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ 19 \overline{)76} \\ \hline 7 \\ \hline \end{array}$
2.	$\begin{array}{r} 6 \\ 14 \overline{)84} \\ \hline 9 \\ \hline \end{array}$	$\begin{array}{r} 24 \\ 18 \overline{)72} \\ \hline 4 \\ \hline \end{array}$	$\begin{array}{r} 42 \\ 13 \overline{)91} \\ \hline 7 \\ \hline \end{array}$
3.	$\begin{array}{r} 20 \\ 51 \overline{)459} \\ \hline 20 \\ \hline \end{array}$	$\begin{array}{r} 20 \\ 41 \overline{)984} \\ \hline 20 \\ \hline \end{array}$	$\begin{array}{r} 23 \\ 12 \overline{)504} \\ \hline 23 \\ \hline \end{array}$
4.	$\begin{array}{r} 78 \\ 42 \overline{)840} \\ \hline 78 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ 33 \overline{)924} \\ \hline 15 \\ \hline \end{array}$	$\begin{array}{r} 64 \\ 18 \overline{)414} \\ \hline 64 \\ \hline \end{array}$
5.	$\begin{array}{r} 48 \\ 51 \overline{)3978} \\ \hline 48 \\ \hline \end{array}$	$\begin{array}{r} 79 \\ 87 \overline{)1305} \\ \hline 79 \\ \hline \end{array}$	$\begin{array}{r} 36 \\ 78 \overline{)4992} \\ \hline 36 \\ \hline \end{array}$
6.	$\begin{array}{r} 48 \\ 73 \overline{)3504} \\ \hline 48 \\ \hline \end{array}$	$\begin{array}{r} 61 \\ 24 \overline{)1896} \\ \hline 61 \\ \hline \end{array}$	$\begin{array}{r} 43 \\ 87 \overline{)3132} \\ \hline 43 \\ \hline \end{array}$
7.	$\begin{array}{r} 48 \\ 45 \overline{)2160} \\ \hline 48 \\ \hline \end{array}$	$\begin{array}{r} 61 \\ 61 \overline{)3721} \\ \hline 61 \\ \hline \end{array}$	$\begin{array}{r} 43 \\ 73 \overline{)3139} \\ \hline 43 \\ \hline \end{array}$

Part 3 Copy. Estimate each of the following products. Find the exact product. Compare the estimated product with the exact product.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 56 \\ \times 91 \\ \hline 5096 \\ \hline \end{array}$	$\begin{array}{r} 31 \\ \times 12 \\ \hline 372 \\ \hline \end{array}$	$\begin{array}{r} 89 \\ \times 23 \\ \hline 2047 \\ \hline \end{array}$
2.	$\begin{array}{r} 593 \\ \times 68 \\ \hline 40324 \\ \hline \end{array}$	$\begin{array}{r} 396 \\ \times 22 \\ \hline 8712 \\ \hline \end{array}$	$\begin{array}{r} 592 \\ \times 31 \\ \hline 18352 \\ \hline \end{array}$
3.	$\begin{array}{r} 498 \\ \times 202 \\ \hline 100596 \\ \hline \end{array}$	$\begin{array}{r} 603 \\ \times 198 \\ \hline 119394 \\ \hline \end{array}$	$\begin{array}{r} 301 \\ \times 494 \\ \hline 148694 \\ \hline \end{array}$

Part 4 Find the greatest common factor of each pair of numbers below. Then find the least common multiple.

	<i>a</i>	<i>b</i>
1.	12 and 16 4; 48	24 and 32 8; 96
2.	10 and 14 2; 70	6 and 14 2; 42

Part 5 Write an open sentence for each problem. Solve the open sentence. Answer the problem.

1. Glenn paid 96 cents for some candy bars. Each candy bar cost him 8 cents. How many candy bars did he buy? $96 \div 8 = \square$; 12 candy bars

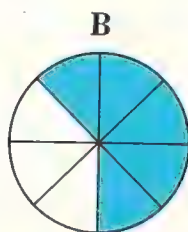
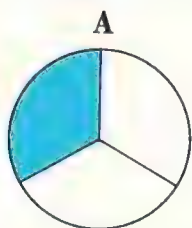
2. Al delivered 3552 newspapers in 48 days. He delivered the same number each day. How many newspapers did he deliver on each day? $3552 \div 48 = \square$; 74 newspapers

Chapter 7

FRACTIONAL NUMBERS

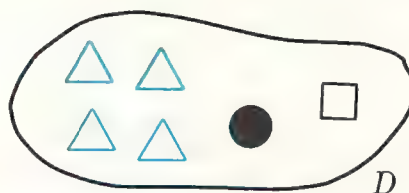
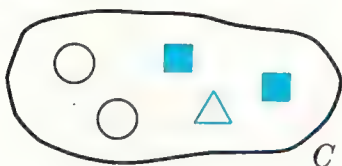
Parts of the Same Size

The inside or interior of each circle below is separated into parts of the same size.



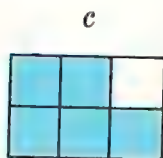
How many parts are there in A? How many of these parts are colored? ³1 You can say that 1 out of 3 parts are colored. In B, 5 out of 8 parts are colored.

How many members are there in set C below? ⁵5 How many of these members are circles? ²2 You can say that 2 out of 5 members are circles.

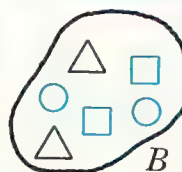
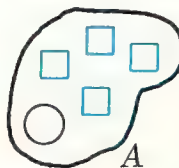


In set D, 4 out of 6 members are triangles.

Oral For each figure below, give a sentence like “___ out of ___ parts are colored.” See below.



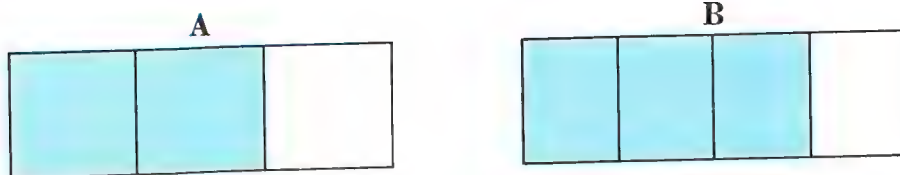
Written For each set below, write a sentence like “___ out of ___ members are circles.” See below.



- a. 1 out of 4 parts are colored.
b. 4 out of 4 parts are colored.
c. 5 out of 6 parts are colored.

- In A, 1 out of 5 members are circles.
In B, 2 out of 6 members are circles.
In C, 2 out of 4 members are circles.

Meaning of a Fraction



The interior of the rectangle in **A** is separated into 3 parts of the same size. How many of these parts are colored?² Can you say that 2 out of 3 parts are colored? You can express 2 out of 3 in two ways as shown below.

Two thirds or $\frac{2}{3}$

The interior of the rectangle in **B** is separated into how many parts of the same size?⁴ How many of these parts are colored?³ Can you say that 3 out of 4 parts are colored? You can express 3 out of 4 in two ways as shown below.

Three fourths or $\frac{3}{4}$

$\frac{2}{3}$ and $\frac{3}{4}$ are **fractions**. They are numerals that name **fractional numbers**. Which fraction could you use to tell how much of the interior in **A** is not colored? Which fraction could you use to tell how much of the interior in **B** is not colored? $\frac{1}{3}$; $\frac{1}{4}$

See the fraction below.

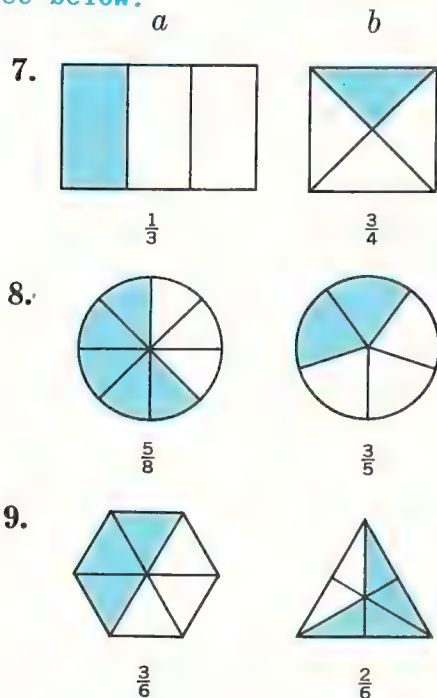
Fraction line $\longrightarrow \frac{7}{8}$ \longleftarrow Numerator
 \longleftarrow Denominator

What do you call the 8 of $\frac{7}{8}$? The *denominator* tells how many parts of the same size the whole has been separated into. What do you call the 7 of $\frac{7}{8}$? The *numerator* tells the number of parts you wish to name. The line between the two numerals of a fraction is called the *fraction line*.

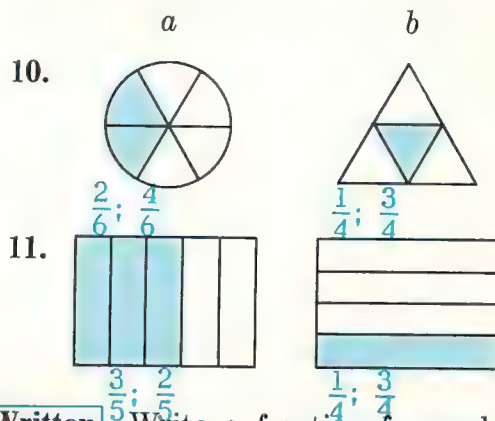
Oral Tell how you would read each of the following fractions. See page T163.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
1.	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{4}{3}$
2.	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
3.	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{5}{5}$
4.	$\frac{1}{6}$	$\frac{3}{6}$	$\frac{5}{6}$	$\frac{6}{6}$
5.	$\frac{3}{7}$	$\frac{5}{7}$	$\frac{6}{7}$	$\frac{7}{7}$
6.	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$

Study each figure shown below. What does each numeral in the fraction tell you about the figure? See below.



Which fraction tells how much of the interior is colored in each figure below? Which fraction tells how much of the interior is *not* colored?



Written Write a fraction for each of the following.

	<i>a</i>	<i>b</i>
1.	one half $\frac{1}{2}$	two thirds $\frac{2}{3}$
2.	four fifths $\frac{4}{5}$	five sixths $\frac{5}{6}$
3.	one eighth $\frac{1}{8}$	six ninths $\frac{6}{9}$
4.	five fourths $\frac{5}{4}$	eight sixths $\frac{8}{6}$
5.	four fourths $\frac{4}{4}$	six sixths $\frac{6}{6}$

Express each fraction below in words. See below.

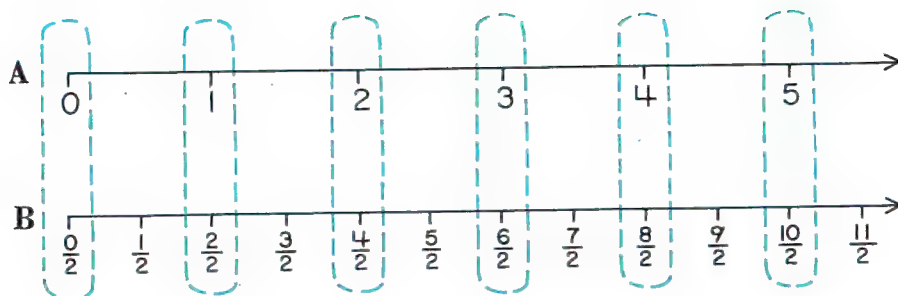
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
6.	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{5}{7}$	$\frac{7}{9}$
7.	$\frac{6}{10}$	$\frac{3}{3}$	$\frac{4}{9}$	$\frac{9}{7}$
8.	$\frac{10}{6}$	$\frac{5}{5}$	$\frac{5}{3}$	$\frac{8}{6}$

Written
 6. two thirds; three fifths, five sevenths; seven ninths
 7. six tenths; three thirds, four ninths; nine sevenths
 8. ten sixths; five fifths; five thirds; eight sixths

Oral

- 7a. 1 out of 3 parts is colored.
 b. 3 out of 4 parts are not colored.
 8a. 5 out of 8 parts are colored.
 b. 3 out of 5 parts are colored.
 9a. 3 out of 6 parts are colored or not colored.
 b. 2 out of 6 parts are not colored.

Fractional Numbers



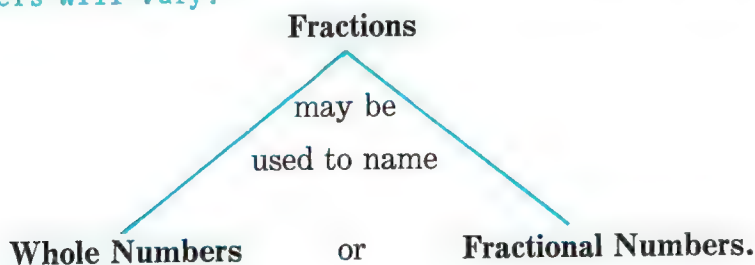
Compare the two number lines above. How are the number lines alike? How are they different? Both have the same direction; In A, the scale is wholes, while in B the scale is halves.

The number line in **A** is in the scale of wholes because only whole numbers are represented on the number line. Which numbers are represented on the number line in **B**? Can you say that the number line in **B** is in the scale of halves? Why? halves; Yes; Only fractions which express halves are used in B.

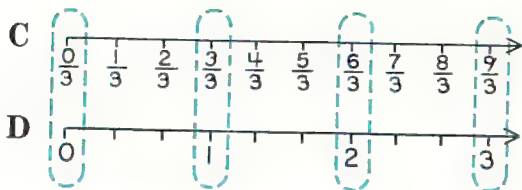
Which fraction in **B** names the whole number zero? Which fraction in **B** names the whole number one? What other fractions in **B** name whole numbers? Which whole number is named by $\frac{0}{2}$? By $\frac{2}{2}$? By $\frac{4}{2}$? By $\frac{6}{2}$? By $\frac{8}{2}$? By $\frac{10}{2}$? $\frac{0}{2}$; $\frac{2}{2}$; $\frac{4}{2}$; $\frac{6}{2}$; $\frac{8}{2}$; and $\frac{10}{2}$; 0; 2; 3; 4; 5

Do all the fractions in **B** name whole numbers? Which fractions in **B** do not name whole numbers? The fractions $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$, and $\frac{11}{2}$ are names for fractional numbers.

Does $\frac{1}{3}$ name a whole number? Does $\frac{3}{4}$ name a whole number? Tell some other fractions which *do not* name whole numbers. Answers will vary.



Oral Study the two number lines below.



Answer the following questions.

1. The number line in C is drawn in which scale? **thirds**

2. Which fraction in C names the whole number zero? The whole number one? **$\frac{0}{3}$; $\frac{3}{3}$**

3. Which fraction in C names the whole number two? The whole number three? **$\frac{6}{3}$; $\frac{9}{3}$**

4. Do all the fractions in C name whole numbers? **No**

5. Which fractions in C do not name whole numbers? **$\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, $\frac{5}{3}$, $\frac{7}{3}$, $\frac{8}{3}$**

6. How do you know that the fractions $\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, and $\frac{5}{3}$ name fractional numbers? **They do not name whole numbers.**

Tell whether each fraction below names a whole number or a fractional number. **Fractions for whole numbers are underlined.**

a	b	c	d
7. <u>$\frac{4}{1}$</u>	$\frac{3}{4}$	$\frac{2}{3}$	<u>$\frac{6}{2}$</u>
8. $\frac{5}{4}$	$\frac{7}{5}$	<u>$\frac{5}{5}$</u>	$\frac{3}{6}$
9. <u>$\frac{6}{3}$</u>	<u>$\frac{8}{4}$</u>	$\frac{4}{8}$	$\frac{5}{7}$

Written Answer the following.
Typical answers given.

1. Use 2 as the denominator to write 7 fractions that name whole numbers. **$\frac{0}{2}$, $\frac{2}{2}$, $\frac{4}{2}$, $\frac{6}{2}$, $\frac{8}{2}$, $\frac{10}{2}$, $\frac{12}{2}$**

2. Use 3 as the denominator to write 7 fractions that name whole numbers. **$\frac{0}{3}$, $\frac{3}{3}$, $\frac{6}{3}$, $\frac{9}{3}$, $\frac{12}{3}$, $\frac{15}{3}$, $\frac{18}{3}$**

3. Use 4 as the denominator to write 7 fractions that name whole numbers. **$\frac{0}{4}$, $\frac{4}{4}$, $\frac{8}{4}$, $\frac{12}{4}$, $\frac{16}{4}$, $\frac{20}{4}$, $\frac{24}{4}$**

4. Write 5 fractions that name the whole number 1. **$\frac{1}{1}$, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$**

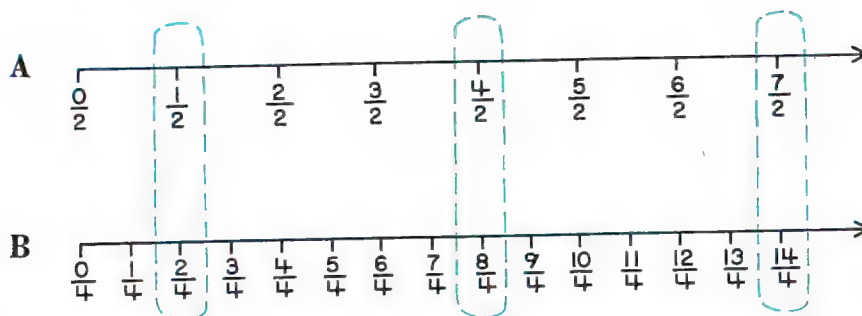
5. Write 5 fractions that name the whole number 2. **$\frac{2}{1}$, $\frac{4}{2}$, $\frac{6}{3}$, $\frac{8}{4}$, $\frac{10}{5}$**

Use each pair of numerals below to write 2 different fractions. Then tell which fraction names a whole number and which names a fractional number. **Fractions for whole numbers are listed first.**

2. <u>$\frac{2}{1}$</u> ; $\frac{a}{2}$	3. <u>$\frac{3}{1}$</u> ; $\frac{b}{3}$	6. <u>$\frac{6}{2}$</u> ; $\frac{c}{6}$
6. 2 and 1	3 and 1	6 and 2
9. <u>$\frac{9}{3}$</u> ; $\frac{3}{9}$	6. <u>$\frac{6}{1}$</u> ; $\frac{1}{6}$	$\frac{10}{2}$; <u>$\frac{2}{10}$</u>
7. 3 and 9	6 and 1	10 and 2
8. <u>$\frac{8}{1}$</u> ; $\frac{1}{8}$	<u>$\frac{8}{4}$</u> ; $\frac{4}{8}$	$\frac{4}{2}$; <u>$\frac{2}{4}$</u>
8. 8 and 1	4 and 8	4 and 2
<u>$\frac{12}{4}$</u> ; $\frac{4}{12}$	<u>$\frac{8}{2}$</u> ; $\frac{2}{8}$	$\frac{6}{3}$; <u>$\frac{3}{6}$</u>
9. 4 and 12	2 and 8	6 and 3

Can you do this? Draw a number line in the scale of fourths, starting with $\frac{0}{4}$ and ending with $\frac{25}{4}$. Circle those fractions that name whole numbers. **$\frac{0}{4}$, $\frac{4}{4}$, $\frac{8}{4}$, $\frac{12}{4}$, $\frac{16}{4}$, $\frac{20}{4}$ and $\frac{24}{4}$ should be circled.**

Equivalent Fractions



Compare the two number lines above. How are the number lines alike? How are they different? Both have the same direction; In A, the scale is halves, while in B the scale is fourths.

The fraction $\frac{1}{2}$ on the number line in A names the same number as the fraction $\frac{2}{4}$ on the number line in B. The fractions $\frac{1}{2}$ and $\frac{2}{4}$ are two different names for the same number. We can express this idea as follows.

$$\frac{1}{2} = \frac{2}{4}$$

Do $\frac{4}{2}$ and $\frac{8}{4}$ both name the same number? Can you say that $\frac{4}{2}$ and $\frac{8}{4}$ are two different names for the same number? Can you express this idea as follows? Yes; Yes; Yes

$$\frac{4}{2} = \frac{8}{4}$$

Do $\frac{7}{2}$ and $\frac{14}{4}$ both name the same number? Can you say that the following is true? Yes; Yes

$$\frac{7}{2} = \frac{14}{4}$$

Name some other pairs of fractions on the number lines above that name the same number. $\frac{0}{2} = \frac{0}{4}$, $\frac{2}{2} = \frac{4}{4}$, $\frac{3}{2} = \frac{6}{4}$, $\frac{5}{2} = \frac{10}{4}$, and $\frac{6}{2} = \frac{12}{4}$

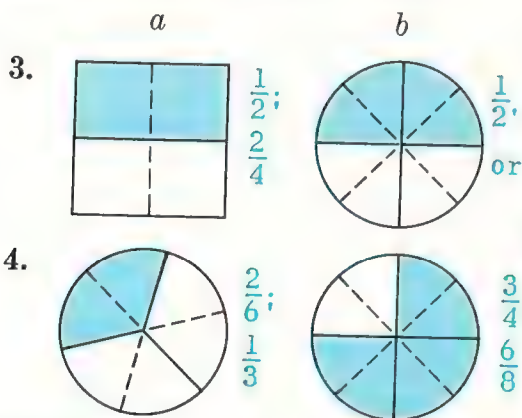
Fractions that name the same number are called **equivalent fractions**.

Oral Answer these questions.

1. The fractions $\frac{1}{2}$ and $\frac{2}{4}$ name the same number. In what way is $\frac{1}{2}$ of a candy bar different from $\frac{2}{4}$ of a candy bar? **See below.**

2. The fractions $\frac{1}{2}$ and $\frac{3}{6}$ name the same number. In what way is $\frac{1}{2}$ of an apple different from $\frac{3}{6}$ of an apple? **See below.**

Tell two different fractions for the colored part of each figure below.



Written Copy. Discover the pattern in each exercise below. Write the next three fractions.

$$1. \frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$$

$$2. \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = \frac{10}{5} = \frac{12}{6}$$

$$3. \frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18}$$

$$4. \frac{3}{1} = \frac{6}{2} = \frac{9}{3} = \frac{12}{4} = \frac{15}{5} = \frac{18}{6}$$

$$5. \frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25} = \frac{18}{30}$$

Oral 1. $\frac{1}{2}$ of a candy bar refers to 1 out of 2 parts of the same size and $\frac{2}{4}$ refers to 2 out of 4 parts of the same size. 167

2. $\frac{1}{2}$ of an apple refers to 1 out of 2 parts of the same size and $\frac{3}{6}$ refers to 3 out of 6 parts of the same size.

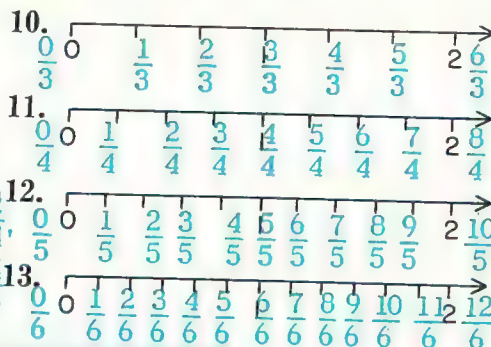
$$6. \frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24}$$

$$7. \frac{4}{1} = \frac{8}{2} = \frac{12}{3} = \frac{16}{4} = \frac{20}{5} = \frac{24}{6}$$

$$8. \frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18}$$

$$9. \frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24}$$

Copy the number lines below. Label each mark with a fraction.



Can you do this? In $\frac{4}{2} = \frac{8}{4}$, find the products shown below.

$$4 \times 8 = 32 \rightarrow 2 \times 8 = 16$$

$$2 \times 4 = 8 \rightarrow 4 \times 4 = 16$$

What do you discover about the two products? Is this true for $\frac{7}{2} = \frac{14}{4}$? **They are the same; Yes**

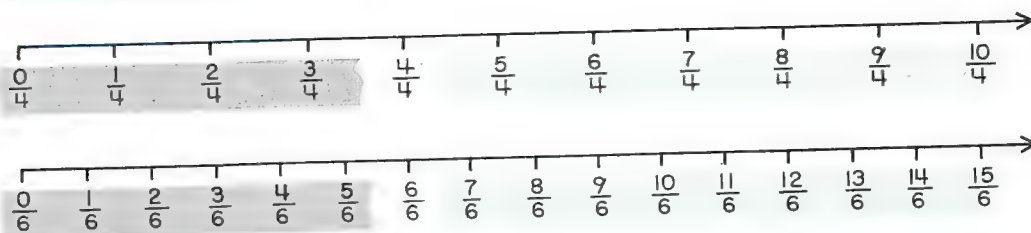
Copy. Write the numeral that should replace the \square to make each sentence true.

a $\frac{2}{2} = \frac{\square}{4}$ b $\frac{3}{\square} = \frac{6}{4}$ c $\frac{10}{4} = \frac{5}{\square}$

1. $\frac{2}{2} = \frac{\square}{4}$ 2. $\frac{16}{4} = \frac{\square}{2}$ 3. $\frac{9}{2} = \frac{\square}{4}$ 4. $\frac{2}{6} = \frac{\square}{3}$

5. $\frac{12}{4} = \frac{\square}{2}$ 6. $\frac{14}{\square} = \frac{7}{2}$ 7. $\frac{6}{10} = \frac{3}{\square}$ 8. $\frac{2}{3} = \frac{4}{\square}$

Kinds of Fractions



Notice the fractions in the gray portion of each number line. Is the numerator greater or less than the denominator in $\frac{1}{4}$? ^{*1} In $\frac{3}{4}$? ^{*1} In $\frac{5}{6}$? ^{*2} In $\frac{5}{6}$? ^{*1} Name other fractional numbers whose numerator is less than the denominator. If the numerator is less than the denominator, the fraction is called a **proper fraction**. A *proper fraction* names a number less than 1.

Notice the fraction $\frac{4}{4}$. Is the numerator greater than, less than, or equal to the denominator? ^{*2} Name another fractional number whose numerator is equal to the denominator. If the numerator and denominator are equal, what number does the fraction name? ₁

Notice the fractions $\frac{5}{4}$, $\frac{8}{4}$, $\frac{10}{4}$, $\frac{7}{6}$, $\frac{10}{6}$, and $\frac{15}{6}$. Is the numerator of each of these greater than or less than the denominator? ^{*3} If the numerator is greater than or equal to the denominator, the fraction is an **improper fraction**. An *improper fraction* names the number 1 or names a number greater than 1. Name some other improper fractions.

Oral Tell whether each fraction below names a number less than one, greater than one, or the number one.

$$a \quad b \quad c \quad d \quad e \quad f$$

$$1. \quad \frac{11}{4} > 1 \quad \frac{1}{4} < 1 \quad \frac{3}{5} < 1 \quad \frac{3}{7} < 1 \quad \frac{10}{9} > 1 \quad \frac{9}{9} = 1$$

$$2. \quad \frac{9}{10} < 1 \quad \frac{4}{1} > 1 \quad \frac{6}{6} = 1 \quad \frac{3}{8} < 1 \quad \frac{8}{8} = 1 \quad \frac{10}{8} > 1$$

Written Do the following.

1. Use 9 as a denominator to write 8 fractions that name a number less than one. That name a number greater than one. **See below.**

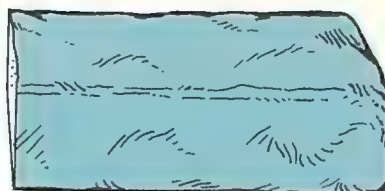
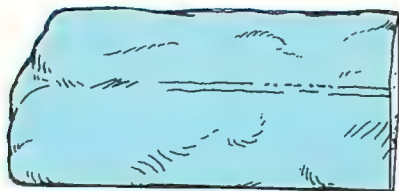
2. Write 6 fractions that name the number one. $\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \frac{6}{6}$, and so on

168

- *1 less than the denominator
- *2 equal to the denominator
- *3 greater than the denominator

Written 1. Any eight of $\frac{0}{9}, \frac{1}{9}, \frac{2}{9}$, and so on to $\frac{8}{9}$ may be accepted; any eight of $\frac{10}{9}, \frac{11}{9}, \frac{12}{9}$, and so on may be accepted

Fractions Indicate Division



Think of the picture above as showing a candy bar being separated into 2 parts of the same size. In mathematics, you can think of this as 1 divided by 2. Which division numeral can you use to express this idea? $1 \div 2$

Does the picture above suggest that you can represent each of the two parts of the same size by a fraction? Which fraction? $\frac{1}{2}$ Since $\frac{1}{2}$ and $1 \div 2$ both name the same number, the fraction $\frac{1}{2}$ may also be thought of as $1 \div 2$. You can express this idea as follows.

$$1 \div 2 = \frac{1}{2}$$

A division numeral may be thought of as a fraction in which the dividend is the numerator and the divisor is the denominator.

Oral Express each of the following fractions as a division numeral.

- | | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|----|---------------|---------------|---------------|---------------|
| 1. | $\frac{1}{6}$ | $\frac{2}{3}$ | $\frac{4}{5}$ | $\frac{8}{9}$ |
| | $1 \div 6$ | $2 \div 3$ | $4 \div 5$ | $8 \div 9$ |
| 2. | $\frac{1}{5}$ | $\frac{6}{5}$ | $\frac{8}{8}$ | $\frac{3}{2}$ |
| | $1 \div 5$ | $6 \div 5$ | $8 \div 8$ | $3 \div 2$ |

Express each of the following division numerals as a fraction.

- | | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|----|---------------|---------------|---------------|---------------|
| 3. | $2 \div 5$ | $4 \div 7$ | $8 \div 9$ | $9 \div 7$ |
| | $\frac{2}{5}$ | $\frac{4}{7}$ | $\frac{8}{9}$ | $\frac{9}{7}$ |
| 4. | $6 \div 5$ | $4 \div 4$ | $1 \div 9$ | $5 \div 5$ |
| | $\frac{6}{5}$ | $\frac{4}{4}$ | $\frac{1}{9}$ | $\frac{5}{5}$ |

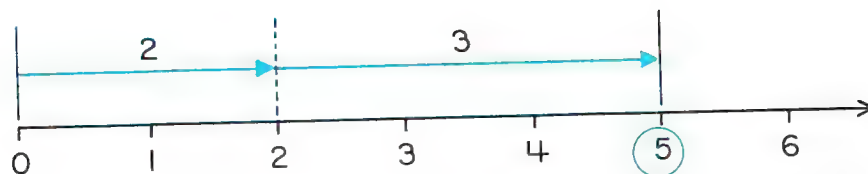
Written Write a fraction for each division numeral below.

- | | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|----|---------------|---------------|---------------|---------------|
| 1. | $1 \div 3$ | $1 \div 4$ | $3 \div 4$ | $6 \div 8$ |
| | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{6}{8}$ |
| 2. | $6 \div 6$ | $5 \div 4$ | $6 \div 3$ | $7 \div 9$ |
| | $\frac{6}{6}$ | $\frac{5}{4}$ | $\frac{6}{3}$ | $\frac{7}{9}$ |

Write a division numeral for each fraction below.

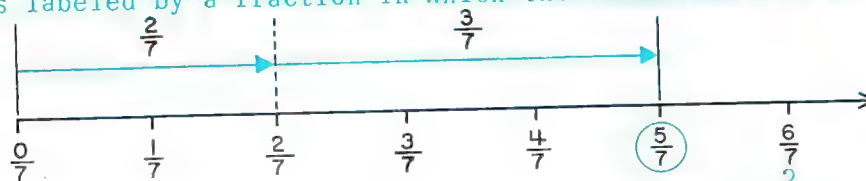
- | | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|----|---------------|---------------|---------------|---------------|
| 3. | $\frac{1}{5}$ | $\frac{2}{7}$ | $\frac{3}{8}$ | $\frac{5}{8}$ |
| | $1 \div 5$ | $2 \div 7$ | $3 \div 8$ | $5 \div 8$ |
| 4. | $\frac{9}{9}$ | $\frac{4}{3}$ | $\frac{7}{5}$ | $\frac{2}{2}$ |
| | $9 \div 9$ | $4 \div 3$ | $7 \div 5$ | $2 \div 2$ |

Adding Fractional Numbers



What addition sentence is suggested by the number line above? You can also use a number line to show the addition of fractional numbers. $2+3=5$

Study the number line shown below. In what scale is the number line drawn? How do you know? *sevenths; Each point is labeled by a fraction in which the denominator is 7.*



What addend is shown by the arrow from 0 to $\frac{2}{7}$? $\frac{2}{7}$ What addend is shown by the arrow from $\frac{2}{7}$ to $\frac{5}{7}$? $\frac{3}{7}$ What is the sum of $\frac{2}{7}$ and $\frac{3}{7}$? $\frac{5}{7}$ Does this number line suggest the following? *Yes*

$$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$$

What is alike about the addends $\frac{2}{7}$ and $\frac{3}{7}$? How can you find the simplest numeral for the numerator of the sum? What is the denominator of the sum? Do $\frac{2}{7} + \frac{3}{7}$ and $\frac{5}{7}$ both name the same number? *the denominator; Add the numerators; 7; Yes*

This addition example can be expressed as shown below.

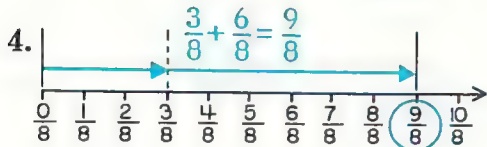
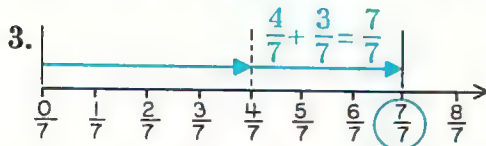
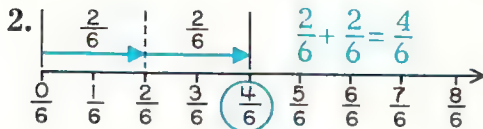
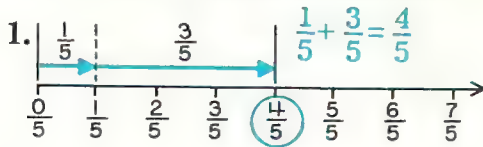
Add the numerators.

$$\begin{array}{r} \frac{2}{7} \\ + \frac{3}{7} \\ \hline \frac{5}{7} \end{array}$$

$2+3=5$

Use the same denominator.

Oral Tell a closed addition sentence for each number line below.



Answer the following.

5. In adding fractional numbers like $\frac{3}{10}$ and $\frac{5}{10}$, how do you decide the denominator for the sum? **Use the denominator of the addends.**

6. In adding fractional numbers like $\frac{3}{10}$ and $\frac{5}{10}$, how do you find the numerator for the sum? **Add the numerators of the addends.**

In each of the following, which single fraction should replace each \square ?

a	b
7. $\frac{2}{9} + \frac{4}{9} = \square \frac{6}{9}$	$\frac{3}{7} + \frac{3}{7} = \square \frac{6}{7}$
8. $\frac{7}{15} + \frac{8}{15} = \square \frac{15}{15}$	$\frac{9}{17} + \frac{8}{17} = \square \frac{17}{17}$
9. $\frac{7}{25} + \frac{3}{25} = \square \frac{10}{25}$	$\frac{9}{18} + \frac{7}{18} = \square \frac{16}{18}$

Written Draw a number line to illustrate each of the following addition sentences.

a	b
1. $\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$	$\frac{1}{6} + \frac{3}{6} = \frac{4}{6}$
2. $\frac{3}{8} + \frac{3}{8} = \frac{6}{8}$	$\frac{2}{7} + \frac{5}{7} = \frac{7}{7}$

Copy. Solve each open sentence below.

a	b
3. $\frac{1}{8} + \frac{3}{8} = \square \frac{4}{8}$	$\frac{1}{9} + \frac{3}{9} = \square \frac{4}{9}$
4. $\frac{2}{6} + \frac{2}{6} = \square \frac{4}{6}$	$\frac{4}{10} + \frac{4}{10} = \square \frac{8}{10}$
5. $\frac{7}{18} + \frac{6}{18} = \square \frac{13}{18}$	$\frac{9}{12} + \frac{2}{12} = \square \frac{11}{12}$
6. $\frac{4}{15} + \frac{9}{15} = \square \frac{13}{15}$	$\frac{9}{42} + \frac{9}{42} = \square \frac{18}{42}$

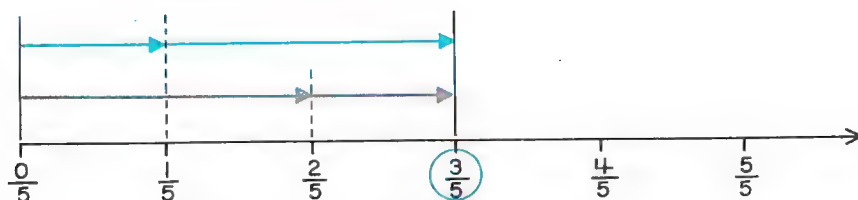
Copy. Find each sum.

a	b	c	d
7. $\frac{3}{10} + \frac{6}{10} = \frac{9}{10}$	$\frac{4}{14} + \frac{8}{14} = \frac{12}{14}$	$\frac{7}{15} + \frac{6}{15} = \frac{13}{15}$	$\frac{8}{16} + \frac{8}{16} = \frac{16}{16}$
8. $\frac{7}{14} + \frac{5}{14} = \frac{12}{14}$	$\frac{8}{13} + \frac{5}{13} = \frac{13}{13}$	$\frac{7}{14} + \frac{7}{14} = \frac{14}{14}$	$\frac{6}{20} + \frac{4}{20} = \frac{10}{20}$
9. $\frac{8}{21} + \frac{2}{21} = \frac{10}{21}$	$\frac{7}{19} + \frac{4}{19} = \frac{11}{19}$	$\frac{6}{24} + \frac{6}{24} = \frac{12}{24}$	$\frac{4}{13} + \frac{9}{13} = \frac{13}{13}$

Can you do this? Draw a number line to illustrate each of the following addition sentences.

a	b
$\frac{1}{8} + \frac{2}{8} + \frac{3}{8} = \frac{6}{8}$	$\frac{4}{9} + \frac{2}{9} + \frac{1}{9} = \frac{7}{9}$

Addition Is Commutative



Study the number line above. In what scale is it drawn? *fifths*

Notice the gray arrows above the number line. Do the gray arrows suggest the closed addition sentence shown below? *Yes*

$$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$

What is the first addend in the closed addition sentence above? What is the second addend? At which mark on the number line does the second gray arrow stop? *$\frac{2}{5}$; $\frac{1}{5}$; $\frac{3}{5}$*

Notice the blue arrows above the number line. Do the blue arrows suggest the closed addition sentence shown below? *Yes*

$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$

What is the first addend in the closed addition sentence above? What is the second addend? At which mark on the number line does the second blue arrow stop? *$\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{5}$*

Why can you say that the following is true? *$\frac{2}{5} + \frac{1}{5}$ and $\frac{1}{5} + \frac{2}{5}$ name the same number.*

$$\frac{2}{5} + \frac{1}{5} = \frac{1}{5} + \frac{2}{5}$$

What number do both $\frac{2}{5} + \frac{1}{5}$ and $\frac{1}{5} + \frac{2}{5}$ name? Does changing the order of the addends change the sum? *$\frac{3}{5}$; No*

Changing the order of two addends does not change the sum. For all numbers that can be named by fractions, addition is commutative.

Oral Tell the simplest numeral that should replace the \square so each sentence becomes true.

1. $\frac{2}{5} + \frac{3}{5} = \frac{3}{5} + \frac{\square}{5}$ ²

2. $\frac{3}{7} + \frac{4}{7} = \frac{4}{7} + \frac{\square}{7}$ ³

3. $\frac{1}{8} + \frac{7}{8} = \frac{7}{8} + \frac{\square}{8}$ ¹

4. $\frac{6}{17} + \frac{7}{17} = \frac{\square}{17} + \frac{6}{17}$ ⁷

5. $\frac{7}{15} + \frac{9}{15} = \frac{\square}{15} + \frac{7}{15}$ ⁹

For each fractional number below, tell a closed addition sentence as shown below. **Answers will vary.** Representative answers for column a only are $\frac{5}{8} = \frac{4}{8} + \frac{1}{8} = \frac{1}{8} + \frac{4}{8}$ shown below.

a	b	c	d
6. $\frac{7}{8}$	$\frac{5}{6}$	$\frac{8}{9}$	$\frac{6}{10}$
7. $\frac{14}{16}$	$\frac{13}{15}$	$\frac{16}{18}$	$\frac{17}{20}$
8. $\frac{9}{20}$	$\frac{11}{15}$	$\frac{14}{17}$	$\frac{18}{25}$
9. $\frac{12}{24}$	$\frac{10}{40}$	$\frac{15}{20}$	$\frac{4}{19}$
10. $\frac{21}{22}$	$\frac{24}{30}$	$\frac{27}{36}$	$\frac{32}{40}$

Written Copy. Replace each \square by the simplest numeral to make each sentence true.

a	b
1. $\frac{\square}{18} + \frac{7}{18} = \frac{7}{18} + \frac{9}{18}$ ⁹	$\frac{0}{20} + \frac{19}{20} = \frac{19}{20} + \frac{\square}{20}$ ⁰
2. $\frac{7}{8} + \frac{2}{8} = \frac{\square}{8} + \frac{7}{8}$ ²	$\frac{8}{9} + \frac{1}{9} = \frac{1}{9} + \frac{\square}{9}$ ⁸
3. $\frac{9}{10} + \frac{4}{10} = \frac{\square}{10} + \frac{9}{10}$ ⁴	$\frac{4}{12} + \frac{\square}{12} = \frac{8}{12} + \frac{4}{12}$ ⁸

Oral

6. $\frac{7}{8} = \frac{1}{8} + \frac{6}{8} = \frac{6}{8} + \frac{1}{8}$

7. $\frac{14}{16} = \frac{4}{16} + \frac{10}{16} = \frac{10}{16} + \frac{4}{16}$

8. $\frac{9}{20} = \frac{5}{20} + \frac{4}{20} = \frac{4}{20} + \frac{5}{20}$

4. $\frac{12}{24} + \frac{\square}{24} = \frac{9}{24} + \frac{12}{24}$ ⁹ $\frac{14}{25} + \frac{8}{25} = \frac{\square}{25} + \frac{14}{25}$ ⁸

5. $\frac{11}{20} + \frac{9}{20} = \frac{9}{20} + \frac{\square}{20}$ ¹¹ $\frac{\square}{30} + \frac{12}{30} = \frac{12}{30} + \frac{13}{30}$ ¹³

Copy. Replace n in each of the following sentences to make each sentence true.

6. If $\frac{8}{10} + \frac{1}{10} = \frac{9}{10}$, then $\frac{1}{10} + \frac{8}{10} = n$. $\frac{9}{10}$

7. If $\frac{6}{12} + \frac{3}{12} = \frac{9}{12}$, then $\frac{3}{12} + \frac{6}{12} = n$. $\frac{9}{12}$

8. If $\frac{8}{15} + \frac{4}{15} = \frac{12}{15}$, then $\frac{4}{15} + \frac{8}{15} = n$. $\frac{12}{15}$

9. If $\frac{9}{20} + \frac{6}{20} = \frac{15}{20}$, then $\frac{6}{20} + \frac{9}{20} = n$. $\frac{15}{20}$

10. If $\frac{18}{40} + \frac{13}{40} = \frac{31}{40}$, then $\frac{13}{40} + \frac{18}{40} = n$. $\frac{31}{40}$

11. If $\frac{16}{35} + \frac{17}{35} = \frac{33}{35}$, then $\frac{17}{35} + \frac{16}{35} = n$. $\frac{33}{35}$

12. If $\frac{19}{20} + \frac{8}{20} = \frac{27}{20}$, then $\frac{8}{20} + \frac{19}{20} = n$. $\frac{27}{20}$

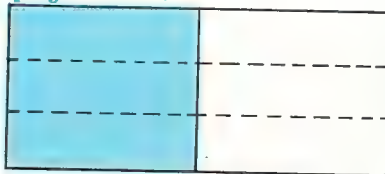
13. If $\frac{22}{30} + \frac{7}{30} = \frac{29}{30}$, then $\frac{7}{30} + \frac{22}{30} = n$. $\frac{29}{30}$

14. If $\frac{18}{40} + \frac{14}{40} = \frac{32}{40}$, then $\frac{14}{40} + \frac{18}{40} = n$. $\frac{32}{40}$

15. If $\frac{38}{90} + \frac{46}{90} = \frac{84}{90}$, then $\frac{46}{90} + \frac{38}{90} = n$. $\frac{84}{90}$

Can you do this? Count by $\frac{1}{2}$'s to 4. ($\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, and so on.) How many $\frac{1}{2}$'s are there in 4? $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, $\frac{5}{2}$, $\frac{6}{2}$, $\frac{7}{2}$, $\frac{8}{2}$; 8

Tell how. The figure below shows that $\frac{1}{2}$ is equivalent to $\frac{3}{6}$. See page T173.



9. $\frac{12}{24} = \frac{0}{24} + \frac{12}{24} = \frac{12}{24} + \frac{0}{24}$

10. $\frac{21}{22} = \frac{7}{22} + \frac{14}{22} = \frac{14}{22} + \frac{7}{22}$

Addition Is Associative

A

Whole Numbers

$$(2+3)+4=2+(3+4)$$

B

Fractional Numbers

$$\left(\frac{2}{10}+\frac{3}{10}\right)+\frac{4}{10}=\frac{2}{10}+\left(\frac{3}{10}+\frac{4}{10}\right)$$

In **A**, what do the parentheses indicate in $(2+3)+4$? In $2+(3+4)$? Does the way in which we group the addends change the sum? What number do both $(2+3)+4$ and $2+(3+4)$ name? What property of addition of whole numbers is illustrated in **A**?
that 2+3 is to be found first; that 3+4 is to be found first;
No; 9; the commutative property

In **B**, what do the parentheses in $\left(\frac{2}{10}+\frac{3}{10}\right)+\frac{4}{10}$ mean? What do the parentheses in $\frac{2}{10}+\left(\frac{3}{10}+\frac{4}{10}\right)$ mean? Think about the examples shown below. that $\frac{2}{10}+\frac{3}{10}$ is to be found first;
that $\frac{3}{10}+\frac{4}{10}$ is to be found first

C

D

$$\begin{aligned}\left(\frac{2}{10}+\frac{3}{10}\right)+\frac{4}{10} &= \frac{2+3}{10}+\frac{4}{10} \\ &= \frac{5}{10}+\frac{4}{10} \\ &= \frac{5+4}{10} \\ &= \frac{9}{10}\end{aligned}$$

$$\begin{aligned}\frac{2}{10}+\left(\frac{3}{10}+\frac{4}{10}\right) &= \frac{2}{10}+\frac{3+4}{10} \\ &= \frac{2}{10}+\frac{7}{10} \\ &= \frac{2+7}{10} \\ &= \frac{9}{10}\end{aligned}$$

In **C**, what number is named by $\left(\frac{2}{10}+\frac{3}{10}\right)+\frac{4}{10}$? In **D**, what number is named by $\frac{2}{10}+\left(\frac{3}{10}+\frac{4}{10}\right)$? Do both $\left(\frac{2}{10}+\frac{3}{10}\right)+\frac{4}{10}$ and $\frac{2}{10}+\left(\frac{3}{10}+\frac{4}{10}\right)$ name the same number? Does the way in which you group the addends change the sum? $\frac{9}{10}$; $\frac{9}{10}$; Yes; No

Do you think addition of fractional numbers is associative? Why? Yes; In **C** and **D** above, the way in which addends are grouped does not change the sum.

The way in which you group the addends does not change the sum. For all numbers that can be named by fractions, addition is associative.

Oral Answer the following.

1. What does "addition is associative" mean? The way in which addends are grouped does not change the sum.

2. Which two numbers are to be added first in $(\frac{2}{5} + \frac{1}{5}) + \frac{3}{5}$? $\frac{2}{5}$ and $\frac{1}{5}$

3. Which two numbers are to be added first in $\frac{2}{5} + (\frac{1}{5} + \frac{3}{5})$? $\frac{1}{5}$ and $\frac{3}{5}$

What single fraction should replace n to make each of the following a true sentence?

4. $(\frac{2}{9} + \frac{3}{9}) + \frac{4}{9} = n + (\frac{3}{9} + \frac{4}{9})$ $\frac{2}{9}$

5. $(\frac{3}{10} + \frac{4}{10}) + \frac{2}{10} = \frac{3}{10} + (n + \frac{2}{10})$ $\frac{4}{10}$

6. $\frac{1}{8} + (\frac{2}{8} + \frac{4}{8}) = (\frac{1}{8} + n) + \frac{4}{8}$ $\frac{2}{8}$

7. $(\frac{3}{12} + \frac{5}{12}) + \frac{1}{12} = \frac{3}{12} + (\frac{5}{12} + n)$ $\frac{1}{12}$

Tell where you would place parentheses to show the associative property of addition in each of the following.

8. $(\frac{3}{12} + \frac{4}{12}) + \frac{5}{12} = \frac{3}{12} + (\frac{4}{12} + \frac{5}{12})$

9. $\frac{4}{20} + (\frac{6}{20} + \frac{8}{20}) = (\frac{4}{20} + \frac{6}{20}) + \frac{8}{20}$

10. $(\frac{5}{25} + \frac{9}{25}) + \frac{8}{25} = \frac{5}{25} + (\frac{9}{25} + \frac{8}{25})$

11. $\frac{6}{24} + (\frac{8}{24} + \frac{7}{24}) = (\frac{6}{24} + \frac{8}{24}) + \frac{7}{24}$

12. $(\frac{7}{32} + \frac{3}{32}) + \frac{5}{32} = \frac{7}{32} + (\frac{3}{32} + \frac{5}{32})$

13. $(\frac{8}{36} + \frac{7}{36}) + \frac{3}{36} = \frac{8}{36} + (\frac{7}{36} + \frac{3}{36})$

14. $\frac{6}{40} + (\frac{4}{40} + \frac{8}{40}) = (\frac{6}{40} + \frac{4}{40}) + \frac{8}{40}$

Written Copy. Use the method below to show that each of the following is a true sentence. See page T175.

$$\begin{aligned}(\frac{1}{7} + \frac{2}{7}) + \frac{3}{7} &= \frac{1}{7} + (\frac{2}{7} + \frac{3}{7}) \\ \frac{3}{7} + \frac{3}{7} &= \frac{1}{7} + \frac{5}{7} \\ \frac{6}{7} &= \frac{6}{7}\end{aligned}$$

1. $(\frac{2}{12} + \frac{3}{12}) + \frac{4}{12} = \frac{2}{12} + (\frac{3}{12} + \frac{4}{12})$

2. $(\frac{3}{20} + \frac{9}{20}) + \frac{8}{20} = \frac{3}{20} + (\frac{9}{20} + \frac{8}{20})$

3. $(\frac{4}{24} + \frac{6}{24}) + \frac{8}{24} = \frac{4}{24} + (\frac{6}{24} + \frac{8}{24})$

4. $\frac{5}{25} + (\frac{9}{25} + \frac{8}{25}) = (\frac{5}{25} + \frac{9}{25}) + \frac{8}{25}$

5. $\frac{9}{18} + (\frac{7}{18} + \frac{1}{18}) = (\frac{9}{18} + \frac{7}{18}) + \frac{1}{18}$

6. $\frac{8}{20} + (\frac{2}{20} + \frac{9}{20}) = (\frac{8}{20} + \frac{2}{20}) + \frac{9}{20}$

7. $\frac{6}{21} + (\frac{8}{21} + \frac{2}{21}) = (\frac{6}{21} + \frac{8}{21}) + \frac{2}{21}$

8. $(\frac{7}{24} + \frac{3}{24}) + \frac{8}{24} = \frac{7}{24} + (\frac{3}{24} + \frac{8}{24})$

9. $\frac{9}{20} + (\frac{4}{20} + \frac{6}{20}) = (\frac{9}{20} + \frac{4}{20}) + \frac{6}{20}$

Copy. Solve each open sentence by replacing n with a single fraction.

	a	b
10.	$\frac{6}{24} + \frac{4}{24} + \frac{8}{24} = n$ $\frac{18}{24}$	$\frac{9}{25} + \frac{3}{25} + \frac{7}{25} = n$ $\frac{19}{25}$
11.	$\frac{2}{30} + \frac{8}{30} + \frac{6}{30} = n$ $\frac{16}{30}$	$\frac{6}{20} + \frac{7}{20} + \frac{3}{20} = n$ $\frac{16}{20}$
12.	$\frac{8}{30} + \frac{9}{30} + \frac{1}{30} = n$ $\frac{18}{30}$	$\frac{6}{50} + \frac{4}{50} + \frac{9}{50} = n$ $\frac{19}{50}$

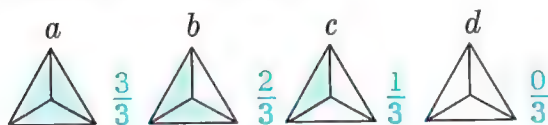
Can you do this? Draw a number line to show that the following is a true sentence.

$$(\frac{8}{18} + \frac{2}{18}) + \frac{4}{18} = (\frac{4}{18} + \frac{2}{18}) + \frac{8}{18}$$

Review and Practice

Part 1 Answer the following.

1. Write a fraction which tells how much of the interior of each triangle below is colored.



2. Study the picture below.



Three out of five of the birds are blue jays. Which fraction can you use to express this idea? $\frac{3}{5}$

3. Which pair or pairs of fractions below have the same denominator? $\frac{2}{6}$ and $\frac{3}{6}$, $\frac{5}{16}$ and $\frac{9}{16}$

a	b	c
$\frac{2}{6}, \frac{3}{6}$	$\frac{1}{4}, \frac{3}{8}$	$\frac{5}{16}, \frac{9}{16}$

4. Write the denominator of $\frac{5}{8}$. 6

5. Which of the fractions below name numbers less than one? $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{7}{8}$

$\frac{1}{2}, \frac{2}{3}, \frac{4}{4}, \frac{6}{5}, \frac{7}{8}, \frac{9}{9}, \frac{10}{9}$

6. Which of the fractions below name numbers greater than one? $\frac{6}{5}$ and $\frac{10}{9}$

$\frac{1}{2}, \frac{2}{3}, \frac{4}{4}, \frac{6}{5}, \frac{7}{8}, \frac{9}{9}, \frac{10}{9}$

7. Which of the fractions below name the number one? $\frac{4}{4}$ and $\frac{9}{9}$

$\frac{1}{2}, \frac{2}{3}, \frac{4}{4}, \frac{6}{5}, \frac{7}{8}, \frac{9}{9}, \frac{10}{9}$

Part 2 Copy. Solve each equation.

a	b
1. $\frac{3}{5} + \frac{1}{5} = n$ $\frac{4}{5}$	$\frac{3}{8} + \frac{2}{8} = n$ $\frac{5}{8}$
2. $\frac{4}{15} + \frac{9}{15} = n$ $\frac{13}{15}$	$\frac{7}{16} + \frac{9}{16} = n$ $\frac{16}{16} = 1$
3. $\frac{8}{18} + \frac{7}{18} = n$ $\frac{15}{18}$	$\frac{5}{19} + \frac{9}{19} = n$ $\frac{14}{19}$
4. $\frac{6}{20} + \frac{7}{20} = n$ $\frac{13}{20}$	$\frac{8}{21} + \frac{8}{21} = n$ $\frac{16}{21}$
5. $\frac{9}{24} + \frac{4}{24} = n$ $\frac{13}{24}$	$\frac{7}{25} + \frac{7}{25} = n$ $\frac{14}{25}$
6. $\frac{16}{32} + \frac{5}{32} = n$ $\frac{21}{32}$	$\frac{14}{36} + \frac{19}{36} = n$ $\frac{33}{36}$
7. $\frac{18}{42} + \frac{22}{42} = n$ $\frac{40}{42}$	$\frac{16}{45} + \frac{15}{45} = n$ $\frac{31}{45}$

Part 3 Copy. Solve each equation.

a	b
1. $\frac{1}{5} + \frac{1}{5} + \frac{2}{5} = n$ $\frac{4}{5}$	$\frac{1}{4} + \frac{2}{4} + \frac{1}{4} = n$ $\frac{4}{4} = 1$
2. $\frac{3}{8} + \frac{2}{8} + \frac{2}{8} = n$ $\frac{7}{8}$	$\frac{3}{10} + \frac{2}{10} + \frac{4}{10} = n$ $\frac{9}{10}$
3. $\frac{9}{16} + \frac{1}{16} + \frac{5}{16} = n$ $\frac{15}{16}$	$\frac{8}{18} + \frac{2}{18} + \frac{7}{18} = n$ $\frac{17}{18}$
4. $\frac{7}{20} + \frac{3}{20} + \frac{7}{20} = n$ $\frac{17}{20}$	$\frac{8}{24} + \frac{5}{24} + \frac{11}{24} = n$ $\frac{24}{24} = 1$
5. $\frac{14}{32} + \frac{6}{32} + \frac{9}{32} = n$ $\frac{29}{32}$	$\frac{13}{36} + \frac{17}{36} + \frac{5}{36} = n$ $\frac{35}{36}$
6. $\frac{22}{40} + \frac{8}{40} + \frac{9}{40} = n$ $\frac{39}{40}$	$\frac{17}{41} + \frac{13}{41} + \frac{6}{41} = n$ $\frac{36}{41}$
7. $\frac{15}{45} + \frac{20}{45} + \frac{9}{45} = n$ $\frac{44}{45}$	$\frac{14}{50} + \frac{14}{50} + \frac{13}{50} = n$ $\frac{41}{50}$
8. $\frac{18}{50} + \frac{12}{50} + \frac{8}{50} = n$ $\frac{38}{50}$	$\frac{19}{60} + \frac{11}{60} + \frac{9}{60} = n$ $\frac{39}{60}$

Solving Problems

Write an open sentence for each problem. Solve the open sentence. Then write an answer for the problem.

1. Ron gave $\frac{1}{5}$ of his candy to Carol and $\frac{2}{5}$ of it to Al. How much of his candy did Ron give away? $\frac{1}{5} + \frac{2}{5} = \square$; $\frac{3}{5}$ of his candy

2. Sally, Herb, and Jill were each given $\frac{2}{8}$ of a candy bar. How much of the candy bar did the three children receive? $\frac{2}{8} + \frac{2}{8} + \frac{2}{8} = \square$; $\frac{6}{8}$ of the candy bar

3. Jane used $\frac{3}{8}$ of a box of cereal and spilled $\frac{1}{8}$ of the box of cereal on the floor. How much of the box of cereal did she use and spill? $\frac{3}{8} + \frac{1}{8} = \square$; $\frac{4}{8}$ of the box of cereal

4. Georgiann read $\frac{5}{10}$ of a book on Saturday and $\frac{3}{10}$ of the book on Sunday. How much of the book did she read on these two days? $\frac{5}{10} + \frac{3}{10} = \square$; $\frac{8}{10}$ of the book

5. Bob read $\frac{3}{8}$ of a book on Friday and $\frac{4}{8}$ of the book on Saturday. How much of the book did he read on these two days? $\frac{3}{8} + \frac{4}{8} = \square$; $\frac{7}{8}$ of the book

6. Peter dug $\frac{2}{9}$ of a ditch from the road to the house. His father dug $\frac{7}{9}$ of the ditch. Together how much of the ditch did Peter and his father dig? $\frac{2}{9} + \frac{7}{9} = \square$; $\frac{9}{9}$ or all of the ditch

7. Laura, Carol, and Sue each used $\frac{1}{6}$ yard of ribbon in sewing class. In all, how much ribbon did the three girls use? $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \square$; $\frac{3}{6}$ of a yard of ribbon

8. $\frac{3}{10} + \frac{2}{10} + \frac{4}{10} = \square$; $\frac{9}{10}$ of the lawn

9. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \square$; $\frac{3}{4}$ of a quart of milk

8. Jack mowed $\frac{3}{10}$ of the lawn. John mowed $\frac{2}{10}$ of the lawn. Their dad mowed $\frac{4}{10}$ of the lawn. How much of the lawn did they mow?

See below.

9. John, Glenn, and Georgie each drank $\frac{1}{4}$ of a quart of milk. How much milk did the three boys drink?

See below.

10. Jim mowed $\frac{4}{9}$ of the lawn. Pat mowed $\frac{2}{9}$ of the lawn. Their dad mowed $\frac{3}{9}$ of the lawn. How much of the lawn is mowed? $\frac{4}{9} + \frac{2}{9} + \frac{3}{9} = \square$; $\frac{9}{9}$ or all of the lawn

11. Thelma walked $\frac{3}{10}$ of a mile to Sara's house. Then she walked $\frac{2}{10}$ of a mile home. In all, how far did she walk? $\frac{3}{10} + \frac{2}{10} = \square$; $\frac{5}{10}$ of a mile

12. Nancy wants to make three arm bands. She needs $\frac{1}{3}$ yard of material for each one. How much material does she need? $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \square$; $\frac{3}{3}$ or 1 yard

13. Tom, Jerry, and Arthur painted a picket fence. Tom and Jerry each painted $\frac{2}{5}$ of the fence. Arthur painted $\frac{1}{5}$ of the fence. In all, how much of the fence did the three boys paint? $\frac{2}{5} + \frac{2}{5} + \frac{1}{5} = \square$; $\frac{5}{5}$ or all of the fence

Can you do this? Compose a story problem for each open sentence.

Answers will vary.

a

b

1. $\frac{3}{8} + \frac{4}{8} = n$

$\frac{1}{5} + \frac{1}{5} + \frac{3}{5} = n$

2. $\frac{2}{6} + \frac{2}{6} = n$

$\frac{4}{10} + \frac{3}{10} + \frac{3}{10} = n$

Checkup Time

The numerals in () tell the pages where you can turn for help.

Important Ideas

1. In $\frac{3}{5}$, the numerator is 3 and the denominator is 5. (162)

2. Fractions can name whole numbers or fractional numbers. (164)

3. Equivalent fractions name the same number. (166)

4. A fraction can be expressed as a division numeral. (169)

5. In solving an open sentence like $\frac{2}{8} + \frac{3}{8} = n$, you add the numerators 2 and 3 and record their sum over the denominator 8. (170)

6. For all numbers that can be named by fractions, addition is commutative. (172)

7. For all numbers that can be named by fractions, addition is associative. (174)

Words to Know

1. Fraction, fractional number, numerator, denominator, fraction line (162)

2. Equivalent fractions (166)

3. Proper fraction, improper fraction (168)

Questions to Discuss

See page T178.

1. Tell how fractions are used to name whole numbers. To name fractional numbers. (164)

2. How can you tell when two fractions are equivalent? (166)

3. Tell how fractions are used to name numbers less than one. To name numbers greater than one. To name the number one. (168)

4. Tell how you would find the sum of $\frac{2}{8}$ and $\frac{5}{8}$. (170)

5. Illustrate that the commutative property of addition holds for all numbers that can be named by fractions. (172)

Written Practice

Do the following.

1. Draw a rectangle to show the meaning of $\frac{3}{8}$. (162)

2. Write 5 different fractions that name the number 3. The number $\frac{3}{4}$. The number $\frac{4}{3}$. (164) **Typical answers are shown below.**

3. Draw a number line to show that $\frac{1}{2} = \frac{2}{4}$. (166)

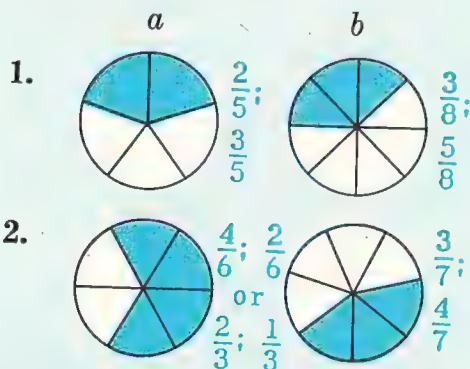
4. Find the sum of $\frac{3}{8}$ and $\frac{4}{8}$. Of $\frac{4}{15}$ and $\frac{7}{15}$. (170) $\frac{3}{8} + \frac{4}{8} = \frac{3+4}{8} = \frac{7}{8}$

$$\frac{4}{15} + \frac{7}{15} = \frac{4+7}{15} = \frac{11}{15}$$

$$2. \quad \frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \frac{12}{3}, \frac{15}{5}, \frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \\ \frac{4}{3}, \frac{8}{6}, \frac{12}{9}, \frac{16}{12}, \frac{20}{15}$$

Self-Evaluation

Part 1 Write a fraction to tell how much of each interior is colored. Then write a fraction to tell how much is not colored.



Part 2 Draw a rectangle and separate its interior into parts of the same size to illustrate each fraction below.

a	b	c	d
1. $\frac{3}{5}$	$\frac{2}{6}$	$\frac{4}{7}$	$\frac{6}{8}$
2. $\frac{4}{8}$	$\frac{5}{9}$	$\frac{7}{10}$	$\frac{8}{12}$

Part 3 Write a fraction for each quotient below.

a	b	c	d
1. $3 \div 4$	$\frac{3}{4}$	$2 \div 3$	$\frac{2}{3}$
2. $4 \div 5$	$\frac{4}{5}$	$6 \div 6$	$\frac{6}{6}$
3. $2 \div 5$	$\frac{2}{5}$	$3 \div 3$	$\frac{3}{3}$
4. $0 \div 10$	$\frac{0}{10}$	$8 \div 3$	$\frac{8}{3}$
		$12 \div 5$	$\frac{12}{5}$
		$9 \div 10$	$\frac{9}{10}$

Part 4 Write a division numeral for each fraction below.

a	b	c	d
1. $\frac{2}{5}$	$\frac{3}{7}$	$\frac{4}{8}$	$\frac{8}{9}$
2. $\frac{2}{5}$	$\frac{3}{7}$	$\frac{4}{8}$	$\frac{8}{9}$
3. $\frac{0}{8}$	$\frac{1}{4}$	$\frac{2}{7}$	$\frac{1}{10}$
	$0 \div 8$	$1 \div 4$	$2 \div 7$
			$1 \div 10$

Part 5 Copy. Solve each open sentence.

a	b
1. $\frac{1}{6} + \frac{3}{6} = n$	$\frac{2}{7} + \frac{3}{7} = n$
2. $\frac{2}{7} + \frac{5}{7} = n$	$\frac{6}{9} + \frac{1}{9} = n$
3. $\frac{8}{16} + \frac{5}{16} = n$	$\frac{9}{17} + \frac{8}{17} = n$
4. $\frac{6}{17} + \frac{7}{17} = n$	$\frac{8}{16} + \frac{4}{16} = n$
5. $\frac{3}{12} + \frac{8}{12} = n$	$\frac{4}{11} + \frac{6}{11} = n$

Part 6 Copy. Solve each open sentence.

a	b
1. $\frac{9}{15} + \frac{1}{15} + \frac{4}{15} = n$	$\frac{3}{18} + \frac{8}{18} + \frac{2}{18} = n$
2. $\frac{6}{20} + \frac{4}{20} + \frac{9}{20} = n$	$\frac{7}{21} + \frac{6}{21} + \frac{4}{21} = n$
3. $\frac{5}{25} + \frac{5}{25} + \frac{8}{25} = n$	$\frac{6}{24} + \frac{3}{24} + \frac{7}{24} = n$
4. $\frac{3}{22} + \frac{7}{22} + \frac{4}{22} = n$	$\frac{2}{30} + \frac{9}{30} + \frac{1}{30} = n$
5. $\frac{2}{16} + \frac{8}{16} + \frac{5}{16} = n$	$\frac{8}{19} + \frac{3}{19} + \frac{7}{19} = n$

Midyear Review

The numerals in () tell the pages where you can turn for help.
See page T180 for answers.

Number Meanings

1. What do the following sets have in common? (7)

- the set of legs on a dog
- the set of legs on a cat
- the set of legs on a cow

2. The union of sets makes you think of which operation on numbers? (10)

3. Removing a subset makes you think of which operation on numbers? (13)

4. For the open sentence $5+2=\square$, write a true closed sentence and a false closed sentence. (17)

5. Write 13,924 in expanded notation. (90)

Addition

1. Name the number 10 as a sum of two whole numbers in as many ways as you can. (33)

2. For the number 12, write two addition numerals that illustrate the commutative property of addition. (34)

3. Why is zero called the identity number of addition? (35)

4. Show that $(3+4)+5=3+(4+5)$ is a true sentence. (40)

5. How would you find the simplest numeral for $365+224$? (44)

6. How would you find the simplest numeral for $652+185$? (52)

Subtraction

1. Write three subtraction numerals to name the number 5. (36)

2. How would you find the simplest numeral for $481-271$? (46)

3. In which of the following must you rename the minuend so that you can subtract ones? (50)

<i>a</i>	<i>b</i>	<i>c</i>
$\begin{array}{r} 28 \\ -14 \\ \hline \end{array}$	$\begin{array}{r} 38 \\ -28 \\ \hline \end{array}$	$\begin{array}{r} 64 \\ -46 \\ \hline \end{array}$

4. In which of the following must you rename the minuend so that you can subtract tens? (54)

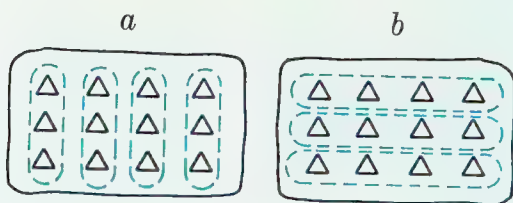
<i>a</i>	<i>b</i>	<i>c</i>
$\begin{array}{r} 491 \\ -280 \\ \hline \end{array}$	$\begin{array}{r} 434 \\ -239 \\ \hline \end{array}$	$\begin{array}{r} 862 \\ -291 \\ \hline \end{array}$

5. To subtract 1844 from 4236, name the minuend in expanded notation so you can subtract in every place-value position. (102)

See page T181 for answers.

Multiplication

1. Tell an addition sentence and a multiplication sentence for each set below. (60)



2. Since $6 \times 4 = 24$, how do you know that $24 \div 4 = 6$? (66)

3. For the number 12, write two multiplication numerals that illustrate the commutative property of multiplication. (68)

4. Show that $(2 \times 3) \times 4 = 2 \times (3 \times 4)$ is a true sentence. What property of multiplication does this illustrate? (78)

5. Explain how the product of 4 and 123 is found in the example below. (110)

$$\begin{aligned} 4 \times 123 &= 4 \times (100 + 20 + 3) \\ &= (4 \times 100) + (4 \times 20) + (4 \times 3) \\ &= 400 + 80 + 12 \\ &= 492 \end{aligned}$$

6. How would you estimate the product 5×89 ? (118)

Division

1. Use repeated subtraction on a number line to find the simplest numeral for $12 \div 4$. (65)

2. Explain how the quotient of 492 divided by 4 is found in the example below. (122)

$$\begin{aligned} 492 \div 4 &= (400 + 80 + 12) \div 4 \\ &= (400 \div 4) + (80 \div 4) + (12 \div 4) \\ &= 100 + 20 + 3 \\ &= 123 \end{aligned}$$

3. How would you find the simplest numeral for $72 \div 18$? (146)

4. Does the simplest numeral for $3367 \div 91$ have two or three digits? Why? (150)

Fractions

1. What fraction tells how much of the interior of the figure below is colored? What fraction tells how much is *not* colored? (161)



2. Which pair of fractions, $\frac{1}{2}$ and $\frac{2}{6}$ or $\frac{1}{3}$ and $\frac{2}{6}$, name the same number? How do you know? (166)

3. In what way is a proper fraction different from an improper fraction? (168)

4. How would you add $\frac{3}{8}$ and $\frac{1}{8}$? (170)

5. Do $\frac{3}{9} + \frac{4}{9}$ and $\frac{4}{9} + \frac{3}{9}$ name the same number? How do you know? (172)

6. Show that $(\frac{2}{8} + \frac{1}{8}) + \frac{4}{8} = \frac{2}{8} + (\frac{1}{8} + \frac{4}{8})$ is a true sentence. (174)

Midyear Tests

Test 1 Select the best of the three answers for each sentence. Each best answer is underscored.

1. The 9 in 8924 means 9 (thousands, hundreds, tens).

2. To make $4 \bullet 5 - 1$ a true sentence, replace \bullet by ($>$, $=$, $<$).

3. A numeral like 101 is called a (one-digit, two-digit, or three-digit) numeral.

4. The Roman numeral for 9 is (XI, VIII, IX).

5. The numeral for five thousand two hundred three has (3, 4, 5) digits.

Test 2 Copy. Solve each equation.

- | a | b |
|---|--|
| 1. $16 + 7 = n$ <u>23</u> | $85 - 14 = n$ <u>71</u> |
| 2. $28 + 15 = n$ <u>43</u> | $283 - 162 = n$ <u>121</u> |
| 3. $324 + 189 = n$ <u>513</u> | $6215 - 1014 = n$ <u>5201</u> |
| 4. $4263 + 5224 = n$ <u>9487</u> | $3914 - 1885 = n$ <u>2029</u> |
| 5. $2744 + 2381 = n$ <u>5125</u> | $3241 - 1038 = n$ <u>2203</u> |
| 6. $3718 + 1493 = n$ <u>5211</u> | $6241 - 1839 = n$ <u>4402</u> |
| 7. $7109 + 1899 = n$ <u>9008</u> | $8019 - 4862 = n$ <u>3157</u> |
| 8. $4390 + 3989 = n$ <u>8379</u> | $7402 - 3293 = n$ <u>4109</u> |

Test 3 Copy. Find each product.

- | | a | b | c | d |
|----|---|---|---|---|
| 1. | $\begin{array}{r} 32 \\ \times 4 \\ \hline 128 \end{array}$ | $\begin{array}{r} 18 \\ \times 60 \\ \hline 1080 \end{array}$ | $\begin{array}{r} 419 \\ \times 22 \\ \hline 9218 \end{array}$ | $\begin{array}{r} 4312 \\ \times 13 \\ \hline 56056 \end{array}$ |
| 2. | $\begin{array}{r} 28 \\ \times 6 \\ \hline 168 \end{array}$ | $\begin{array}{r} 32 \\ \times 14 \\ \hline 448 \end{array}$ | $\begin{array}{r} 619 \\ \times 48 \\ \hline 29712 \end{array}$ | $\begin{array}{r} 1309 \\ \times 32 \\ \hline 41888 \end{array}$ |
| 3. | $\begin{array}{r} 41 \\ \times 7 \\ \hline 287 \end{array}$ | $\begin{array}{r} 59 \\ \times 12 \\ \hline 708 \end{array}$ | $\begin{array}{r} 408 \\ \times 62 \\ \hline 25296 \end{array}$ | $\begin{array}{r} 3019 \\ \times 48 \\ \hline 144912 \end{array}$ |

Test 4 Copy. Find each quotient and remainder.

- | | a | b | c | d |
|----|---|---|--|---|
| 1. | $\begin{array}{r} 32 \\ 3 \overline{)96} \end{array}$ | $\begin{array}{r} 16 \\ 6 \overline{)96} \end{array}$ | $\begin{array}{r} 192 \\ 4 \overline{)768} \end{array}$ | $\begin{array}{r} 347 \\ 5 \overline{)1735} \end{array}$ |
| 2. | $\begin{array}{r} 5 \\ 16 \overline{)80} \end{array}$ | $\begin{array}{r} 4 \\ 12 \overline{)48} \end{array}$ | $\begin{array}{r} 24 \\ 22 \overline{)528} \end{array}$ | $\begin{array}{r} 49 \\ 73 \overline{)3577} \end{array}$ |
| 3. | $\begin{array}{r} 3 \\ 24 \overline{)72} \end{array}$ | $\begin{array}{r} 4r1 \\ 13 \overline{)53} \end{array}$ | $\begin{array}{r} 4r2 \\ 63 \overline{)254} \end{array}$ | $\begin{array}{r} 79r14 \\ 24 \overline{)1910} \end{array}$ |

Test 5 Answer each of the following.

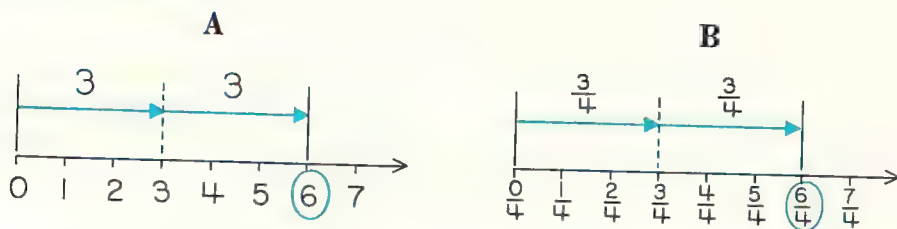
- Express $3 \div 7$ as a fraction. $\frac{3}{7}$
- Write a division numeral for the fraction $\frac{4}{9}$. $4 \div 9$
- Which of the following fractions name numbers less than one?
 $\frac{3}{4}$ and $\frac{4}{5}$ ($\frac{7}{6}$, $\frac{3}{4}$, $\frac{5}{5}$, $\frac{4}{5}$)
- Solve each equation below.

a	b
$\frac{3}{8} + \frac{4}{8} = n$ $\frac{7}{8}$	$\frac{2}{9} + \frac{3}{9} + \frac{7}{9} = n$ $1\frac{1}{3}$

Chapter 8

MULTIPLICATION OF FRACTIONAL NUMBERS

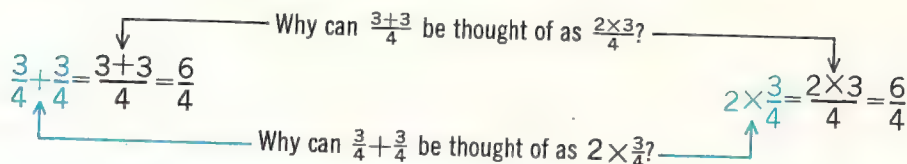
Repeated Addition



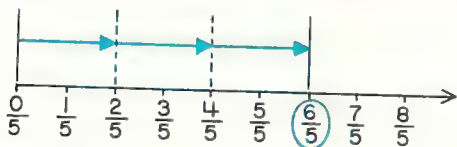
How does the number line in **A** show that you can think of the multiplication of whole numbers as repeated addition? $2 \times 3 = 3 + 3$
 You can extend this idea to show that the multiplication of fractional numbers can be thought of as repeated addition.

In **B**, the arrows above the number line show what closed addition sentence? What closed multiplication sentence? $\frac{3}{4} + \frac{3}{4} = \frac{6}{4}$; $2 \times \frac{3}{4} = \frac{6}{4}$

Study how the examples below show that $\frac{3}{4} + \frac{3}{4}$ and $2 \times \frac{3}{4}$ both name the same number.



Oral Answer the following.



The arrows above show what closed addition sentence? What closed multiplication sentence?

$$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5}; 3 \times \frac{2}{5} = \frac{6}{5}$$

Written Copy. State each product as a repeated addition and as a single fraction. See below.

a	b	c
1. $4 \times \frac{2}{3}$	$5 \times \frac{3}{8}$	$2 \times \frac{1}{5}$
2. $3 \times \frac{3}{4}$	$5 \times \frac{2}{5}$	$4 \times \frac{1}{3}$
3. $2 \times \frac{1}{2}$	$8 \times \frac{2}{9}$	$3 \times \frac{4}{5}$

Written

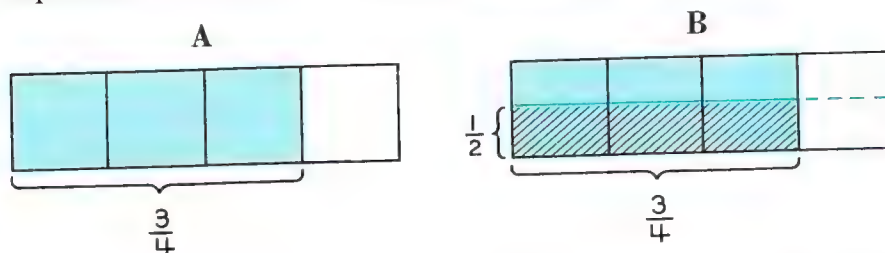
1a. $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}; \frac{8}{3}$ 1b. $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}; \frac{15}{8}$ 1c. $\frac{1}{5} + \frac{1}{5}; \frac{2}{5}$

2a. $\frac{3}{4} + \frac{3}{4} + \frac{3}{4}; \frac{9}{4}$ 2b. $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}; \frac{10}{5}$ 2c. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}; \frac{4}{3}$

3a. $\frac{1}{2} + \frac{1}{2}; \frac{2}{2}$ 3b. $\frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9}; \frac{16}{9}$ 3c. $\frac{4}{5} + \frac{4}{5} + \frac{4}{5}; \frac{12}{5}$

Multiplication of Fractional Numbers

To name a product like $\frac{1}{2} \times \frac{3}{4}$ as a single fraction, you can separate the interior of a figure as shown below.



In **A**, the interior of the figure is separated into how many parts of the same size? How many of these parts are colored? What fraction tells how much of the interior is colored? 4 ; 3 ; $\frac{3}{4}$

In **B**, the colored part of the interior of the figure is separated into two parts of the same size by the solid blue line. Then 1 out of 2 parts or $\frac{1}{2}$ of the colored part is marked diagonal lines . That part of the interior of the figure marked diagonal lines is $\frac{1}{2}$ of $\frac{3}{4}$ or $\frac{1}{2} \times \frac{3}{4}$ of the interior.

How many small rectangles are shown in **B**? How many of the small rectangles are marked diagonal lines ? What fraction tells how much of the interior of the large rectangle is marked diagonal lines ? 8 ; 3 ; $\frac{3}{8}$

A more convenient way to name a product like $\frac{1}{2} \times \frac{3}{4}$ as a single fraction is shown below.

Multiply the numerators.

$$\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$$

Multiply the denominators.

How is the 3 in $\frac{3}{8}$ obtained? How is the 8 in $\frac{3}{8}$ obtained?

To name a product like $\frac{1}{2} \times \frac{3}{4}$ as a single fraction, multiply the numerators to get the numerator of the product, and then multiply the denominators to get the denominator of the product.

Oral Think about naming $\frac{3}{5} \times \frac{2}{7}$ as a single fraction. Answer the following questions.

1. How can you find the numerator of the product? **3×2**

2. How can you find the denominator of the product? **5×7**

3. What single fraction names the product $\frac{3}{5} \times \frac{2}{7}$? **$\frac{6}{35}$**

Think about naming $\frac{3}{4} \times \frac{5}{8}$ as a single fraction. Answer the following questions.

4. How can you find the numerator of the product? **3×5**

5. How can you find the denominator of the product? **4×8**

6. What single fraction names the product $\frac{3}{4} \times \frac{5}{8}$? **$\frac{15}{32}$**

Tell what single fraction you would use to replace the letter to make each of the following a true sentence.

a

7. $\frac{1}{2} \times \frac{2}{5} = a$ **$\frac{2}{10}$**

8. $\frac{1}{4} \times \frac{2}{3} = b$ **$\frac{2}{12}$**

9. $\frac{3}{4} \times \frac{1}{2} = c$ **$\frac{3}{8}$**

10. $\frac{2}{6} \times \frac{1}{3} = d$ **$\frac{2}{18}$**

11. $\frac{2}{7} \times \frac{3}{4} = e$ **$\frac{6}{28}$**

b

$\frac{2}{3} \times \frac{1}{4} = f$ **$\frac{2}{12}$**

$\frac{1}{3} \times \frac{2}{5} = g$ **$\frac{2}{15}$**

$\frac{4}{5} \times \frac{2}{5} = h$ **$\frac{8}{25}$**

$\frac{3}{5} \times \frac{4}{8} = i$ **$\frac{12}{40}$**

$\frac{2}{3} \times \frac{6}{7} = j$ **$\frac{12}{21}$**

Written Draw a figure to represent each closed multiplication sentence below. **Figures will vary.**

<i>a</i>	<i>b</i>
1. $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$	$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$
2. $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$	$\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

Copy. Find the single fraction that you could use to name each product below.

<i>a</i>	<i>b</i>	<i>c</i>
3. $\frac{1}{2} \times \frac{4}{5}$ $\frac{4}{10}$	$\frac{2}{5} \times \frac{3}{7}$ $\frac{6}{35}$	$\frac{2}{4} \times \frac{3}{8}$ $\frac{6}{32}$
4. $\frac{7}{9} \times \frac{8}{9}$ $\frac{56}{81}$	$\frac{6}{7} \times \frac{8}{9}$ $\frac{48}{63}$	$\frac{4}{8} \times \frac{6}{9}$ $\frac{24}{72}$
5. $\frac{5}{7} \times \frac{5}{8}$ $\frac{25}{56}$	$\frac{4}{9} \times \frac{4}{9}$ $\frac{16}{81}$	$\frac{1}{2} \times \frac{1}{2}$ $\frac{1}{4}$
6. $\frac{1}{3} \times \frac{1}{3}$ $\frac{1}{9}$	$\frac{1}{2} \times \frac{5}{6}$ $\frac{5}{12}$	$\frac{2}{8} \times \frac{3}{4}$ $\frac{6}{32}$
7. $\frac{6}{9} \times \frac{6}{8}$ $\frac{36}{72}$	$\frac{5}{8} \times \frac{3}{8}$ $\frac{15}{64}$	$\frac{4}{7} \times \frac{4}{7}$ $\frac{16}{49}$

Tell why Since $\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$, you can think of $\frac{8}{15} = \frac{4 \times 2}{5 \times 3} = \frac{4}{5} \times \frac{2}{3}$.

Express each of the following fractional numbers as a product of two fractional numbers. **Answers may differ from those shown.**

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
$\frac{6}{15}$	$\frac{2}{3} \times \frac{3}{5}$	$\frac{4}{3} \times \frac{2}{3}$	$\frac{4}{12} \times \frac{2}{3}$
	$\frac{2}{3} \times \frac{2}{3}$	$\frac{4}{3} \times \frac{2}{3}$	$\frac{5}{12} \times \frac{1}{2}$
	$\frac{2}{3} \times \frac{2}{3}$	$\frac{4}{3} \times \frac{2}{3}$	$\frac{5}{12} \times \frac{1}{2}$

Tell how How would you solve the open sentences below? **See page T185.**

$(\frac{1}{2} \times \frac{2}{3}) \times \frac{3}{4} = n$ $\frac{1}{2} \times (\frac{2}{3} \times \frac{3}{4}) = m$

Multiplication Is Commutative

Whole Numbers

$$3 \times 4 = 4 \times 3$$

Fractional Numbers

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{4} \times \frac{1}{2}$$

Yes

Do 3×4 and 4×3 both name the same number? Does changing the order or commuting the two factors change the product? **No**
What property of multiplication of whole numbers is illustrated by the sentence " $3 \times 4 = 4 \times 3$ "? **commutative property of multiplication**

Study the examples below to see if multiplication of fractional numbers is commutative.

$$\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$$

Do 1×3 and 3×1 both name the same number? Why?

$$\frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}$$

Do 2×4 and 4×2 both name the same number? Why?

What single fraction would you use to name $\frac{1}{2} \times \frac{3}{4}$? To name $\frac{3}{4} \times \frac{1}{2}$? Do both $\frac{1}{2} \times \frac{3}{4}$ and $\frac{3}{4} \times \frac{1}{2}$ name the same number? Does changing the order of the factors change the product? $\frac{3}{8}$; $\frac{3}{8}$; **Yes**; **No**

For all numbers that can be named by fractions, multiplication is commutative.

Oral Commute the factors to form another multiplication numeral for each of the following. Then change the multiplication numeral to a single fraction. **See below.**

Written For each fractional number below, write a closed multiplication sentence as follows. **See page T186.**

M P
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R E C
T I C
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	a	b	c
1.	$\frac{2}{3} \times \frac{3}{4}$	$\frac{4}{5} \times \frac{2}{4}$	$\frac{3}{7} \times \frac{2}{5}$
2.	$\frac{1}{3} \times \frac{3}{4}$	$\frac{6}{7} \times \frac{1}{2}$	$\frac{4}{5} \times \frac{1}{4}$
3.	$\frac{3}{5} \times \frac{2}{6}$	$\frac{1}{8} \times \frac{2}{3}$	$\frac{1}{2} \times \frac{3}{4}$

186
Oral

	a	b
1.	$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$	$\frac{2}{4} \times \frac{4}{5} = \frac{8}{20}$
2.	$\frac{3}{4} \times \frac{1}{3} = \frac{3}{12}$	$\frac{1}{2} \times \frac{6}{7} = \frac{6}{14}$
3.	$\frac{2}{6} \times \frac{3}{5} = \frac{6}{30}$	$\frac{2}{3} \times \frac{1}{8} = \frac{2}{24}$

	a	b	c	d
1.	$\frac{10}{15}$	$\frac{3}{8}$	$\frac{4}{9}$	$\frac{4}{12}$
2.	$\frac{8}{12}$	$\frac{15}{20}$	$\frac{18}{20}$	$\frac{6}{9}$
3.	$\frac{6}{8}$	$\frac{9}{12}$	$\frac{14}{16}$	$\frac{1}{4}$

	c
1.	$\frac{2}{3} \times \frac{3}{7} = \frac{6}{21}$
2.	$\frac{1}{4} \times \frac{4}{5} = \frac{4}{20}$
3.	$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

Identity Number of Multiplication

If one of two whole numbers is the number one, what can you say about their product? Why do you call *one* the identity number of multiplication of whole numbers? *Multiplying a number by 1 leaves the identity of that number unchanged.*

The examples below show how you can find the product of a fractional number and the number one.

A	B	C
$\frac{1}{4} \times 1 = \frac{1}{4} \times \frac{1}{1}$ $= \frac{1 \times 1}{4 \times 1}$ $= \frac{1}{4}$	$\frac{2}{5} \times 1 = \frac{2}{5} \times \frac{1}{1}$ $= \frac{2 \times 1}{5 \times 1}$ $= \frac{2}{5}$	$1 \times \frac{7}{13} = \frac{1}{1} \times \frac{7}{13}$ $= \frac{1 \times 7}{1 \times 13}$ $= \frac{7}{13}$

In A, why can 1 be replaced by $\frac{1}{1}$? Why can $\frac{1}{4} \times \frac{1}{1}$ be changed to $\frac{1 \times 1}{4 \times 1}$? How is $\frac{1}{4}$ obtained from $\frac{1 \times 1}{4 \times 1}$? Explain the examples in B and C above.

If a fractional number is multiplied by the number one, what can you say about the product? *The product is the same as that fractional number.*

A fractional number is not changed when it is multiplied by the number one. You call the number *one* the **identity number of multiplication**.

Oral What single fraction would you use to name each of the following products?

<i>a</i>	<i>b</i>	<i>c</i>
1. $\frac{3}{8} \times 1$	$1 \times \frac{4}{5}$	$\frac{2}{3} \times 1$
2. $\frac{4}{9} \times 1$	$1 \times \frac{3}{9}$	$\frac{4}{7} \times 1$
3. $\frac{2}{6} \times 1$	$1 \times \frac{4}{8}$	$\frac{6}{9} \times 1$

Written Copy. Find each product as a single fraction using the method shown above. See page T187.

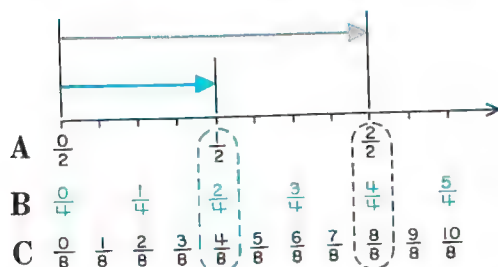
<i>a</i>	<i>b</i>	<i>c</i>
1. $\frac{2}{8} \times 1$	$1 \times \frac{2}{9}$	$\frac{3}{5} \times 1$
2. $\frac{5}{8} \times 1$	$1 \times \frac{5}{9}$	$\frac{4}{6} \times 1$
3. $\frac{2}{7} \times 1$	$1 \times \frac{1}{8}$	$\frac{5}{7} \times 1$

*1 Their product is the same as the other number.

*2 $\frac{1}{4} \times \frac{1}{1} = \frac{1 \times 1}{4 \times 1}$

*3 $\frac{1 \times 1}{4 \times 1} = \frac{1}{4}$

Renaming a Fractional Number



This number line is labeled in three different but equivalent ways. What fraction would you use to express the number represented by the blue arrow by using the names listed in A? By using the names listed in B? By using the names listed in C? Do

$\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ name the same number? What three fractions would you use to express the number represented by the gray arrow? $\frac{2}{2}$, $\frac{4}{4}$, $\frac{8}{8}$

You can find many names for every fractional number without using a number line. You can multiply a fractional number by the number one and then rename one in a suitable manner to get another name for the number. See how $\frac{1}{2}$ is changed to $\frac{2}{4}$ and $\frac{4}{8}$ below.

A

$$\begin{aligned}\frac{1}{2} &= \frac{1}{2} \times 1 \\ &= \frac{1}{2} \times \frac{2}{2} \\ &= \frac{1 \times 2}{2 \times 2} \\ &= \frac{2}{4}\end{aligned}$$

B

$$\begin{aligned}\frac{1}{2} &= \frac{1}{2} \times 1 \\ &= \frac{1}{2} \times \frac{4}{4} \\ &= \frac{1 \times 4}{2 \times 4} \\ &= \frac{4}{8}\end{aligned}$$

In A, how is 1 renamed in changing $\frac{1}{2}$ to $\frac{2}{4}$? Why is 1 changed to $\frac{2}{2}$ instead of $\frac{3}{3}$ or some other fraction? Tell how $\frac{1}{2} \times \frac{2}{2}$ is changed to $\frac{2}{4}$.

In B, how is 1 renamed in changing $\frac{1}{2}$ to $\frac{4}{8}$? Why is 1 now changed to $\frac{4}{4}$ instead of $\frac{2}{2}$, $\frac{3}{3}$, or some other fraction? Tell how $\frac{1}{2} \times \frac{4}{4}$ is changed to $\frac{4}{8}$.

How can you find many names for every fractional number? See the procedure presented above.

The identity number of multiplication helps you find many names for every fractional number.

*1 $\frac{2}{2}$ is the most convenient name to use to change $\frac{1}{2}$ to $\frac{2}{4}$.

*2 $\frac{4}{4}$ is the most convenient name to use to change $\frac{1}{2}$ to $\frac{4}{8}$.

Oral Study how $\frac{2}{5}$ is changed to $\frac{8}{20}$. Then answer the questions that follow.

$$\begin{aligned}\frac{2}{5} &= \frac{2}{5} \times 1 \\ &= \frac{2}{5} \times \frac{4}{4} \\ &= \frac{2 \times 4}{5 \times 4} \\ &= \frac{8}{20}\end{aligned}$$

- How do you know that $\frac{2}{5} = \frac{2}{5} \times 1$? **1 is the identity number of multiplication.**
- How is 1 renamed in $\frac{2}{5} \times \frac{4}{4}$?

3. Why is 1 renamed as $\frac{4}{4}$ instead of $\frac{2}{2}$, $\frac{3}{3}$, or $\frac{5}{5}$? **$\frac{4}{4}$ is the most convenient name to use.**

4. How do you change $\frac{2}{5} \times \frac{4}{4}$ to $\frac{8}{20}$? **See example above.**

Written Do the following. **See page T189.**

- | a | b |
|--|---|
| 1. Rename $\frac{1}{2}$ as $\frac{5}{10}$. | Rename $\frac{2}{3}$ as $\frac{4}{6}$. |
| 2. Rename $\frac{3}{4}$ as $\frac{9}{12}$. | Rename $\frac{2}{5}$ as $\frac{12}{30}$. |
| 3. Rename $\frac{4}{7}$ as $\frac{12}{21}$. | Rename $\frac{3}{8}$ as $\frac{9}{24}$. |
| 4. Rename $\frac{4}{5}$ as $\frac{20}{25}$. | Rename $\frac{1}{8}$ as $\frac{4}{32}$. |
| 5. Rename $\frac{1}{4}$ as $\frac{3}{12}$. | Rename $\frac{4}{9}$ as $\frac{16}{36}$. |

Copy. Write three more fractions for each fractional number. **Answers may vary from those shown below.**

- | a | b | c | d |
|------------------|---------------|---------------|---------------|
| 6. $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{1}{5}$ | $\frac{2}{3}$ |
| 7. $\frac{3}{4}$ | $\frac{2}{5}$ | $\frac{3}{8}$ | $\frac{1}{6}$ |
| 8. $\frac{2}{9}$ | $\frac{5}{8}$ | $\frac{4}{7}$ | $\frac{5}{6}$ |

- Written**
- | | |
|---|---|
| 6. $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}$ | $\frac{2}{8}, \frac{3}{12}, \frac{4}{16}$ |
| 7. $\frac{6}{8}, \frac{9}{12}, \frac{12}{16}$ | $\frac{4}{10}, \frac{6}{15}, \frac{8}{20}$ |
| 8. $\frac{4}{18}, \frac{6}{27}, \frac{8}{36}$ | $\frac{10}{16}, \frac{15}{24}, \frac{20}{32}$ |

Draw a number line to show that the fractions in each pair below name the same number. **See page 188 for style.**

- | a | b | c |
|-------------------------------------|----------------------------------|----------------------------------|
| 9. $\frac{2}{3}$ and $\frac{4}{6}$ | $\frac{3}{4}$ and $\frac{6}{8}$ | $\frac{1}{5}$ and $\frac{2}{10}$ |
| 10. $\frac{1}{3}$ and $\frac{3}{9}$ | $\frac{2}{3}$ and $\frac{6}{9}$ | $\frac{2}{8}$ and $\frac{1}{4}$ |
| 11. $\frac{3}{6}$ and $\frac{1}{2}$ | $\frac{2}{5}$ and $\frac{4}{10}$ | $\frac{1}{4}$ and $\frac{3}{12}$ |

Copy. Discover the pattern in each exercise below. Write the next three fractions.

- $\frac{1}{8} = \frac{2}{16} = \frac{3}{24} = \frac{4}{32} = \frac{5}{40} = \frac{6}{48}$
- $\frac{1}{6} = \frac{2}{12} = \frac{3}{18} = \frac{4}{24} = \frac{5}{30} = \frac{6}{36}$
- $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25} = \frac{12}{30}$
- $\frac{2}{7} = \frac{4}{14} = \frac{6}{21} = \frac{8}{28} = \frac{10}{35} = \frac{12}{42}$
- $\frac{3}{8} = \frac{6}{16} = \frac{9}{24} = \frac{12}{32} = \frac{15}{40} = \frac{18}{48}$

Can you do this? To find the simplest numeral to replace the \square so that $\frac{3}{4} = \frac{\square}{8}$ is a true sentence you can do the following.

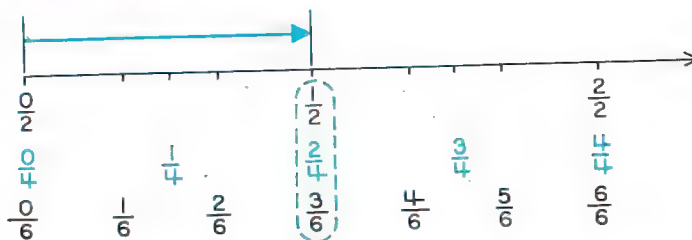
- Divide 8 by 4.
- Multiply 3 by 2.
- Replace the \square by 6.

Copy. Write the numeral that should replace the \square to make each sentence true.

- | a | b | c |
|--|---|---|
| 1. $\frac{2}{3} = \frac{\square}{9}$ 6 | $\frac{1}{2} = \frac{\square}{10}$ 5 | $\frac{\square}{16} = \frac{2}{8}$ 4 |
| 2. $\frac{3}{8} = \frac{\square}{16}$ 6 | $\frac{1}{3} = \frac{\square}{15}$ 5 | $\frac{\square}{12} = \frac{3}{4}$ 9 |

- | a | b | c |
|---|---|-----|
| 1. $\frac{2}{10}, \frac{3}{15}, \frac{4}{20}$ | $\frac{4}{6}, \frac{6}{9}, \frac{8}{12}$ | |
| 2. $\frac{6}{16}, \frac{9}{24}, \frac{12}{32}$ | $\frac{2}{12}, \frac{3}{18}, \frac{4}{24}$ | |
| 3. $\frac{8}{14}, \frac{12}{21}, \frac{16}{28}$ | $\frac{10}{12}, \frac{15}{18}, \frac{20}{24}$ | |

Changing Fractions to Simplest Form



What fractional number is represented by the blue arrow above? What fractions above can be used to name that fractional number? $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$

Since you can find many names for a fractional number, it is helpful to consider one of these fractions as a fraction in **simplest form**.

A fraction is in *simplest form* if the greatest common factor of the numerator and the denominator is the number one.

Consider the fraction $\frac{1}{2}$ in the row of equivalent fractions below.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

What is the greatest common factor of 1 and 2? Is $\frac{1}{2}$ in simplest form? Is the fraction $\frac{2}{4}$ in simplest form? Why or why not? Are $\frac{3}{6}$, $\frac{4}{8}$, or $\frac{5}{10}$ in simplest form? How can you tell?

You can change a fraction to simplest form by renaming the fractional number as shown below.

$$\begin{aligned} \frac{4}{8} &= \frac{1 \times 4}{2 \times 4} & \left\{ \begin{array}{l} \text{What is the greatest common factor of 4 and 8?} \\ \text{Is 4 a factor in } 1 \times 4? \text{ In } 2 \times 4? \end{array} \right. \\ &= \frac{1}{2} \times \frac{4}{4} & \left\{ \begin{array}{l} \text{How does knowing } \frac{1}{2} \times \frac{4}{4} = \frac{1 \times 4}{2 \times 4} \text{ help you change} \\ \frac{1 \times 4}{2 \times 4} \text{ to } \frac{1}{2} \times \frac{4}{4}? \end{array} \right. \\ &= \frac{1}{2} \times 1 & \text{How is } \frac{4}{4} \text{ renamed in changing } \frac{1}{2} \times \frac{4}{4} \text{ to } \frac{1}{2} \times 1? \\ &= \frac{1}{2} & \text{Why can } \frac{1}{2} \times 1 \text{ be changed to } \frac{1}{2}? \end{aligned}$$

190 *1 Yes

*2 No

*3 The greatest common factor of 2 and 4 is 2.

*4 No

*5 The greatest common factor of the numerator and the denominator of each fraction is not the number 1.

Oral Answer the following.

1. Consider the numerator and denominator of the fraction $\frac{8}{12}$. What is the greatest common factor of 8 and 12? Is $\frac{8}{12}$ in simplest form? Why or why not? **4; No; greatest common factor of 8 and 12 is 4**

2. Consider the numerator and denominator of the fraction $\frac{6}{8}$. What is the greatest common factor of 6 and 8? Is $\frac{6}{8}$ in simplest form? Why or why not? **2; No; greatest common factor of 6 and 8 is 2**

3. How can you tell whether or not a fraction is in simplest form? **See below.**

Tell whether or not each fraction below is in simplest form. **Those in simplest form are circled.**

a	b	c	d
4. $\frac{6}{9}$	$\frac{10}{12}$	$\frac{4}{8}$	$\frac{3}{4}$
5. $\frac{3}{5}$	$\frac{3}{6}$	$\frac{4}{7}$	$\frac{4}{9}$
6. $\frac{5}{7}$	$\frac{6}{7}$	$\frac{6}{15}$	$\frac{3}{9}$

Study how $\frac{8}{20}$ is changed to $\frac{2}{5}$. Then answer questions 7–11.

$$\begin{aligned}\frac{8}{20} &= \frac{2 \times 4}{5 \times 4} \\ &= \frac{2}{5} \times \frac{4}{4} \\ &= \frac{2}{5} \times 1 \\ &= \frac{2}{5}\end{aligned}$$

7. What is the greatest common factor of 8 and 20? **4**

8. Is the greatest common factor a factor in 2×4 ? In 5×4 ? **Yes**

9. How does knowing $\frac{2}{5} \times \frac{4}{4} = \frac{2 \times 4}{5 \times 4}$ help you change $\frac{2 \times 4}{5 \times 4}$ to $\frac{2}{5} \times \frac{4}{4}$? **$\frac{2}{5} \times \frac{4}{4}$ and $\frac{2 \times 4}{5 \times 4}$ name the same number.**

Oral 3. A fraction is in simplest form if the greatest common factor of the numerator and the denominator is the number one.

10. How is $\frac{4}{4}$ renamed in changing $\frac{2}{5} \times \frac{4}{4}$ to $\frac{2}{5} \times 1$? **$\frac{4}{4}$ is named as 1.**

11. Why can $\frac{2}{5} \times 1$ be changed to $\frac{2}{5}$? **1 is the identity number of multiplication.**

Written Copy. Change each fraction to simplest form.

a	b	c	d
1. $\frac{2}{4} \frac{1}{2}$	$\frac{6}{9} \frac{2}{3}$	$\frac{10}{12} \frac{5}{6}$	$\frac{6}{15} \frac{2}{5}$
2. $\frac{4}{8} \frac{1}{2}$	$\frac{3}{6} \frac{1}{2}$	$\frac{6}{12} \frac{1}{2}$	$\frac{3}{9} \frac{1}{3}$
3. $\frac{4}{12} \frac{1}{3}$	$\frac{5}{15} \frac{1}{3}$	$\frac{8}{12} \frac{2}{3}$	$\frac{9}{12} \frac{3}{4}$
4. $\frac{10}{15} \frac{2}{3}$	$\frac{16}{18} \frac{8}{9}$	$\frac{20}{25} \frac{4}{5}$	$\frac{8}{20} \frac{2}{5}$
5. $\frac{9}{15} \frac{3}{5}$	$\frac{6}{20} \frac{3}{10}$	$\frac{4}{20} \frac{1}{5}$	$\frac{4}{10} \frac{2}{5}$

Another way Another way to change a fraction to simplest form is shown below.

$$\frac{8}{20} = \frac{8}{20} \div 1$$

How is $\frac{8}{20}$ renamed?

$$= \frac{8}{20} \div 4$$

{ What is the greatest common factor of 8 and 20?
Why is 1 renamed as $\frac{4}{4}$ instead of $\frac{2}{2}$ or $\frac{3}{3}$?

$$= \frac{8 \div 4}{20 \div 4}$$

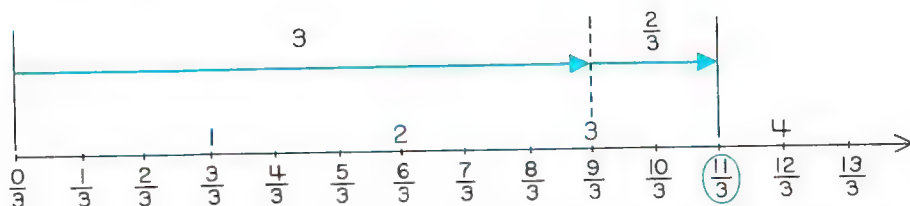
{ As a simplest numeral, what is $8 \div 4$? $20 \div 4$?

$$= \frac{2}{5}$$

Use the method shown above to change each fraction in *Written* 1–5 to simplest form. **See page T191.**

Can you do this? Draw a number line to show that $\frac{1}{4}$, $\frac{2}{8}$, and $\frac{3}{12}$ all name the same number. **See page 190 for style.**

Mixed Numerals



What addition numeral is suggested by the number line above? A more convenient way to express $3 + \frac{2}{3}$ is shown below.

$$3 + \frac{2}{3}$$

$$3 + \frac{2}{3} = 3\frac{2}{3}$$

You call $3\frac{2}{3}$ a **mixed numeral**. Why do you suppose $3\frac{2}{3}$ is called a mixed numeral? The 3 is a numeral for a whole number; $\frac{2}{3}$ is a numeral for a fractional number.

The numeral $3\frac{2}{3}$ is read *three and two thirds*. It means $3 + \frac{2}{3}$. The plus sign may be inserted in a mixed numeral whenever it is convenient to do so.

A mixed numeral names the sum of two numbers. One of the numbers is named by a numeral for a whole number and the other is named by a fraction.

Oral Read each mixed numeral below and express each as a sum.
See page T192.

- | | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|----|-----------------|-------------------|------------------|-----------------|
| 1. | $3\frac{3}{4}$ | $2\frac{2}{3}$ | $6\frac{4}{5}$ | $8\frac{6}{9}$ |
| 2. | $7\frac{3}{8}$ | $4\frac{9}{10}$ | $5\frac{14}{15}$ | $6\frac{9}{8}$ |
| 3. | $9\frac{10}{6}$ | $12\frac{20}{24}$ | $10\frac{8}{9}$ | $15\frac{1}{2}$ |
| 4. | $25\frac{1}{3}$ | $14\frac{7}{8}$ | $13\frac{3}{5}$ | $36\frac{3}{5}$ |
| 5. | $22\frac{1}{8}$ | $9\frac{5}{4}$ | $7\frac{3}{4}$ | $14\frac{2}{5}$ |
| 6. | $18\frac{3}{9}$ | $17\frac{12}{9}$ | $12\frac{1}{2}$ | $13\frac{3}{2}$ |

Written Copy. Write each mixed numeral as an addition numeral.

- | | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|----|-----------------|--------------------|----------------|-------------------|
| 1. | $4\frac{1}{5}$ | $4 + \frac{1}{5}$ | $6\frac{3}{4}$ | $6 + \frac{3}{4}$ |
| 2. | $6\frac{3}{10}$ | $6 + \frac{3}{10}$ | $7\frac{1}{9}$ | $7 + \frac{1}{9}$ |
| | | | $8\frac{2}{3}$ | $8 + \frac{2}{3}$ |
| | | | $4\frac{3}{8}$ | $4 + \frac{3}{8}$ |

Copy. Write a mixed numeral for each addition numeral.

- | | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|----|-------------------|----------------|--------------------|-----------------|
| 3. | $6 + \frac{2}{3}$ | $6\frac{2}{3}$ | $8 + \frac{7}{8}$ | $8\frac{7}{8}$ |
| | | | $9 + \frac{4}{7}$ | $9\frac{4}{7}$ |
| | | | $14 + \frac{1}{3}$ | $14\frac{1}{3}$ |
| 4. | $5 + \frac{5}{6}$ | $5\frac{5}{6}$ | $2 + \frac{3}{4}$ | $2\frac{3}{4}$ |
| | | | $1 + \frac{1}{2}$ | $1\frac{1}{2}$ |
| | | | $32 + \frac{5}{6}$ | $32\frac{5}{6}$ |

Review and Practice

You should be able to do the work on this page without any help.

Part 1 Copy. Write a single fraction and a multiplication numeral for each of the following. See page T193.

- | a | b |
|--|---|
| 1. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ | $\frac{2}{8} + \frac{2}{8} + \frac{2}{8}$ |
| 2. $\frac{3}{8} + \frac{3}{8}$ | $\frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$ |
| 3. $\frac{3}{4} + \frac{3}{4} + \frac{3}{4}$ | $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$ |

Part 2 Copy. Write a single fraction for each of the following.

- | a | b | c |
|---------------------------------------|-------------------------------------|-------------------------------------|
| 1. $3 \times \frac{1}{5} \frac{3}{5}$ | $4 \times \frac{5}{6} \frac{10}{3}$ | $7 \times \frac{3}{9} \frac{7}{3}$ |
| 2. $2 \times \frac{3}{8} \frac{3}{4}$ | $4 \times \frac{2}{9} \frac{8}{9}$ | $5 \times \frac{1}{3} \frac{5}{3}$ |
| 3. $6 \times \frac{1}{8} \frac{3}{4}$ | $9 \times \frac{4}{7} \frac{36}{7}$ | $8 \times \frac{2}{5} \frac{16}{5}$ |
| 4. $2 \times \frac{3}{4} \frac{3}{2}$ | $3 \times \frac{5}{6} \frac{5}{2}$ | $6 \times \frac{7}{8} \frac{21}{4}$ |

Part 3 Write three more fractions for each number listed below. See page T193.

- | a | b | c | d |
|-------------------|----------------|----------------|----------------|
| 1. $\frac{2}{8}$ | $\frac{2}{4}$ | $\frac{4}{5}$ | $\frac{2}{6}$ |
| 2. $\frac{4}{7}$ | $\frac{4}{8}$ | $\frac{3}{6}$ | $\frac{6}{8}$ |
| 3. $\frac{3}{11}$ | $\frac{3}{12}$ | $\frac{4}{15}$ | $\frac{6}{21}$ |
| 4. $\frac{2}{3}$ | $\frac{2}{5}$ | $\frac{2}{9}$ | $\frac{8}{9}$ |

Part 4 Copy. Change each fraction to simplest form.

- | a | b | c | d |
|------------------------------|----------------------------|-----------------------------|-----------------------------|
| 1. $\frac{2}{4} \frac{1}{2}$ | $\frac{4}{8} \frac{1}{2}$ | $\frac{6}{12} \frac{1}{2}$ | $\frac{8}{16} \frac{1}{2}$ |
| 2. $\frac{3}{9} \frac{1}{3}$ | $\frac{5}{15} \frac{1}{3}$ | $\frac{7}{21} \frac{1}{3}$ | $\frac{9}{27} \frac{1}{3}$ |
| 3. $\frac{4}{6} \frac{2}{3}$ | $\frac{8}{12} \frac{2}{3}$ | $\frac{12}{18} \frac{2}{3}$ | $\frac{16}{24} \frac{2}{3}$ |

Part 5 Copy. Find each product. Name each product as a single fraction in simplest form.

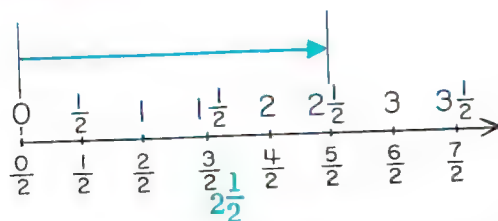
- | a | b | c |
|--|--|---|
| 1. $\frac{1}{3} \times \frac{4}{6} \frac{2}{9}$ | $\frac{1}{2} \times \frac{1}{4} \frac{1}{8}$ | $\frac{1}{2} \times \frac{1}{3} \frac{1}{6}$ |
| 2. $\frac{2}{3} \times \frac{3}{4} \frac{1}{2}$ | $\frac{3}{4} \times \frac{4}{5} \frac{3}{5}$ | $\frac{4}{5} \times \frac{1}{2} \frac{2}{5}$ |
| 3. $\frac{3}{8} \times \frac{5}{6} \frac{5}{16}$ | $\frac{2}{7} \times \frac{1}{4} \frac{1}{14}$ | $\frac{3}{9} \times \frac{1}{8} \frac{1}{24}$ |
| 4. $\frac{5}{6} \times \frac{1}{3} \frac{5}{18}$ | $\frac{6}{7} \times \frac{2}{5} \frac{12}{35}$ | $\frac{4}{7} \times \frac{3}{4} \frac{3}{7}$ |

Part 6 Write an open sentence for each problem. Solve the open sentence. Answer the problem.

1. Joe's mother bought a dozen bananas. The family ate $\frac{1}{3}$ of them at lunch. How many of the bananas were eaten? $\frac{1}{3} \times 12 = \square$; 4 bananas

2. Mrs. Wilson has $\frac{2}{3}$ of a pie left. Her son, Tom, who has not eaten any pie yet, wants to eat $\frac{1}{2}$ of the pie that is left. What part of a whole pie will Tom eat? $\frac{1}{2} \times \frac{2}{3} = \square$; $\frac{1}{3}$ of the pie

Mixed Numerals and Fractions



This number line is labeled in two different but equivalent ways. Using the names above the number line, how would you express the number represented by the blue arrow? Using the names below the number line, how would you express the same number? $\frac{5}{2}$

A convenient way to change a mixed numeral like $2\frac{1}{2}$ to a fraction is shown below. Study the example.

$$2\frac{1}{2} = 2 + \frac{1}{2}$$

How is $2\frac{1}{2}$ renamed?

$$= \frac{4}{2} + \frac{1}{2}$$

Why is 2 named as $\frac{4}{2}$ in $\frac{4}{2} + \frac{1}{2}$?

$$= \frac{4+1}{2}$$

Tell why $\frac{4}{2} + \frac{1}{2}$ can be changed to $\frac{4+1}{2}$.

$$= \frac{5}{2}$$

How is $\frac{4+1}{2}$ changed to $\frac{5}{2}$?

To change $2\frac{1}{2}$ to a fraction, rename $2\frac{1}{2}$ as $2 + \frac{1}{2}$. Then rename the whole number 2 as a fraction with the denominator of 2. Finally, add the fractional numbers $\frac{4}{2}$ and $\frac{1}{2}$.

In changing a fraction like $\frac{5}{2}$ to a mixed numeral you reverse the steps shown above.

$$\frac{5}{2} = \frac{4+1}{2}$$

Why is 5 renamed as $4+1$ in $\frac{4+1}{2}$?

$$= \frac{4}{2} + \frac{1}{2}$$

{ Do $\frac{4+1}{2}$ and $\frac{4}{2} + \frac{1}{2}$ name the same number?
How do you know?

$$= 2 + \frac{1}{2}$$

{ Do $\frac{4}{2}$ and 2 name the same number?
How do you know?

$$= 2\frac{1}{2}$$

Why do $2 + \frac{1}{2}$ and $2\frac{1}{2}$ name the same number?

To change $\frac{5}{2}$ to a mixed numeral, rename 5 as $4+1$. Then change $\frac{4+1}{2}$ to a sum of two fractional numbers $\frac{4}{2}$ and $\frac{1}{2}$. Then rename $\frac{4}{2}$ as 2. Finally, rename $2 + \frac{1}{2}$ as $2\frac{1}{2}$.

Oral Study the example shown below. Then answer the questions that follow.

$$\begin{aligned} 5\frac{3}{4} &= 5 + \frac{3}{4} \\ &= \frac{20}{4} + \frac{3}{4} \\ &= \frac{20+3}{4} \\ &= \frac{23}{4} \end{aligned}$$

- How is $5\frac{3}{4}$ renamed? $5 + \frac{3}{4}$
- Why is 5 named as $\frac{20}{4}$, instead of $\frac{10}{2}$, $\frac{15}{3}$, or $\frac{25}{5}$? $\frac{20}{4}$ is the most convenient name to use.
- Tell how $\frac{20}{4} + \frac{3}{4}$ is changed to $\frac{23}{4}$. $\frac{20+3}{4} = \frac{23}{4}$
- Tell how you would change $4\frac{2}{3}$ to $\frac{14}{3}$. See page T195.

Study the example below. Then answer questions 5–10.

$$\begin{aligned} \frac{22}{3} &= \frac{21+1}{3} \\ &= \frac{21}{3} + \frac{1}{3} \\ &= 7 + \frac{1}{3} \\ &= 7\frac{1}{3} \end{aligned}$$

- Why is 22 renamed as $21+1$ instead of $20+2$, $19+3$, or $18+4$? See page T195.

- Why can $\frac{21+1}{3}$ be changed to $\frac{21}{3} + \frac{1}{3}$? $\frac{21+1}{3} = \frac{21}{3} + \frac{1}{3}$

- Why can $\frac{21}{3} + \frac{1}{3}$ be changed to $7 + \frac{1}{3}$? $\frac{21}{3} + \frac{1}{3} = 7 + \frac{1}{3}$

- Why can $7 + \frac{1}{3}$ be changed to $7\frac{1}{3}$? See page T195.

- Tell how you would change $\frac{13}{4}$ to $3\frac{1}{4}$. $\frac{12+1}{4} = \frac{12}{4} + \frac{1}{4} = 3\frac{1}{4}$

- Tell how you would change $\frac{16}{3}$ to $5\frac{1}{3}$. $\frac{15+1}{3} = \frac{15}{3} + \frac{1}{3} = 5\frac{1}{3}$

Tell why

$$\begin{aligned} 4\frac{2}{3} &= \frac{(4 \times 3) + 2}{3} \\ &= \frac{12+2}{3} \\ &= \frac{14}{3} \end{aligned}$$

Written Copy. Change each mixed numeral to a fraction.

- | | a | b | c | d |
|----|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 1. | $5\frac{1}{5} \frac{26}{5}$ | $6\frac{2}{3} \frac{20}{3}$ | $8\frac{1}{4} \frac{33}{4}$ | $7\frac{1}{2} \frac{15}{2}$ |
| 2. | $9\frac{1}{3} \frac{28}{3}$ | $4\frac{3}{4} \frac{19}{4}$ | $3\frac{3}{8} \frac{27}{8}$ | $5\frac{5}{6} \frac{35}{6}$ |
| 3. | $2\frac{4}{7} \frac{18}{7}$ | $6\frac{1}{5} \frac{31}{5}$ | $4\frac{2}{5} \frac{22}{5}$ | $2\frac{1}{8} \frac{17}{8}$ |
| 4. | $4\frac{1}{3} \frac{13}{3}$ | $2\frac{5}{8} \frac{21}{8}$ | $7\frac{3}{5} \frac{38}{5}$ | $9\frac{4}{9} \frac{85}{9}$ |
| 5. | $8\frac{1}{2} \frac{17}{2}$ | $1\frac{1}{3} \frac{4}{3}$ | $9\frac{1}{8} \frac{73}{8}$ | $4\frac{1}{2} \frac{9}{2}$ |

Copy. Change each fraction to a mixed numeral.

- | | a | b | c | d |
|-----|-----------------------------|---|-----------------------------|---|
| 6. | $\frac{7}{4} 1\frac{3}{4}$ | $\frac{7}{2} 3\frac{1}{2}$ | $\frac{9}{4} 2\frac{1}{4}$ | $\frac{10}{3} 3\frac{1}{3}$ |
| 7. | $\frac{12}{5} 2\frac{2}{5}$ | $\frac{26}{5} 5\frac{1}{5}$ | $\frac{11}{4} 2\frac{3}{4}$ | $\frac{14}{6} 2\frac{2}{3}$ or $2\frac{1}{3}$ |
| 8. | $\frac{13}{7} 1\frac{6}{7}$ | $\frac{19}{6} 3\frac{1}{6}$ | $\frac{17}{8} 2\frac{1}{8}$ | $\frac{25}{9} 2\frac{7}{9}$ |
| 9. | $\frac{11}{3} 3\frac{2}{3}$ | $\frac{14}{4} 3\frac{2}{4}$ or $3\frac{1}{2}$ | $\frac{9}{2} 4\frac{1}{2}$ | $\frac{16}{3} 5\frac{1}{3}$ |
| 10. | $\frac{20}{3} 6\frac{2}{3}$ | $\frac{29}{4} 7\frac{1}{4}$ | $\frac{32}{5} 6\frac{2}{5}$ | $\frac{29}{6} 4\frac{5}{6}$ |

Tell why You can change the mixed numeral $5\frac{3}{4}$ to $\frac{23}{4}$ as shown below.

$$\begin{aligned} 5\frac{3}{4} &= \frac{(5 \times 4) + 3}{4} \\ &= \frac{20+3}{4} \\ &= \frac{23}{4} \end{aligned}$$

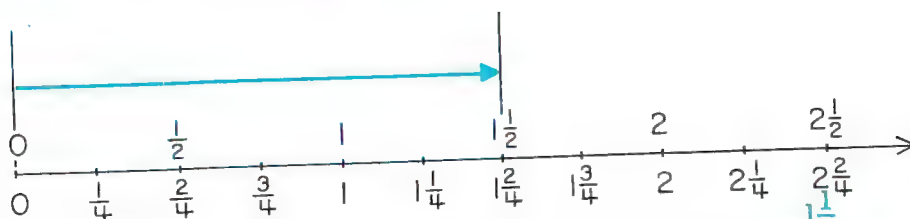
Change $4\frac{2}{3}$ to $\frac{14}{3}$ as shown above. See below.

Tell how How can you change $\frac{22}{3}$ to $7\frac{1}{3}$ by division? $22 \div 3 = 7 + (1 \div 3) = 7\frac{1}{3}$

Change $\frac{25}{4}$ to $6\frac{1}{4}$ by division.

$$\begin{aligned} 25 \div 4 &= 6 + (1 \div 4) \\ &= 6 + \frac{1}{4} \\ &= 6\frac{1}{4} \end{aligned}$$

Mixed Numerals in Simplest Form



What number is represented by the blue arrow above? What two mixed numerals above name that number? What is different about the mixed numerals $1\frac{1}{2}$ and $1\frac{2}{4}$? $1\frac{1}{2}$ and $1\frac{2}{4}$; The fractions are different. Yes

Consider only the fraction in $1\frac{1}{2}$. Is $\frac{1}{2}$ in simplest form? How do you know? Since $\frac{1}{2}$ is in simplest form, you say that $1\frac{1}{2}$ is in simplest form.

Consider only the fraction in $1\frac{2}{4}$. Is $\frac{2}{4}$ in simplest form? How do you know? Since $\frac{2}{4}$ is not in simplest form, you say that $1\frac{2}{4}$ is not in simplest form. No

A convenient way to change a mixed numeral to simplest form is shown below.

$$\begin{aligned}
 1\frac{2}{4} &= 1 + \frac{2}{4} && \text{How is } 1\frac{2}{4} \text{ renamed?} \\
 &= 1 + \left(\frac{1 \times 2}{2 \times 2}\right) && \left\{ \begin{array}{l} \text{What is the greatest common factor of 2 and 4?} \\ \text{Is 2 a factor in } 1 \times 2? \text{ In } 2 \times 2? \end{array} \right. \\
 &= 1 + \left(\frac{1}{2} \times \frac{2}{2}\right) && \left\{ \begin{array}{l} \text{How does knowing } \frac{1}{2} \times \frac{2}{2} = \frac{1 \times 2}{2 \times 2} \text{ help you change} \\ \frac{1 \times 2}{2 \times 2} \text{ to } \frac{1}{2} \times \frac{2}{2}? \end{array} \right. \\
 &= 1 + \left(\frac{1}{2} \times 1\right) && \text{How is } \frac{2}{2} \text{ renamed in changing } \frac{1}{2} \times \frac{2}{2} \text{ to } \frac{1}{2} \times 1? \\
 &= 1 + \frac{1}{2} && \text{Why can } 1 + \left(\frac{1}{2} \times 1\right) \text{ be changed to } 1 + \frac{1}{2}? \\
 &= 1\frac{1}{2} && \text{How is } 1 + \frac{1}{2} \text{ renamed?}
 \end{aligned}$$

A mixed numeral is in simplest form if the fraction is in simplest form and names a number less than one.

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- *1 The greatest common factor of the numerator and the denominator is the number one.
- *2 The greatest common factor of the numerator and the denominator is not the number one.

Oral Answer the following.

1. Consider the fraction in the mixed numeral $5\frac{6}{8}$. Is $\frac{6}{8}$ in simplest form? How do you know? Is $5\frac{6}{8}$ in simplest form? Why or why not? **See page T197.**

2. Consider the fraction in the mixed numeral $7\frac{3}{8}$. Is $\frac{3}{8}$ in simplest form? How do you know? Is $7\frac{3}{8}$ in simplest form? Why or why not? **See page T197.**

3. How can you tell whether or not a mixed numeral like $4\frac{2}{3}$ is in simplest form? **See page T197.**

Tell whether or not each mixed numeral is in simplest form. **Those in simplest form are circled.**

- | | a | b | c | d |
|----|----------------|-----------------|-----------------|-----------------|
| 4. | $3\frac{4}{8}$ | $2\frac{1}{3}$ | $5\frac{5}{8}$ | $6\frac{2}{4}$ |
| 5. | $8\frac{3}{9}$ | $7\frac{2}{10}$ | $4\frac{3}{10}$ | $2\frac{6}{10}$ |
| 6. | $1\frac{8}{9}$ | $4\frac{3}{5}$ | $2\frac{3}{6}$ | $3\frac{8}{12}$ |

Study how $7\frac{9}{12}$ is changed to $7\frac{3}{4}$. Then answer questions 7–13.

$$\begin{aligned}
 7\frac{9}{12} &= 7 + \frac{9}{12} \\
 &= 7 + \left(\frac{3 \times 3}{4 \times 3}\right) \\
 &= 7 + \left(\frac{3}{4} \times \frac{3}{3}\right) \\
 &= 7 + \left(\frac{3}{4} \times 1\right) \\
 &= 7 + \frac{3}{4} \\
 &= 7\frac{3}{4}
 \end{aligned}$$

7. How is $7\frac{9}{12}$ renamed? $7 + \frac{9}{12}$
8. Is the $\frac{9}{12}$ of $7 + \frac{9}{12}$ in simplest form? How do you know? **See page T197.**
9. Why is the $\frac{9}{12}$ of $7 + \frac{9}{12}$ renamed as $\frac{3 \times 3}{4 \times 3}$ in $7 + \left(\frac{3 \times 3}{4 \times 3}\right)$? **So the identity number of multiplication can be used to change $\frac{9}{12}$ to simplest form.**

10. How does knowing $\frac{3}{4} \times \frac{3}{3} = \frac{3 \times 3}{4 \times 3}$ help you change $7 + \left(\frac{3 \times 3}{4 \times 3}\right)$ to $7 + \left(\frac{3}{4} \times \frac{3}{3}\right)$? **See page T197.**

11. Why can $\frac{3}{4} \times \frac{3}{3}$ in $7 + \left(\frac{3}{4} \times \frac{3}{3}\right)$ be renamed as $\frac{3}{4} \times 1$ in $7 + \left(\frac{3}{4} \times 1\right)$? $\frac{3}{3} = 1$

12. How is $\frac{3}{4} \times 1$ in $7 + \left(\frac{3}{4} \times 1\right)$ renamed in $7 + \frac{3}{4}$? Why? **See page T197.**

13. How is $7 + \frac{3}{4}$ renamed? $7\frac{3}{4}$

How would you change each mixed numeral below to simplest form? **See page T197.**

- | | a | b | c | d |
|-----|----------------|----------------|----------------|----------------|
| 14. | $5\frac{2}{4}$ | $8\frac{3}{6}$ | $4\frac{3}{9}$ | $7\frac{4}{6}$ |

Written Copy. Change each mixed numeral to simplest form.

- | | a | b | c | d |
|----|------------------|----------------|------------------|----------------|
| 1. | $3\frac{6}{9}$ | $3\frac{2}{3}$ | $2\frac{6}{8}$ | $2\frac{3}{4}$ |
| | $4\frac{9}{12}$ | $4\frac{3}{4}$ | $7\frac{8}{12}$ | $7\frac{2}{3}$ |
| 2. | $8\frac{10}{15}$ | $8\frac{2}{3}$ | $9\frac{12}{15}$ | $9\frac{4}{5}$ |
| | $6\frac{3}{6}$ | $6\frac{1}{2}$ | $1\frac{2}{4}$ | $1\frac{1}{2}$ |
| 3. | $5\frac{12}{16}$ | $5\frac{3}{4}$ | $3\frac{8}{16}$ | $3\frac{1}{2}$ |
| | $8\frac{10}{12}$ | $8\frac{5}{6}$ | $6\frac{4}{8}$ | $6\frac{1}{2}$ |
| 4. | $9\frac{15}{20}$ | $9\frac{3}{4}$ | $1\frac{12}{20}$ | $1\frac{3}{5}$ |
| | $7\frac{12}{24}$ | $7\frac{1}{2}$ | $2\frac{18}{24}$ | $2\frac{3}{4}$ |
| 5. | $1\frac{6}{30}$ | $1\frac{1}{5}$ | $8\frac{5}{30}$ | $8\frac{1}{6}$ |
| | $3\frac{8}{32}$ | $3\frac{1}{4}$ | $5\frac{9}{36}$ | $5\frac{1}{4}$ |

Tell why Answer the following.

1. Is $4\frac{0}{3}$ a mixed numeral? Why or why not? **See page T197.**

2. Is $4\frac{0}{3}$ in simplest form? Why or why not? **See page T197.**

3. What is the simplest form of $4\frac{0}{3}$?
4

Solving Problems

Write an open sentence for each problem below. Solve each open sentence. Answer the problem.

1. Mrs. Virgil is going to make a Halloween costume for her daughter. She bought $\frac{5}{8}$ of a yard of red material, $\frac{2}{8}$ of a yard of blue material, and $\frac{2}{8}$ of a yard of white material. How much material did she buy in all? $\frac{5}{8} + \frac{2}{8} + \frac{2}{8} = \square$; $1\frac{1}{8}$ yards

2. On Friday Jim walked $\frac{5}{10}$ of a mile. On Saturday he walked $\frac{8}{10}$ of a mile. On Sunday he walked $\frac{4}{10}$ of a mile. How many miles did he walk altogether on these three days?

See below.

3. Ronnie had 24 concert tickets to sell. He was able to sell $\frac{2}{3}$ of them. How many of the concert tickets was he able to sell? $\frac{2}{3} \times 24 = \square$; 16 tickets

4. Sue had 48 pages in her stamp book. She had enough stamps to fill $\frac{1}{4}$ of the pages. How many pages could she fill? $\frac{1}{4} \times 48 = \square$; 12 pages

5. Carol had 28 sea shells in a collection. She obtained $\frac{3}{4}$ of them in Florida. How many of the shells came from Florida? $\frac{3}{4} \times 28 = \square$; 21 shells

6. Mrs. James had $\frac{2}{6}$ of a cake left. Her son ate $\frac{1}{2}$ of what was left. How much of the whole cake did he eat? $\frac{1}{2} \times \frac{2}{6} = \square$; $\frac{1}{6}$ of the whole cake

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2. $\frac{5}{10} + \frac{8}{10} + \frac{4}{10} = \square$; $1\frac{7}{10}$ miles

7. Sally had $\frac{3}{4}$ yard of ribbon. She bought another $\frac{3}{4}$ yard of ribbon. How much ribbon does she now have? $2 \times \frac{3}{4} = \square$; $1\frac{1}{2}$ yards

8. Mrs. Ellis had $\frac{2}{3}$ of a dozen cookies. She gave $\frac{1}{2}$ of them to Ann. What part of a dozen cookies did she give to Ann? $\frac{1}{2} \times \frac{2}{3} = \square$; $\frac{1}{3}$ of a dozen

9. Bill rode his bicycle $\frac{4}{10}$ of a mile to the store. Then he rode it $\frac{6}{10}$ of a mile to Ted's house. In all, how far did he ride his bicycle? $\frac{4}{10} + \frac{6}{10} = \square$; 1 mile

10. Mrs. Seaton had 6 boxes of holiday cookies. She put $\frac{1}{2}$ pound of cookies in each box. How many pounds of cookies did she have? $6 \times \frac{1}{2} = \square$; 3 pounds

Can you do this? Solve each problem below. Answer each problem.

1. Tom used $\frac{3}{10}$ of some rope for his boat and $\frac{5}{10}$ of the rope for his tent. How much of the rope does he have left? (Hint: Think of the whole amount of rope as $\frac{10}{10}$.) $\frac{2}{10}$ of the rope is left

2. Harry ate $\frac{1}{4}$ of a box of cereal for breakfast. Tim also ate $\frac{1}{4}$ of the box of cereal. How much cereal was still in the box? (Hint: Think of the whole box of cereal as $\frac{4}{4}$.) $\frac{1}{2}$ of the cereal was still in the box

3. Pete used $\frac{3}{8}$ of a can of paint to paint a table. He used $\frac{2}{8}$ of the can of paint to paint a chair. How much of the paint is still left? $\frac{3}{8}$ of the paint is left

Checkup Time

The numerals in () tell the pages where you can turn for help.

Important Ideas

1. To multiply two fractional numbers like $\frac{2}{3}$ and $\frac{5}{6}$, multiply their numerators and then multiply their denominators. (184)
2. Multiplication is commutative for all numbers that can be named by fractions. (186)
3. The number one is the identity number of multiplication for fractional numbers. (187)
4. You can find many names for every fractional number. (188)
5. A fraction is in simplest form if the greatest common factor of the numerator and the denominator is one. (190)
6. A mixed numeral like $2\frac{3}{4}$ means $2 + \frac{3}{4}$. (192)
7. A mixed numeral names the sum of two numbers. One of the numbers is named by a numeral for a whole number and the other is named by a fraction. (192)

Words to Know

1. Identity number of multiplication (187)

2. Simplest form (190)

3. Mixed numeral (192)

Questions to Discuss

See page T199.

1. How would you solve the open sentence $\frac{2}{3} \times \frac{1}{4} = n$? (184)
2. Does commuting or changing the order of the fractional numbers in $\frac{2}{3} \times \frac{3}{5} = n$ change the product? Why? (186)
3. How would you change $\frac{1}{2}$ to $\frac{4}{8}$? (188)
4. How would you change $\frac{5}{10}$ to $\frac{1}{2}$? (190)
5. How would you change $\frac{16}{3}$ to $5\frac{1}{3}$? (194)
6. How would you change $4\frac{5}{6}$ to $\frac{29}{6}$? (194)

Written Practice

See page T199.

Do the following.

1. Write a single fraction for $\frac{3}{8} \times \frac{7}{9}$. (184)
2. Change $\frac{2}{3}$ to $\frac{12}{18}$. (188)
3. Change $\frac{20}{24}$ to $\frac{5}{6}$. (190)
4. Change $3\frac{7}{8}$ to $\frac{31}{8}$. (194)
5. Change $\frac{27}{4}$ to $6\frac{3}{4}$. (194)

Self-Evaluation

Part 1 Write each addition numeral as a multiplication numeral. Then write a single fraction for each multiplication numeral. See below for multiplication numerals.

- | | a | b |
|----|---|--------------------------------|
| 1. | $\frac{2}{8} + \frac{2}{8} + \frac{2}{8}$ | $\frac{6}{8}$ or $\frac{3}{4}$ |
| 2. | $\frac{1}{4} + \frac{1}{4}$ | $\frac{2}{4}$ or $\frac{1}{2}$ |
| 3. | $\frac{3}{6} + \frac{3}{6} + \frac{3}{6}$ | $\frac{9}{6}$ or $\frac{3}{2}$ |
| 4. | $\frac{4}{5} + \frac{4}{5}$ | $\frac{8}{5}$ |

Part 2 Find each product in simplest form.

- | | a | b | c |
|----|-----------------------------------|---------------|----------------------------------|
| 1. | $\frac{1}{3} \times \frac{2}{3}$ | $\frac{2}{9}$ | $\frac{1}{5} \times \frac{1}{5}$ |
| 2. | $\frac{4}{5} \times \frac{1}{2}$ | $\frac{2}{5}$ | $\frac{3}{7} \times \frac{3}{7}$ |
| 3. | $\frac{9}{10} \times \frac{2}{3}$ | $\frac{3}{5}$ | $\frac{5}{6} \times \frac{2}{5}$ |
| 4. | $\frac{1}{2} \times \frac{1}{2}$ | $\frac{1}{4}$ | $\frac{4}{8} \times \frac{3}{4}$ |

Part 3 Write five more fractions for each number named below. Answers may vary.

- | | a | b | c | d |
|----|---------------|----------------|---------------|----------------|
| 1. | $\frac{2}{3}$ | $\frac{3}{4}$ | $\frac{3}{5}$ | $\frac{4}{7}$ |
| 2. | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{10}$ |
| 3. | $\frac{5}{6}$ | $\frac{3}{9}$ | $\frac{5}{8}$ | $\frac{2}{6}$ |
| 4. | $\frac{1}{2}$ | $\frac{3}{12}$ | $\frac{4}{9}$ | $\frac{5}{7}$ |

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Part 1

- 1a. $3 \times \frac{2}{8}$
3b. $2 \times \frac{2}{3}$

- 1b. $4 \times \frac{3}{9}$
4a. $2 \times \frac{4}{5}$

- 2a. $2 \times \frac{1}{4}$
4b. $4 \times \frac{7}{8}$

Part 4 Change each of the following fractions to simplest form.

- | | a | b | c | d |
|----|-----------------|---------------|-----------------|----------------|
| 1. | $\frac{6}{18}$ | $\frac{1}{3}$ | $\frac{5}{15}$ | $\frac{1}{3}$ |
| 2. | $\frac{3}{12}$ | $\frac{1}{4}$ | $\frac{4}{16}$ | $\frac{1}{4}$ |
| 3. | $\frac{20}{32}$ | $\frac{5}{8}$ | $\frac{14}{20}$ | $\frac{7}{10}$ |
| 4. | $\frac{24}{36}$ | $\frac{2}{3}$ | $\frac{28}{32}$ | $\frac{7}{8}$ |

Part 5 Copy. Change each mixed numeral to a fraction.

- | | a | b | c | d |
|----|----------------|----------------|----------------|----------------|
| 1. | $3\frac{1}{5}$ | $\frac{16}{5}$ | $6\frac{2}{3}$ | $\frac{20}{3}$ |
| 2. | $9\frac{1}{3}$ | $\frac{28}{3}$ | $4\frac{3}{4}$ | $\frac{19}{4}$ |

Copy. Change each fraction to a mixed numeral.

- | | a | b | c | d |
|----|----------------|----------------|----------------|----------------|
| 3. | $\frac{7}{4}$ | $1\frac{3}{4}$ | $\frac{26}{5}$ | $5\frac{1}{5}$ |
| 4. | $\frac{14}{6}$ | $2\frac{1}{3}$ | $\frac{13}{7}$ | $1\frac{6}{7}$ |

Part 6 Copy. Find each product in simplest form.

- | | a | b | c |
|----|-------------------------|----------------|-------------------------|
| 1. | $8 \times \frac{3}{4}$ | 6 | $7 \times \frac{2}{3}$ |
| 2. | $10 \times \frac{1}{2}$ | 5 | $15 \times \frac{1}{3}$ |
| 3. | $22 \times \frac{2}{5}$ | $8\frac{4}{5}$ | $25 \times \frac{3}{8}$ |

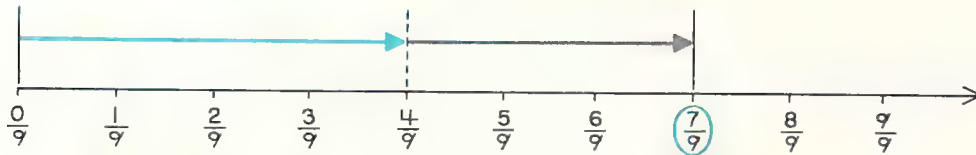
- | | a | b | c |
|----|-------------------------|----------------|-------------------------|
| 1. | $8 \times \frac{3}{4}$ | 6 | $7 \times \frac{2}{3}$ |
| 2. | $10 \times \frac{1}{2}$ | 5 | $15 \times \frac{1}{3}$ |
| 3. | $22 \times \frac{2}{5}$ | $8\frac{4}{5}$ | $25 \times \frac{3}{8}$ |

- 3a. $3 \times \frac{3}{6}$

Chapter 9

ADDITION AND SUBTRACTION OF FRACTIONAL NUMBERS

Addition of Fractional Numbers



Think of starting at $\frac{0}{9}$ and moving 4 segments or spaces to the right. The blue arrow represents what fractional number? $\frac{4}{9}$
 Now move three more segments to the right. The gray arrow represents what fractional number? $\frac{3}{9}$
 At which mark on the number line do you stop? $\frac{7}{9}$ These two moves suggest the following.

$$\frac{4}{9} + \frac{3}{9} = \frac{4+3}{9} = \frac{7}{9} \quad \text{or} \quad \begin{array}{r} \frac{4}{9} \\ + \frac{3}{9} \\ \hline \frac{7}{9} \end{array}$$

What is the denominator of each addend in $\frac{4}{9} + \frac{3}{9}$? What is the denominator of the sum? What are the numerators of the addends? What do you do with these numerators to get the numerator of the sum? $9; 9; 4 \text{ and } 3; \text{ add the numerators.}$

Oral Tell how to find the numerator and the denominator for the single fraction that names each sum. What is that single fraction?

- $\frac{3}{8} + \frac{2}{8} = \frac{3+2}{8} = \frac{5}{8}$
- $\frac{8}{25} + \frac{7}{25} = \frac{8+7}{25} = \frac{15}{25}$ or $\frac{3}{5}$
- $\frac{7}{15} + \frac{4}{15} = \frac{7+4}{15} = \frac{11}{15}$

Written Do the following.

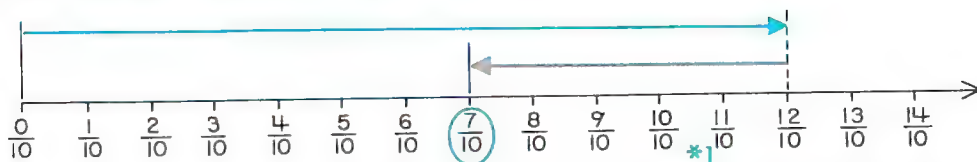
1-3. Copy *Oral* 1-3 and solve.
 See *Oral*.

Copy. Find each sum as a single fraction.

	a	b	c	d
4.	$\frac{2}{10}$	$\frac{3}{8}$	$\frac{6}{12}$	$\frac{6}{14}$
	$+\frac{5}{10}$	$+\frac{4}{8}$	$+\frac{5}{12}$	$+\frac{3}{14}$
	$\frac{7}{10}$	$\frac{7}{8}$	$\frac{11}{12}$	$\frac{9}{14}$
				201

*1 $\frac{3}{9}$

Subtraction of Fractional Numbers



In what scale is this number line drawn? Think of starting at $\frac{0}{10}$ and moving twelve segments or spaces to the right. The blue arrow represents what fractional number? $\frac{12}{10}$

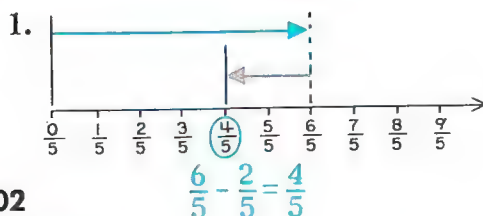
Now move five segments to the left. The gray arrow represents what fractional number? At which mark on the number line do you stop? These two moves suggest the following.

$$\frac{12}{10} - \frac{5}{10} = \frac{12-5}{10} = \frac{7}{10} \quad \text{or} \quad \begin{array}{r} \frac{12}{10} \\ - \frac{5}{10} \\ \hline \frac{7}{10} \end{array}$$

What is the denominator of the minuend? Of the subtrahend? What is the denominator of the difference? What is the numerator of the minuend? What is the numerator of the subtrahend? What do you do with the 12 and 5 to get the numerator of the difference? 10; 10; 10; 12; 5; subtract 5 from 12

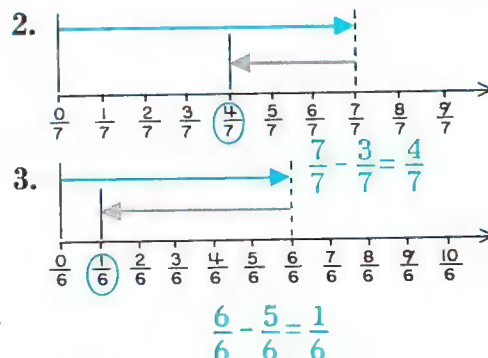
To subtract two fractional numbers having the same denominator, subtract the numerators and record this difference over that same denominator.

Oral Tell the closed subtraction sentence shown by the arrows on each number line.



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*1 tenths
*2 $\frac{5}{10}$



Tell how to find the numerator and the denominator for the single fraction that names each difference. What is that single fraction?

$$4. \quad \frac{4}{8} - \frac{1}{8} = \frac{4-1}{8} = \square \quad \frac{3}{8}$$

$$5. \quad \frac{5}{7} - \frac{2}{7} = \frac{5-2}{7} = \square \quad \frac{3}{7}$$

$$6. \quad \frac{15}{16} - \frac{8}{16} = \frac{15-8}{16} = \square \quad \frac{7}{16}$$

$$7. \quad \frac{13}{18} - \frac{8}{18} = \frac{13-8}{18} = \square \quad \frac{5}{18}$$

$$8. \quad \frac{13}{24} - \frac{6}{24} = \frac{13-6}{24} = \square \quad \frac{7}{24}$$

$$9. \quad \frac{16}{21} - \frac{7}{21} = \frac{16-7}{21} = \square \quad \frac{9}{21} \text{ or } \frac{3}{7}$$

Written Draw a number line to illustrate each of the following subtraction sentences. See page T203.

a

b

$$1. \quad \frac{7}{12} - \frac{5}{12} = \frac{2}{12}$$

$$\frac{9}{9} - \frac{5}{9} = \frac{4}{9}$$

$$2. \quad \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$$

$$\frac{8}{10} - \frac{3}{10} = \frac{5}{10}$$

Copy. Find each difference. Change each difference to simplest form, if needed.

$$3. \quad \frac{8}{9} - \frac{1}{9} = a \quad \frac{7}{9}$$

$$\frac{7}{10} - \frac{2}{10} = b \quad \frac{5}{10} = \frac{1}{2}$$

$$4. \quad \frac{9}{12} - \frac{8}{12} = c \quad \frac{1}{12}$$

$$\frac{8}{8} - \frac{4}{8} = d \quad \frac{4}{8} = \frac{1}{2}$$

$$5. \quad \frac{14}{18} - \frac{9}{18} = e \quad \frac{5}{18}$$

$$\frac{14}{12} - \frac{7}{12} = f \quad \frac{7}{12}$$

$$6. \quad \frac{12}{15} - \frac{8}{15} = g \quad \frac{4}{15}$$

$$\frac{13}{17} - \frac{6}{17} = h \quad \frac{7}{17}$$

$$7. \quad \frac{14}{16} - \frac{6}{16} = i \quad \frac{8}{16} = \frac{1}{2}$$

$$\frac{12}{21} - \frac{9}{21} = j \quad \frac{3}{21} = \frac{1}{7}$$

Tell how

$$\frac{5}{10} - \square = \frac{2}{10}$$

$$\frac{5}{10} - \frac{3}{10} = \frac{2}{10} + \square$$

$$\frac{5}{10} - \frac{2}{10} = \square$$

Thus, subtract $\frac{2}{10}$ from $\frac{5}{10}$.

$$8. \quad \frac{17}{18} - \frac{5}{18} = k \quad \frac{12}{18} = \frac{2}{3}$$

$$\frac{15}{16} - \frac{5}{16} = r \quad \frac{10}{16} = \frac{5}{8}$$

$$9. \quad \frac{24}{25} - \frac{14}{25} = m \quad \frac{10}{25} = \frac{2}{5}$$

$$\frac{32}{36} - \frac{8}{36} = m \quad \frac{24}{36} = \frac{2}{3}$$

$$10. \quad \frac{28}{30} - \frac{8}{30} = p \quad \frac{20}{30} = \frac{2}{3}$$

$$\frac{26}{35} - \frac{19}{35} = q \quad \frac{7}{35} = \frac{1}{5}$$

Copy. Find each difference in simplest form.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
11.	$\frac{3}{4}$ $-\frac{2}{4}$ $\frac{1}{4}$	$\frac{6}{8}$ $-\frac{3}{8}$ $\frac{3}{8}$	$\frac{8}{9}$ $-\frac{4}{9}$ $\frac{4}{9}$	$\frac{6}{10}$ $-\frac{1}{10}$ $\frac{5}{10}$
12.	$\frac{14}{15}$ $-\frac{7}{15}$ $\frac{7}{15}$	$\frac{17}{20}$ $-\frac{9}{20}$ $\frac{8}{20}$	$\frac{16}{16}$ $-\frac{9}{16}$ $\frac{7}{16}$	$\frac{9}{18}$ $-\frac{3}{18}$ $\frac{6}{18}$
13.	$\frac{20}{24}$ $-\frac{12}{24}$ $\frac{8}{24}$	$\frac{21}{25}$ $-\frac{9}{25}$ $\frac{12}{25}$	$\frac{19}{28}$ $-\frac{9}{28}$ $\frac{10}{28}$	$\frac{18}{32}$ $-\frac{9}{32}$ $\frac{9}{32}$
14.	$\frac{18}{24}$ $-\frac{5}{24}$ $\frac{13}{24}$	$\frac{9}{16}$ $-\frac{8}{16}$ $\frac{1}{16}$	$\frac{12}{20}$ $-\frac{8}{20}$ $\frac{4}{20}$	$\frac{16}{32}$ $-\frac{15}{32}$ $\frac{1}{32}$

Can you do this? Find the unnamed addend in each of the following.

a

b

$$1. \quad \frac{3}{9} + a = \frac{6}{9} \quad \frac{3}{9}$$

$$b + \frac{5}{8} = \frac{8}{8} \quad \frac{3}{8}$$

$$2. \quad \frac{5}{10} + c = \frac{8}{10} \quad \frac{3}{10}$$

$$d + \frac{6}{12} = \frac{11}{12} \quad \frac{5}{12}$$

$$3. \quad \frac{8}{15} + e = \frac{12}{15} \quad \frac{4}{15}$$

$$f + \frac{9}{24} = \frac{14}{24} \quad \frac{5}{24}$$

Tell why You can find the single fraction to replace the \square in $\square - \frac{2}{8} = \frac{3}{8}$ by solving the open sentence $\frac{3}{8} + \frac{2}{8} = \square$. Why? Addition is the inverse operation of subtraction.

Tell how How would you find the single fraction to replace the \square in $\frac{5}{10} - \square = \frac{2}{10}$? See below.

Least Common Denominator

Two or more fractions having different denominators can be renamed so that they have the same denominator.

Study the two rows of equivalent fractions below.

$$\frac{1}{6} = \frac{2}{12} = \frac{3}{18} = \frac{4}{24} = \frac{5}{30} = \frac{6}{36} = \frac{7}{42} = \frac{8}{48} = \frac{9}{54} = \frac{10}{60}$$

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24} = \frac{21}{28} = \frac{24}{32} = \frac{27}{36} = \frac{30}{40}$$

Can $\frac{1}{6}$ and $\frac{3}{4}$ both be named with a denominator of 12? With a denominator of 24? With a denominator of 36? With what other numbers as a denominator? **Yes; Yes; Yes; 48, 60, 72, and all other multiples of 12**

The numbers 12, 24, and 36 are called **common denominators** of $\frac{1}{6}$ and $\frac{3}{4}$. When you say that two fractions have a *common denominator* you mean that they have the *same denominator*. Do $\frac{2}{12}$ and $\frac{9}{12}$ have the same denominator? **Yes**

What is the least number that is a common denominator of $\frac{1}{6}$ and $\frac{3}{4}$? ^{*1}You call that number the **least common denominator** of $\frac{1}{6}$ and $\frac{3}{4}$.

A more convenient way to find the least common denominator is described below.

Consider only the denominators of $\frac{1}{6}$ and $\frac{3}{4}$. How can you use the product of 6 and 4 and the greatest common factor ^{*2} of 6 and 4 to find their least common multiple? Do you find the least common multiple of 6 and 4 to be 12? **Yes**

Compare the least common multiple of 6 and 4 with the least common denominator of $\frac{1}{6}$ and $\frac{3}{4}$ found above. What are your conclusions? **They are the same.**

The *least common denominator* of two fractional numbers like $\frac{1}{6}$ and $\frac{3}{4}$ is the least common multiple of their denominators 6 and 4.

Oral Answer the following questions. Use the rows of equivalent fractions listed below.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16}$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18} = \frac{7}{21} = \frac{8}{24}$$

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24} = \frac{7}{28} = \frac{8}{32}$$

$$\frac{1}{5} = \frac{2}{10} = \frac{3}{15} = \frac{4}{20} = \frac{5}{25} = \frac{6}{30} = \frac{7}{35} = \frac{8}{40}$$

1. What is the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$? **6**

2. What is the least common denominator of $\frac{1}{2}$ and $\frac{1}{4}$? **4**

3. What is the least common denominator of $\frac{1}{2}$ and $\frac{1}{5}$? **10**

4. What is the least common denominator of $\frac{1}{3}$ and $\frac{1}{4}$? **12**

Answer the following questions about finding the least common denominator of the fractional numbers $\frac{1}{2}$ and $\frac{3}{8}$.

5. What is the product of 2 and 8? **16**

6. What is the greatest common factor of 2 and 8? **2**

7. How can you use the product of 2 and 8 and the greatest common factor of 2 and 8 to find their least common multiple? **Divide 16 by 2.**

8. What is the least common multiple of 2 and 8? **8**

9. What is the least common denominator of $\frac{1}{2}$ and $\frac{3}{8}$? **8**

Answer the following questions about $\frac{2}{3}$ and $\frac{4}{9}$.

10. What is the product of 3 and 9? **27**

11. What is the greatest common factor of 3 and 9? **3**

12. How can you use the product of 3 and 9 and the greatest common factor of 3 and 9 to find their least common multiple? **Divide 27 by 3.**

13. What is the least common multiple of 3 and 9? **9**

14. What is the least common denominator of $\frac{2}{3}$ and $\frac{4}{9}$? **9**

Tell how you would find the least common denominator for each pair of fractional numbers listed below.

See below.

a

b

c

15. $\frac{1}{5}$ and $\frac{3}{10}$ $\frac{1}{2}$ and $\frac{1}{6}$ $\frac{3}{8}$ and $\frac{5}{12}$

Written Find the least common denominator for each pair of fractional numbers below.

a

b

c

1. $\frac{3}{4}$ and $\frac{1}{12}$ $\frac{1}{3}$ and $\frac{1}{5}$ $\frac{1}{8}$ and $\frac{1}{2}$

2. $\frac{2}{3}$ and $\frac{3}{8}$ $\frac{1}{3}$ and $\frac{3}{5}$ $\frac{1}{4}$ and $\frac{2}{8}$

3. $\frac{5}{6}$ and $\frac{3}{8}$ $\frac{1}{3}$ and $\frac{3}{4}$ $\frac{2}{3}$ and $\frac{3}{7}$

Can you do this? Find the least common denominator for the fractional numbers named below.

$\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$

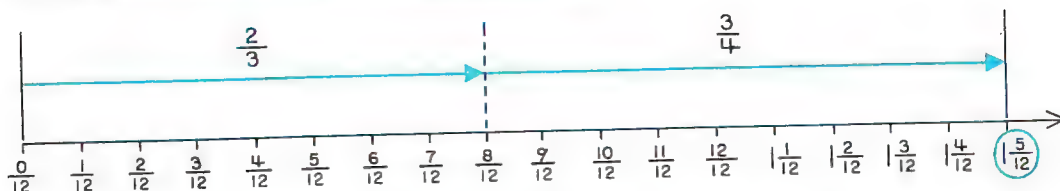
12

Oral 15a. $(5 \times 10) \div 5 = 10$

15b. $(2 \times 6) \div 2 = 6$

15c. $(8 \times 12) \div 4 = 24$

Least Common Denominator in Addition



You can find the simplest numeral for $\frac{2}{3} + \frac{3}{4}$ by using a number line as shown above.

An easier way to find the simplest numeral for $\frac{2}{3} + \frac{3}{4}$ is to rename both addends as fractions with the same denominator.

Consider only the denominators of $\frac{2}{3}$ and $\frac{3}{4}$.	3 and 4
What is the product of 3 and 4?	$3 \times 4 = 12$
What is the greatest common factor of 3 and 4?	1
What is the least common multiple of 3 and 4?	$12 \div 1 = 12$
Then what is the least common denominator of $\frac{2}{3}$ and $\frac{3}{4}$?	12

Now you know that $\frac{2}{3}$ and $\frac{3}{4}$ should both be renamed with a denominator of 12. You can rename $\frac{2}{3}$ and $\frac{3}{4}$ as follows.

$$\frac{2}{3} \times 1 = \frac{2}{3} \times \frac{\square}{\square} = \frac{8}{12}$$

What numeral should replace the \square so that $3 \times \square = 12$ becomes true?⁴
 Since $\frac{8}{12}$ stands for 1, both \square 's must be replaced by the same numeral.
 How is 8 obtained in $\frac{8}{12}$? 2×4

$$\frac{3}{4} \times 1 = \frac{3}{4} \times \frac{\square}{\square} = \frac{9}{12}$$

What numeral should replace the \square so that $4 \times \square = 12$ becomes true?³
 Since $\frac{9}{12}$ stands for 1, both \square 's must be replaced by the same numeral.
 How is 9 obtained in $\frac{9}{12}$? 3×3

You can now express the example in two different ways.

$\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12}$	How is $\frac{2}{3} + \frac{3}{4}$ renamed?	$\begin{array}{r} \frac{2}{3} \\ + \frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} \frac{8}{12} \\ + \frac{9}{12} \\ \hline \end{array}$
$= \frac{8+9}{12}$	What is done with the numerators?		$\frac{8+9}{12} = \frac{17}{12}$
$= \frac{17}{12}$ or $1\frac{5}{12}$	How is $\frac{17}{12}$ changed to $1\frac{5}{12}$?		$= 1\frac{5}{12}$

Oral Answer the following questions about solving the open sentence $\frac{3}{8} + \frac{5}{6} = n$.

1. What is the product of 8 and 6? 48

2. What is the greatest common factor of 8 and 6? 2

3. How can you use the product of 8 and 6 and the greatest common factor of 8 and 6 to find their least common multiple? Divide 48 by 2.

4. What is the least common multiple of 8 and 6? 24

5. What is the least common denominator of $\frac{3}{8}$ and $\frac{5}{6}$? 24

6. How does $\frac{3}{8} = \frac{3}{8} \times 1$ help you rename $\frac{3}{8}$ as $\frac{9}{24}$? See below.

7. How does $\frac{5}{6} = \frac{5}{6} \times 1$ help you rename $\frac{5}{6}$ as $\frac{20}{24}$? See below.

8. Do $\frac{3}{8} + \frac{5}{6}$ and $\frac{9}{24} + \frac{20}{24}$ both name the same number? Yes

9. Which single fraction names the sum $\frac{9}{24} + \frac{20}{24}$? $\frac{29}{24}$

10. Which mixed numeral is equivalent to $\frac{29}{24}$? $1\frac{5}{24}$

Explain how you would solve $\frac{3}{8} + \frac{5}{6} = n$ as shown below.

$$\begin{array}{r} 11. \quad \frac{3}{8} \longrightarrow \frac{9}{24} \\ + \frac{2}{3} \quad \quad + \frac{16}{24} \\ \hline \quad \quad \quad \frac{25}{24} = 1\frac{1}{24} \end{array}$$

Oral 6. $\frac{3}{8} = \frac{3}{8} \times 1 = \frac{3}{8} \times \frac{3}{3} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$

Written Copy. Find each sum as a fraction or a mixed numeral in simplest form.

	a	b	c
1.	$\frac{2}{6} + \frac{1}{2} = \frac{5}{6}$	$\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$	$\frac{1}{2} + \frac{3}{8} = \frac{7}{8}$
2.	$\frac{2}{5} + \frac{3}{6} = \frac{9}{10}$	$\frac{5}{9} + \frac{2}{3} = 1\frac{2}{9}$	$\frac{2}{7} + \frac{3}{8} = \frac{37}{56}$
3.	$\frac{2}{3} + \frac{3}{8} = 1\frac{1}{24}$	$\frac{5}{7} + \frac{7}{21} = 1\frac{1}{21}$	$\frac{3}{5} + \frac{3}{4} = 1\frac{7}{20}$
4.	$\frac{3}{4} + \frac{2}{6} = 1\frac{1}{12}$	$\frac{8}{9} + \frac{7}{18} = 1\frac{5}{18}$	$\frac{2}{4} + \frac{3}{8} = \frac{7}{8}$
5.	$\frac{3}{5} + \frac{2}{10} = \frac{4}{5}$	$\frac{1}{2} + \frac{5}{8} = 1\frac{1}{8}$	$\frac{1}{2} + \frac{3}{4} = 1\frac{1}{4}$
6.	$\frac{3}{8} + \frac{2}{5} = \frac{31}{40}$	$\frac{3}{5} + \frac{3}{10} = \frac{9}{10}$	$\frac{2}{6} + \frac{5}{8} = \frac{23}{24}$

Copy. Find each sum as a fraction or a mixed numeral in simplest form.

	a	b	c	d
7.	$\frac{3}{6} + \frac{1}{3} = 1\frac{1}{6}$	$\frac{5}{8} + \frac{1}{2} = 1\frac{1}{8}$	$\frac{4}{6} + \frac{2}{3} = \frac{3}{4}$	$\frac{3}{4} + \frac{7}{8} = 1\frac{5}{8}$
8.	$\frac{2}{3} + \frac{4}{9} = 1\frac{1}{9}$	$\frac{1}{4} + \frac{4}{16} = \frac{1}{2}$	$\frac{5}{6} + \frac{2}{3} = 1\frac{1}{2}$	$\frac{2}{3} + \frac{5}{8} = 1\frac{7}{24}$

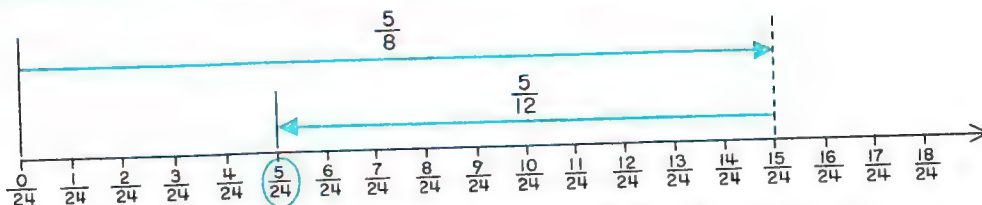
Can you do this? Find the unnamed minuend in each of the following.

	a	b
1.	$n - \frac{2}{8} = \frac{3}{8} \quad \frac{5}{8}$	$n - \frac{1}{5} = \frac{8}{15} \quad \frac{11}{15}$
2.	$n - \frac{3}{4} = \frac{1}{12} \quad \frac{5}{6}$	$n - \frac{1}{6} = \frac{2}{3} \quad \frac{5}{6}$

Tell why If $n - \frac{1}{4} = \frac{1}{2}$, then $n = \frac{1}{2} + \frac{1}{4}$. Why? How can you use this idea to find a replacement for n ? Addition is the inverse operation of subtraction; find the sum of $\frac{1}{2}$ and $\frac{1}{4}$. **207**

7. $\frac{5}{6} = \frac{5}{6} \times 1 = \frac{5}{6} \times \frac{4}{4} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$

Least Common Denominator in Subtraction



You can find the simplest numeral for $\frac{5}{8} - \frac{5}{12}$ by using a number line as shown above.

An easier way to find the simplest numeral for $\frac{5}{8} - \frac{5}{12}$ is to rename both $\frac{5}{8}$ and $\frac{5}{12}$ so that they have the same denominator.

Consider only the denominators of $\frac{5}{8}$ and $\frac{5}{12}$.	8 and 12
What is the product of 8 and 12?	$8 \times 12 = 96$
What is the greatest common factor of 8 and 12?	4
What is the least common multiple of 8 and 12?	$96 \div 4 = 24$
Then what is the least common denominator of $\frac{5}{8}$ and $\frac{5}{12}$?	24

Now you know that $\frac{5}{8}$ and $\frac{5}{12}$ should both be renamed with a denominator of 24. You can rename $\frac{5}{8}$ and $\frac{5}{12}$ as follows.

$$\frac{5}{8} \times 1 = \frac{5}{8} \times \frac{\square}{\square} = \frac{15}{24}$$

What numeral should replace the \square so that $8 \times \square = 24$ becomes true?³
 Since $\frac{3}{3}$ stands for 1, both \square 's must be replaced by the same numeral.
 How is 15 obtained in $\frac{15}{24}$? 5×3

$$\frac{5}{12} \times 1 = \frac{5}{12} \times \frac{\square}{\square} = \frac{10}{24}$$

What numeral should replace the \square so that $12 \times \square = 24$ becomes true?²
 Since $\frac{2}{2}$ stands for 1, both \square 's must be replaced by the same numeral.
 How is 10 obtained in $\frac{10}{24}$? 5×2

You can now express the example in two different ways.

$\frac{5}{8} - \frac{5}{12} = \frac{15}{24} - \frac{10}{24}$	How is $\frac{5}{8} - \frac{5}{12}$ renamed?	$\frac{15}{24}$
		$\frac{10}{24}$
		$\frac{5}{24}$
$= \frac{15-10}{24}$	What is done to 15 and 10 to obtain 5?	$\frac{15-10}{24} = \frac{5}{24}$
$= \frac{5}{24}$		

Oral Answer the following questions about solving the open sentence $\frac{5}{6} - \frac{8}{15} = n$.

1. What is the product of 6 and 15? **90**

2. What is the greatest common factor of 6 and 15? **3**

3. How can you use the product of 6 and 15 and the greatest common factor of 6 and 15 to find their least common multiple? **Divide 90 by 3.**

4. What is the least common multiple of 6 and 15? **30**

5. What is the least common denominator of $\frac{5}{6}$ and $\frac{8}{15}$? **30**

6. How does $\frac{5}{6} = \frac{5}{6} \times 1$ help you rename $\frac{5}{6}$ as $\frac{25}{30}$? **$\frac{5}{6} \times \frac{5}{5} = \frac{25}{30}$**

7. How does $\frac{8}{15} = \frac{8}{15} \times 1$ help you rename $\frac{8}{15}$ as $\frac{16}{30}$? **$\frac{8}{15} \times \frac{2}{2} = \frac{16}{30}$**

8. Do $\frac{5}{6} - \frac{8}{15}$ and $\frac{25}{30} - \frac{16}{30}$ both name the same number? **Yes**

9. What single fraction with a denominator of 30 is named by $\frac{25}{30} - \frac{16}{30}$? **$\frac{9}{30}$**

10. What is $\frac{9}{30}$ in simplest form? **$\frac{3}{10}$**

Explain how you would solve $\frac{3}{4} - \frac{5}{14} = n$ as shown below.

$$\begin{array}{r} 11. \quad \frac{3}{4} \\ - \frac{5}{14} \end{array} \longrightarrow \begin{array}{r} \frac{21}{28} \\ - \frac{10}{28} \\ \hline \frac{11}{28} \end{array}$$

Written Copy. Find each difference in simplest form.

	a	b	c
1.	$\frac{2}{3} - \frac{3}{6} = \frac{1}{6}$	$\frac{5}{16} - \frac{1}{4} = \frac{1}{16}$	$\frac{7}{8} - \frac{1}{2} = \frac{3}{8}$
2.	$\frac{7}{8} - \frac{3}{4} = \frac{1}{8}$	$\frac{2}{3} - \frac{4}{9} = \frac{2}{9}$	$\frac{2}{5} - \frac{1}{10} = \frac{3}{10}$
3.	$\frac{12}{16} - \frac{3}{8} = \frac{3}{8}$	$\frac{5}{8} - \frac{1}{2} = \frac{1}{8}$	$\frac{10}{15} - \frac{1}{3} = \frac{1}{3}$
4.	$\frac{8}{9} - \frac{7}{18} = \frac{1}{2}$	$\frac{11}{12} - \frac{3}{4} = \frac{1}{6}$	$\frac{5}{6} - \frac{1}{2} = \frac{1}{3}$
5.	$\frac{5}{7} - \frac{7}{21} = \frac{8}{21}$	$\frac{3}{6} - \frac{2}{24} = \frac{5}{12}$	$\frac{8}{9} - \frac{5}{6} = \frac{1}{18}$

Copy. Find each difference in simplest form.

	a	b	c	d
6.	$\frac{2}{3} - \frac{3}{9} = \frac{1}{3}$	$\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$	$\frac{5}{8} - \frac{1}{6} = \frac{11}{24}$	$\frac{3}{4} - \frac{3}{8} = \frac{3}{8}$
7.	$\frac{4}{5} - \frac{3}{10} = \frac{1}{2}$	$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$	$\frac{1}{5} - \frac{1}{6} = \frac{1}{30}$	$\frac{3}{4} - \frac{1}{3} = \frac{5}{12}$

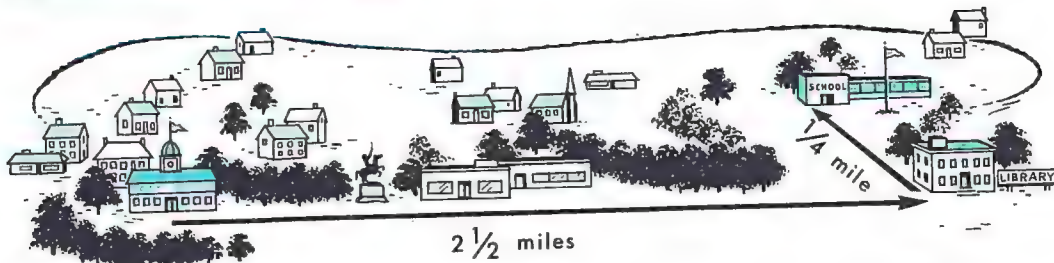
Can you do this? Find the unnamed addend in each of the following in simplest form.

	a	b
1.	$\frac{2}{3} + a = \frac{11}{12} \quad \frac{1}{4}$	$b + \frac{1}{6} = \frac{5}{8} \quad \frac{11}{24}$
2.	$\frac{1}{2} + c = \frac{6}{8} \quad \frac{1}{4}$	$d + \frac{1}{5} = \frac{3}{10} \quad \frac{1}{10}$
3.	$\frac{1}{3} + e = \frac{20}{24} \quad \frac{1}{2}$	$f + \frac{1}{3} = \frac{11}{12} \quad \frac{7}{12}$

Tell how How would you solve the following open sentence? **See page T209.**

$$\frac{5}{8} - \frac{1}{4} = \frac{5}{8}$$

Adding Fractional Numbers



Jerry rode his bicycle from the post office to the library. Then he rode from the library to the school. How far did Jerry ride his bicycle? $2\frac{1}{2} + \frac{1}{4} = \square$

What open sentence would you use to solve this problem?
The sum of $2\frac{1}{2}$ and $\frac{1}{4}$ can be found as follows.

$$\begin{aligned} 2\frac{1}{2} + \frac{1}{4} &= \left(2 + \frac{1}{2}\right) + \frac{1}{4} \\ &= 2 + \left(\frac{1}{2} + \frac{1}{4}\right) \\ &= 2 + \left(\frac{2}{4} + \frac{1}{4}\right) \\ &= 2 + \left(\frac{2+1}{4}\right) \\ &= 2 + \frac{3}{4} \\ &= 2\frac{3}{4} \end{aligned}$$

How is $2\frac{1}{2}$ renamed? $2 + \frac{1}{2}$

{ What property of addition is used to change $(2 + \frac{1}{2}) + \frac{1}{4}$ to $2 + (\frac{1}{2} + \frac{1}{4})$? **associative**

{ How is $\frac{1}{2}$ renamed in changing $2 + (\frac{1}{2} + \frac{1}{4})$ to $2 + (\frac{2}{4} + \frac{1}{4})$? $\frac{2}{4}$

{ Do $\frac{2}{4} + \frac{1}{4}$ and $\frac{2+1}{4}$ both name the same number? How do you know? **Yes; definition of addition of fractional numbers**

What is the simplest numeral for $2 + 1$? **3**

Do $2 + \frac{3}{4}$ and $2\frac{3}{4}$ name the same number? **Yes**

A more convenient way to arrange the numerals for addition is shown below.

$$\begin{array}{r} 2\frac{1}{2} \\ + \frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 2 + \frac{1}{2} \\ + \frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 2 + \frac{2}{4} \\ + \frac{1}{4} \\ \hline 2 + \frac{3}{4} \end{array}$$

$$\begin{array}{r} 2\frac{2}{4} \\ + \frac{1}{4} \\ \hline 2\frac{3}{4} \end{array}$$

How far did Jerry ride his bicycle? $2\frac{3}{4}$ miles

Oral Study the example shown below. Then answer the following questions about solving the open sentence

$$5\frac{1}{4} + \frac{1}{3} = a.$$

$$\begin{aligned} 5\frac{1}{4} + \frac{1}{3} &= (5 + \frac{1}{4}) + \frac{1}{3} \\ &= 5 + (\frac{1}{4} + \frac{1}{3}) \\ &= 5 + (\frac{3}{12} + \frac{4}{12}) \\ &= 5 + (\frac{7}{12}) \\ &= 5\frac{7}{12} \end{aligned}$$

1. Why is $5\frac{1}{4}$ renamed as $5 + \frac{1}{4}$?

See below.

2. What property of addition is used to change $(5 + \frac{1}{4}) + \frac{1}{3}$ to $5 + (\frac{1}{4} + \frac{1}{3})$? **associative**

3. How is $\frac{1}{4}$ renamed in $5 + (\frac{3}{12} + \frac{4}{12})$? How do you change $\frac{1}{4}$ to $\frac{3}{12}$?

See below.

4. How is $\frac{1}{3}$ renamed in $5 + (\frac{3}{12} + \frac{4}{12})$? How do you change $\frac{1}{3}$ to $\frac{4}{12}$?

See below.

5. How do you change $5 + (\frac{3}{12} + \frac{4}{12})$ to $5\frac{7}{12}$? $\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$ and $5 + \frac{7}{12} = 5\frac{7}{12}$

6. What is the simplest numeral for $5\frac{1}{4} + \frac{1}{3}$? $5\frac{7}{12}$

Tell how you would solve each open sentence below. See page T211.

a

b

7. $3\frac{1}{3} + \frac{2}{6} = a$

$5\frac{1}{8} + \frac{1}{2} = b$

8. $2\frac{1}{4} + \frac{3}{6} = c$

$4\frac{1}{3} + \frac{1}{5} = d$

9. $9\frac{1}{3} + \frac{4}{9} = e$

$7\frac{3}{8} + \frac{1}{6} = f$

10. $1\frac{3}{4} + \frac{1}{2} = g$

$2\frac{2}{3} + \frac{1}{2} = h$

Written Copy. Find each sum in simplest form.

a

b

1. $2\frac{1}{2} + \frac{5}{12} = a$ $2\frac{11}{12}$

$9\frac{2}{5} + \frac{3}{10} = b$ $9\frac{7}{10}$

2. $1\frac{3}{4} + \frac{1}{2} = c$ $2\frac{1}{4}$

$8\frac{1}{4} + \frac{1}{8} = d$ $8\frac{3}{8}$

3. $3\frac{1}{3} + \frac{3}{6} = e$ $3\frac{5}{6}$

$5\frac{2}{3} + \frac{1}{12} = f$ $5\frac{3}{4}$

4. $8\frac{5}{8} + \frac{1}{2} = g$ $9\frac{1}{8}$

$2\frac{3}{10} + \frac{1}{5} = h$ $2\frac{1}{2}$

5. $1\frac{6}{12} + \frac{1}{3} = i$ $1\frac{5}{6}$

$2\frac{1}{4} + \frac{2}{3} = j$ $2\frac{11}{12}$

Copy. Use the method shown below to find each sum.

$$\begin{array}{r} 6\frac{5}{12} \\ + \frac{1}{4} \\ \hline \end{array} \longrightarrow \begin{array}{r} 6\frac{5}{12} \\ + \frac{3}{12} \\ \hline 6\frac{8}{12} = 6\frac{2}{3} \end{array}$$

a

b

c

d

6. $\begin{array}{r} 8\frac{3}{8} \\ + \frac{1}{4} \\ \hline 8\frac{7}{8} \end{array}$

$\begin{array}{r} 7\frac{1}{2} \\ + \frac{1}{8} \\ \hline 7\frac{5}{8} \end{array}$

$\begin{array}{r} \frac{1}{9} \\ + 3\frac{2}{3} \\ \hline 3\frac{7}{9} \end{array}$

$\begin{array}{r} \frac{1}{2} \\ + 4\frac{3}{8} \\ \hline 4\frac{7}{8} \end{array}$

7. $\begin{array}{r} 9\frac{3}{4} \\ + \frac{1}{8} \\ \hline 9\frac{7}{8} \end{array}$

$\begin{array}{r} 8\frac{1}{3} \\ + \frac{2}{6} \\ \hline 8\frac{2}{3} \end{array}$

$\begin{array}{r} 2\frac{1}{2} \\ + \frac{5}{10} \\ \hline 3 \end{array}$

$\begin{array}{r} 3\frac{2}{5} \\ + \frac{3}{10} \\ \hline 3\frac{7}{10} \end{array}$

8. $\begin{array}{r} \frac{1}{4} \\ + 2\frac{1}{5} \\ \hline 2\frac{9}{20} \end{array}$

$\begin{array}{r} \frac{1}{6} \\ + 3\frac{1}{3} \\ \hline 3\frac{1}{2} \end{array}$

$\begin{array}{r} \frac{4}{8} \\ + 5\frac{2}{4} \\ \hline 6 \end{array}$

$\begin{array}{r} \frac{6}{9} \\ + 3\frac{2}{6} \\ \hline 4 \end{array}$

9. $\begin{array}{r} 3\frac{1}{5} \\ + \frac{3}{10} \\ \hline 3\frac{1}{2} \end{array}$

$\begin{array}{r} 4\frac{1}{8} \\ + \frac{3}{4} \\ \hline 4\frac{7}{8} \end{array}$

$\begin{array}{r} \frac{1}{7} \\ + 1\frac{1}{14} \\ \hline 1\frac{3}{14} \end{array}$

$\begin{array}{r} \frac{6}{8} \\ + 9\frac{1}{16} \\ \hline 9\frac{13}{16} \end{array}$

10. $\begin{array}{r} 4\frac{3}{8} \\ + \frac{1}{4} \\ \hline 4\frac{7}{8} \end{array}$

$\begin{array}{r} 2\frac{3}{9} \\ + \frac{1}{3} \\ \hline 2\frac{2}{3} \end{array}$

$\begin{array}{r} \frac{1}{6} \\ + 4\frac{6}{6} \\ \hline 5 \end{array}$

$\begin{array}{r} \frac{1}{4} \\ + 5\frac{2}{3} \\ \hline 5\frac{11}{12} \end{array}$

Tell how How is it possible to find the simplest numeral for $2\frac{1}{2} + \frac{1}{4}$ by adding $\frac{10}{4}$ and $\frac{1}{4}$? $2\frac{1}{2}$ and $\frac{10}{4}$ name the same number.

Oral 1. so the numbers can be associated in different ways

3. $\frac{3}{12}; \frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$

4. $\frac{1}{10}; \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

MORE PRACTICE

PAGE 325

Subtracting a Fractional Number from the Number One



Could you use the open sentence $1 - \frac{3}{4} = a$ to find how far the cabin cruiser is from the sailboat? **Yes**

Do you know how to subtract one fractional number from another? Does the row of equivalent fractions below show that 1 can be expressed as a fraction in many ways? **Yes; Yes**

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \frac{7}{7} = \frac{8}{8} = \frac{9}{9}$$

To find the simplest numeral for $1 - \frac{3}{4}$, replace 1 by one of the fractions from the equivalence row as shown below.

$$\begin{aligned} 1 - \frac{3}{4} &= \frac{4}{4} - \frac{3}{4} \\ &= \frac{4-3}{4} \\ &= \frac{1}{4} \end{aligned}$$

Why did you use $\frac{4}{4}$ instead of $\frac{2}{2}$, $\frac{3}{3}$, or one of the other fractions to replace 1? Tell how $\frac{4}{4} - \frac{3}{4}$ is changed to $\frac{1}{4}$.

How far is the cabin cruiser from the sailboat? $\frac{1}{4}$ mile

How would you find the difference in each example below? **Accept any answer that describes what is shown.**

$$\begin{aligned} 1 - \frac{3}{7} &= \frac{7}{7} - \frac{3}{7} \\ &= \frac{7-3}{7} \\ &= \frac{4}{7} \end{aligned}$$

$$\begin{aligned} 1 - \frac{8}{9} &= \frac{9}{9} - \frac{8}{9} \\ &= \frac{9-8}{9} \\ &= \frac{1}{9} \end{aligned}$$

212

*1 $\frac{4}{4}$ is the most convenient name to use in this case since the denominator (4) is the same as the denominator of the fractional number being subtracted.

Oral Study the example shown below. Then answer the following questions about solving the open sentence $1 - \frac{2}{3} = n$.

$$\begin{aligned} 1 - \frac{2}{3} &= \frac{3}{3} - \frac{2}{3} \\ &= \frac{3-2}{3} \\ &= \frac{1}{3} \end{aligned}$$

1. In how many ways can you name 1 as a fraction? Tell some of them.

many ways; $\frac{1}{1}$, $\frac{2}{2}$, $\frac{3}{3}$, and so on

2. How is 1 renamed in $\frac{3}{3} - \frac{2}{3}$? $\frac{3}{3}$

3. Why is 1 renamed as $\frac{3}{3}$ instead of $\frac{2}{2}$, $\frac{4}{4}$, or $\frac{5}{5}$? See below.

4. How is $\frac{3}{3} - \frac{2}{3}$ changed to $\frac{1}{3}$?

See below.

5. What is the simplest numeral for $1 - \frac{2}{3}$? $\frac{1}{3}$

6. Could you find the simplest numeral for $1 - \frac{2}{3}$ by replacing 1 by $\frac{2}{2}$? Yes ($\frac{2}{2} - \frac{2}{3} = \frac{6}{6} - \frac{4}{6} = \frac{2}{6} = \frac{1}{3}$)

7. How is subtracting a fractional number from 1 like subtracting one fractional number from another? It is the same.

Tell how you would solve each open sentence below. See page T213.

a

b

8. $1 - \frac{1}{5} = a$

$1 - \frac{1}{4} = b$

9. $1 - \frac{3}{8} = c$

$1 - \frac{2}{7} = d$

10. $1 - \frac{4}{9} = e$

$1 - \frac{3}{10} = f$

11. $1 - \frac{5}{14} = g$

$1 - \frac{7}{15} = h$

$\frac{15}{15}$ or 1 $a - \frac{1}{15} = \frac{14}{15}$

$b - \frac{3}{5} = \frac{2}{5}$ or 1

Oral 3. The denominator of $\frac{3}{3}$ is the same as the denominator of the fractional number being subtracted. **213**

4. $\frac{3}{3} - \frac{2}{3} = \frac{3-2}{3} = \frac{1}{3}$

Written Copy. Find each difference as discussed in *Oral*.

a

b

1. $1 - \frac{1}{2} = a$ $\frac{1}{2}$

$1 - \frac{1}{3} = b$ $\frac{2}{3}$

2. $1 - \frac{3}{5} = c$ $\frac{2}{5}$

$1 - \frac{5}{8} = d$ $\frac{3}{8}$

3. $1 - \frac{2}{8} = e$ $\frac{3}{4}$

$1 - \frac{6}{10} = f$ $\frac{2}{5}$

4. $1 - \frac{5}{9} = g$ $\frac{4}{9}$

$1 - \frac{1}{9} = h$ $\frac{8}{9}$

5. $1 - \frac{6}{8} = i$ $\frac{1}{4}$

$1 - \frac{2}{5} = j$ $\frac{3}{5}$

6. $1 - \frac{4}{8} = k$ $\frac{1}{2}$

$1 - \frac{5}{7} = r$ $\frac{2}{7}$

7. $1 - \frac{1}{10} = m$ $\frac{9}{10}$

$1 - \frac{7}{10} = n$ $\frac{3}{10}$

8. $1 - \frac{6}{9} = p$ $\frac{1}{3}$

$1 - \frac{7}{9} = q$ $\frac{2}{9}$

Copy. Find each difference as shown below.

$$\begin{array}{r} 1 \\ -\frac{1}{6} \\ \hline \end{array} \longrightarrow \begin{array}{r} \frac{6}{6} \\ -\frac{1}{6} \\ \hline \frac{5}{6} \end{array}$$

a

b

c

d

9. $\frac{1}{-\frac{3}{6}} \frac{1}{2}$

$\frac{1}{-\frac{6}{7}} \frac{1}{7}$

$\frac{1}{-\frac{7}{8}} \frac{1}{8}$

$\frac{1}{-\frac{8}{8}} \frac{7}{8}$

10. $\frac{1}{-\frac{5}{12}} \frac{7}{12}$

$\frac{1}{-\frac{8}{10}} \frac{1}{5}$

$\frac{1}{-\frac{9}{9}} \frac{1}{9}$

$\frac{1}{-\frac{12}{12}} \frac{5}{12}$

Can you do this? Find the unnamed number in each of the following.

a

b

MORE PRACTICE

PAGE

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Review and Practice

Part 1 Copy. Find each sum in simplest form.

$$a$$

$$1. \quad \frac{3}{8} + \frac{4}{8} = a \quad \frac{7}{8}$$

$$2. \quad \frac{7}{9} + \frac{2}{9} = b \quad 1$$

$$3. \quad \frac{6}{12} + \frac{7}{12} = c \quad 1\frac{1}{12}$$

$$4. \quad \frac{4}{15} + \frac{8}{15} = d \quad \frac{4}{5}$$

$$b$$

$$\frac{2}{9} + \frac{4}{9} = f \quad \frac{2}{3}$$

$$\frac{9}{20} + \frac{8}{20} = g \quad \frac{17}{20}$$

$$\frac{6}{21} + \frac{6}{21} = h \quad \frac{4}{7}$$

$$\frac{7}{24} + \frac{7}{24} = i \quad \frac{7}{12}$$

Part 2 Copy. Find each difference in simplest form.

$$a$$

$$1. \quad \frac{4}{8} - \frac{3}{8} = a \quad \frac{1}{8}$$

$$2. \quad \frac{9}{10} - \frac{2}{10} = b \quad \frac{7}{10}$$

$$3. \quad \frac{8}{14} - \frac{4}{14} = c \quad \frac{2}{7}$$

$$4. \quad \frac{13}{21} - \frac{4}{21} = d \quad \frac{3}{7}$$

$$5. \quad \frac{18}{24} - \frac{9}{24} = e \quad \frac{3}{8}$$

$$b$$

$$\frac{7}{9} - \frac{2}{9} = f \quad \frac{5}{9}$$

$$\frac{7}{12} - \frac{6}{12} = g \quad \frac{1}{12}$$

$$\frac{9}{15} - \frac{6}{15} = h \quad \frac{1}{5}$$

$$\frac{16}{20} - \frac{9}{20} = i \quad \frac{7}{20}$$

$$\frac{14}{25} - \frac{8}{25} = j \quad \frac{6}{25}$$

Part 3 Copy. Find each sum in simplest form.

$$a$$

$$1. \quad \frac{2}{3} + \frac{1}{6} = a \quad \frac{5}{6}$$

$$2. \quad \frac{3}{8} + \frac{3}{4} = b \quad 1\frac{1}{8}$$

$$3. \quad \frac{3}{8} + \frac{2}{3} = c \quad 1\frac{1}{24}$$

$$4. \quad \frac{3}{5} + \frac{3}{10} = d \quad \frac{9}{10}$$

$$b$$

$$\frac{5}{12} + \frac{1}{2} = e \quad \frac{11}{12}$$

$$\frac{3}{14} + \frac{3}{7} = f \quad \frac{9}{14}$$

$$\frac{6}{15} + \frac{1}{3} = g \quad \frac{11}{15}$$

$$\frac{8}{21} + \frac{3}{7} = h \quad \frac{17}{21}$$

Part 4 Copy. Find each difference in simplest form.

$$a$$

$$1. \quad \frac{2}{3} - \frac{1}{6} = a \quad \frac{1}{2}$$

$$2. \quad \frac{3}{4} - \frac{1}{5} = b \quad \frac{11}{20}$$

$$3. \quad \frac{3}{5} - \frac{6}{15} = c \quad \frac{1}{5}$$

$$4. \quad \frac{5}{6} - \frac{1}{4} = d \quad \frac{7}{12}$$

$$b$$

$$\frac{2}{3} - \frac{1}{4} = e \quad \frac{5}{12}$$

$$\frac{7}{8} - \frac{3}{4} = f \quad \frac{1}{8}$$

$$\frac{7}{9} - \frac{1}{3} = g \quad \frac{4}{9}$$

$$\frac{7}{10} - \frac{2}{5} = h \quad \frac{3}{10}$$

Part 5 Copy. Find each sum in simplest form.

$$a$$

$$1. \quad 3\frac{1}{2} + \frac{1}{3} = a \quad 3\frac{5}{6}$$

$$2. \quad 8\frac{3}{4} + \frac{1}{2} = b \quad 9\frac{1}{4}$$

$$3. \quad 6\frac{3}{8} + \frac{1}{6} = c \quad 6\frac{13}{24}$$

$$4. \quad 2\frac{3}{5} + \frac{1}{4} = d \quad 2\frac{17}{20}$$

$$5. \quad 1\frac{5}{6} + \frac{1}{2} = e \quad 2\frac{1}{3}$$

$$b$$

$$\frac{1}{3} + 9\frac{7}{9} = f \quad 10\frac{1}{9}$$

$$\frac{2}{3} + 7\frac{1}{6} = g \quad 7\frac{5}{6}$$

$$\frac{1}{4} + 5\frac{1}{2} = h \quad 5\frac{3}{4}$$

$$\frac{5}{8} + 11\frac{5}{8} = i \quad 12\frac{11}{24}$$

$$\frac{4}{7} + 12\frac{2}{3} = j \quad 13\frac{5}{21}$$

Part 6 Copy. Find each difference in simplest form.

$$a$$

$$1. \quad 1 - \frac{4}{6} = a \quad \frac{1}{3}$$

$$2. \quad 1 - \frac{7}{9} = b \quad \frac{2}{9}$$

$$3. \quad 1 - \frac{7}{10} = c \quad \frac{3}{10}$$

$$4. \quad 1 - \frac{8}{11} = d \quad \frac{3}{11}$$

$$b$$

$$1 - \frac{6}{7} = e \quad \frac{1}{7}$$

$$1 - \frac{8}{9} = f \quad \frac{1}{9}$$

$$1 - \frac{9}{10} = g \quad \frac{1}{10}$$

$$1 - \frac{5}{12} = h \quad \frac{7}{12}$$

Checkup Time

The numerals in () tell the pages where you can turn for help.

Important Ideas

1. To subtract $\frac{3}{8}$ from $\frac{7}{8}$, subtract the numerators and record this difference over the denominator. (202)
2. The least common denominator of two fractional numbers is the least common multiple of their denominators. (204)
3. To add two fractional numbers that have different denominators, rename the fractional numbers so that they have the same denominator and then add. (206)
4. To subtract two fractional numbers that have different denominators, rename the fractional numbers so that they have the same denominator and then subtract. (208)
5. To add two numbers like $2\frac{1}{2}$ and $\frac{1}{4}$, rename $2\frac{1}{2}$ as a sum, add the fractional numbers, then express the sum as a mixed numeral. (210)
6. To subtract a fractional number from one, rename one as a fraction and subtract. (212)

Words to Know

Common denominator, least common denominator (204)

Questions to Discuss

See page T215.

1. How would you solve the open sentence $\frac{2}{8} + \frac{5}{8} = a$? (201)
2. How would you solve the open sentence $\frac{7}{9} - \frac{3}{9} = b$? (202)
3. How would you find the least common denominator of $\frac{2}{3}$ and $\frac{3}{4}$? (204)
4. How would you solve the open sentence $\frac{3}{4} + \frac{2}{3} = e$? (206)
5. How would you solve the open sentence $\frac{7}{8} - \frac{3}{4} = d$? (208)
6. How would you solve the open sentence $2\frac{1}{2} + \frac{1}{3} = e$? (210)
7. How would you solve the open sentence $1 - \frac{1}{2} = f$? (212)

Written Practice

Copy. Express each sum or difference as a single fraction or mixed numeral in simplest form.

a

b

1. $\frac{3}{15} + \frac{9}{15} = a$ (201) $\frac{4}{5}$ $\frac{14}{16} - \frac{8}{16} = b$ (202) $\frac{3}{8}$
2. $\frac{3}{8} + \frac{3}{4} = c$ (206) $1\frac{1}{8}$ $\frac{7}{8} - \frac{3}{4} = d$ (208) $\frac{1}{8}$
3. $4\frac{1}{2} + \frac{1}{3} = e$ (210) $4\frac{5}{6}$ $1 - \frac{5}{9} = f$ (212) $\frac{4}{9}$
4. $3\frac{2}{3} + \frac{2}{5} = g$ (210) $4\frac{1}{15}$ $1 - \frac{3}{8} = h$ (212) $\frac{5}{8}$

Self-Evaluation

Part 1 Copy. Find each sum or difference in simplest form.

a	b
1. $\frac{4}{9} + \frac{4}{9} = a$ $\frac{8}{9}$	$\frac{7}{9} - \frac{1}{9} = e$ $\frac{2}{3}$
2. $\frac{2}{12} + \frac{10}{12} = b$ 1	$\frac{3}{8} - \frac{2}{8} = f$ $\frac{1}{8}$
3. $\frac{4}{10} + \frac{2}{10} = c$ $\frac{3}{5}$	$\frac{7}{12} - \frac{2}{12} = g$ $\frac{5}{12}$
4. $\frac{1}{9} + \frac{1}{9} = d$ $\frac{2}{9}$	$\frac{13}{14} - \frac{9}{14} = h$ $\frac{2}{7}$

Part 2 Copy. Find each sum or difference in simplest form.

a	b
1. $\frac{3}{4} + \frac{1}{8} = a$ $\frac{7}{8}$	$\frac{3}{8} - \frac{1}{8} = b$ $\frac{1}{4}$
2. $\frac{2}{5} + \frac{3}{10} = b$ $\frac{7}{10}$	$\frac{2}{5} - \frac{3}{10} = g$ $\frac{1}{10}$
3. $\frac{2}{3} + \frac{1}{9} = c$ $\frac{7}{9}$	$\frac{2}{3} - \frac{1}{9} = h$ $\frac{5}{9}$
4. $\frac{1}{2} + \frac{1}{10} = d$ $\frac{3}{5}$	$\frac{1}{2} - \frac{1}{10} = i$ $\frac{2}{5}$
5. $\frac{6}{8} + \frac{1}{4} = e$ 1	$\frac{6}{8} - \frac{1}{4} = j$ $\frac{1}{2}$

Part 3 Copy. Find each sum or difference in simplest form.

a	b
1. $2\frac{3}{5} + \frac{1}{10} = a$ $2\frac{7}{10}$	$1 - \frac{1}{6} = e$ $\frac{5}{6}$
2. $8\frac{1}{2} + \frac{1}{3} = b$ $8\frac{5}{6}$	$1 - \frac{2}{5} = f$ $\frac{3}{5}$
3. $14\frac{2}{3} + \frac{2}{6} = c$ 15	$1 - \frac{3}{8} = g$ $\frac{5}{8}$
4. $21\frac{1}{8} + \frac{3}{4} = d$ $21\frac{7}{8}$	$1 - \frac{4}{9} = h$ $\frac{5}{9}$

Part 4 Copy. Find each sum or difference in simplest form.

a	b	c	d
1. $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$	$\frac{1}{4} + \frac{1}{5} = \frac{9}{20}$	$\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$	$\frac{3}{8} - \frac{1}{4} = \frac{1}{8}$
2. $\frac{5}{6} + \frac{1}{2} = 1\frac{1}{3}$	$\frac{4}{9} + \frac{2}{3} = 1\frac{1}{9}$	$\frac{7}{8} - \frac{2}{3} = \frac{5}{24}$	$\frac{9}{10} - \frac{1}{2} = \frac{2}{5}$
3. $\frac{9}{12} + \frac{1}{4} = 1$	$\frac{8}{15} + \frac{1}{6} = 1\frac{1}{5}$	$\frac{9}{12} - \frac{2}{3} = \frac{1}{12}$	$\frac{12}{15} - \frac{3}{5} = \frac{1}{5}$

Part 5 Copy. Find each sum or difference in simplest form.

a	b	c	d
1. $2\frac{1}{4} + \frac{2}{3} = 2\frac{11}{12}$	$3\frac{1}{2} + \frac{3}{8} = 3\frac{7}{8}$	$\frac{1}{9} + 2\frac{3}{18} = 2\frac{5}{18}$	$1 - \frac{7}{15} = \frac{8}{15}$
2. $5\frac{1}{6} + \frac{5}{12} = 5\frac{7}{12}$	$6\frac{1}{2} + \frac{5}{8} = 7\frac{1}{8}$	$\frac{3}{14} + 4\frac{1}{7} = 4\frac{5}{14}$	$1 - \frac{8}{16} = \frac{1}{2}$
3. $4\frac{4}{15} + \frac{1}{5} = 4\frac{7}{15}$	$\frac{3}{16} + 4\frac{7}{8} = 5\frac{1}{16}$	$1 - \frac{4}{6} = \frac{1}{3}$	$1 - \frac{5}{8} = \frac{3}{8}$

Part 6 Write an open sentence for each problem. Solve the open sentence. Answer the problem.

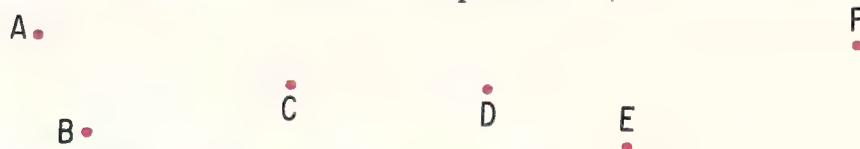
1. Janet walked $\frac{1}{2}$ of a mile to Sue's house. Then she walked $\frac{3}{10}$ of a mile to Jill's house. In all, how far did she walk? $\frac{1}{2} + \frac{3}{10} = \square$; $\frac{8}{10}$ or $\frac{4}{5}$ of a mile

2. Mary had $\frac{3}{4}$ yard of material. She bought $2\frac{1}{2}$ yards more. How much material does she now have? $\frac{3}{4} + 2\frac{1}{2} = \square$; $3\frac{1}{4}$ yards

Chapter 10 NON-METRIC GEOMETRY

Points and Line Segments

You can think of **points** as locations in space. A point has no size. You draw a dot to picture a point.



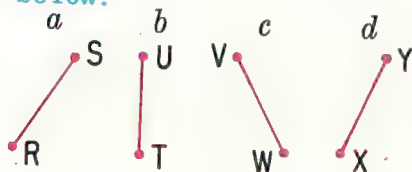
How many dots are labeled above? **6** How many points are pictured above? **6** You use capital letters to label points. Name each of the points pictured above. **point A, point B, point C, point D, point E, and point F**

Two points may be connected as shown below.



The figures formed are called **line segments**. The two points that are connected are called **endpoints** of the line segment. Name the *endpoints* of each line segment above.^{*1} The line segment having endpoints A and B is named *line segment AB* or *line segment BA*. What are the two names for each of the other line segments pictured above? **line segment CD (or DC); line segment EF (or FE)**

Oral Name the endpoints of each line segment below. Then give two names for each line segment. See below.



Written Draw 4 points on your paper as shown below.

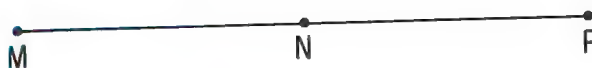


Connect point M to points N, Q, and P by line segments. How many line segments have you drawn? **3**

^{*1} A and B; C and D; E and F

Oral a: R and S, line segment RS (or SR); b: T and U, line segment TU (or UT); c: V and W, line segment VW (or WV); d: X and Y, line segment XY (or YX)

Line Segments and Lines



Think of starting at point M and ending at point P. What would you call the figure? Does line segment MP include points M and P and all the points between them? Can you think of a line segment as a set of points? Is point N one of these points? You can say that line segment MP passes *through* point N; or you can say that point N is *on* line segment MP.



Does the figure above stop at point R? At point S? What do the arrows at each end of the figure indicate? This figure is a picture of *line* RS or *line* SR. When we say *line*, we mean *straight line*. Is a line also a set of points? Does a line have endpoints?



The figure above is called *line* RS or *line* SR. The figure may also be called *line* RW or *line* WR. To name a line you can use any two points on the line. Tell another name you could use for the line above. *line* WS (or *line* SW)

Now think of starting at point R and ending at point S. What would you call this figure? Is line segment RS a part of line RS? Considering that a line segment and a line are sets of points, would you say that a line segment is a subset of a line? What set of points do line RS and line segment RS have in common?

In the future, when we say *draw a line* or *draw a line segment*, we mean to *draw a picture of a line* or to *draw a picture of a line segment*.

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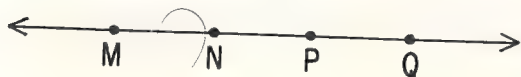
*1 line segment MP (or PM)

*2 Yes

*3 The line continues on and on in both directions.

*4 line segment RS

Oral Use the figure below to answer the following.



1. Can you call the figure above line MQ? Line QM? Line MP? Line PM? What other names can you use for this line? *See page T219.*

2. Think of starting at point M and stopping at point Q. Would you call that figure line MQ or line segment MQ? Why? How else could you name line segment MQ? *See page T219.*

3. Would you say that line segment MQ consists of points M and Q and all the points between them? What set of points are you referring to when you say line segment NQ? Line segment NP? *See page T219.*

4. Would you say that line segment NQ is a part of line MQ? *Yes*

5. Are there points on line segment MQ which are not labeled? Are there points on line MQ which are not labeled? *Yes; Yes*

6. Tell the difference between line MQ and line segment MQ. *See page T219.*

7. Which points are the endpoints of line segment MQ? *M and Q*

8. Does line MQ have endpoints? *No* How is this shown in the figure? *The arrows indicate that the line continues in both directions.*

9. Using points M, N, P, and Q, name as many line segments as possible. There are 6 altogether. *See page T219.*

Written Do the following.

1. Draw a point on your paper. Label the point with the letter X. Draw a line through the point. Draw another line through the point. How many lines can you draw through a point? *any number*

2. Draw 2 points on your paper. Label them A and B. Draw line segment AB. Can you draw more than 1 line segment connecting point A to point B? What is the least number of points necessary to draw a line segment? *No, two*

3. Draw 4 points on your paper as shown below.



Connect each point with the other 3 points by line segments. How many line segments are possible? *6*

4. Draw line segment PQ on your paper. Draw a point on the line segment and label it T. In all, how many line segments are there? *3* Using arrows, extend line segment PQ to the left and to the right. What set of points do line segment PQ and line PQ have in common? *points P and Q and all the points between P and Q*

Planes



Name some objects in the picture above that have a flat surface. Consider the top of the ping pong table. Imagine that you could make it larger and larger. In geometry, we think of the extended top of the ping pong table as a model of a **plane**.

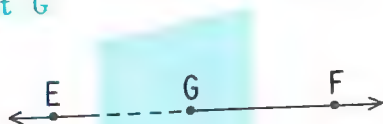
Each flat surface that you named above represents only *part* of a *plane* because each object has a boundary. Like a line has no endpoints, a plane has no boundaries.

Oral Answer the following.

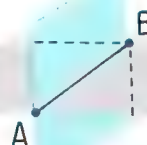
1. Name 5 objects in your classroom that are models of part of a plane. **Answers will vary.**

2. Does a plane have a boundary? **No**

3. How many points are there on both line EF and the plane below? **1**
Name that point or those points.
point G



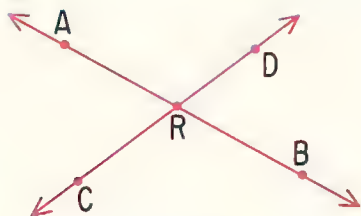
4. Where are all the points that are on both planes below?
on line AB (or BA)



Written A sheet of paper flat on your desk is a model of part of a plane. Do the following.

Draw line AB on your paper. Is every point of line AB also a point of the plane? **Yes**

Intersecting and Parallel Lines

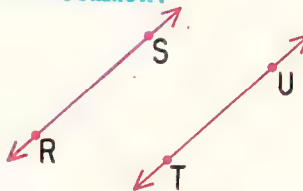


Yes

Do lines AB and CD cross each other? Which point is on both of these lines? Since point R is on both lines, we say it is a *common point* of both lines. We say that lines AB and CD **intersect** each other.

Two lines that have only one point in common are called **intersecting lines**. We also say that the two lines *intersect* each other.

Are lines JK and MN intersecting lines? Why or why not? **Yes; The lines have only one point in common.**



Do lines EF and GH have any common points? If they were extended do you think they would intersect? **No; No**

Two lines in the same plane that do not intersect are called **parallel lines**. We also say the two lines are *parallel* to each other.

Are lines RS and TU parallel lines? Why or why not? **Yes; The two lines are in the same plane and they do not intersect.**

Oral Answer the following.

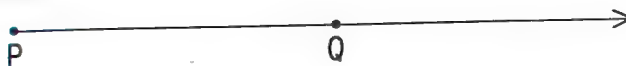
1. Name 5 objects in your classroom that are models of intersecting lines. Of parallel lines. **Answers will vary.**

2. How many points do two intersecting lines have in common? **1**

3. How many points do two parallel lines have in common? **0**

*1 point R

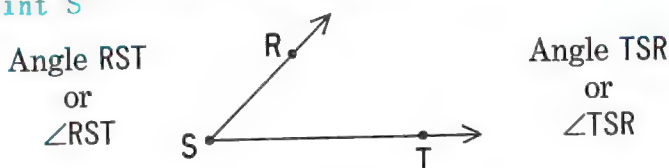
Rays and Angles



In the figure above, think of starting at point P and continuing through point Q without stopping. Such a figure is called a **ray**. How many endpoints does ray PQ have? What does the arrow mean? This ray is named *ray PQ*. Which point is named first when naming a ray? **the endpoint**

Now think of starting at point P and stopping at point Q. Would you call this figure line PQ, line segment PQ, or ray PQ? How does a ray differ from a line segment? How does a ray differ from a line?

Name the two rays in the figure below. Do the two rays start at the same point? Which point is that? **ray SR and ray ST; Yes; point S**

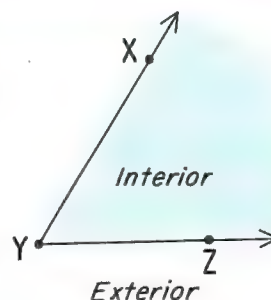


Such a figure is called an **angle**. Each of the rays is called a *side* of the angle. Name the sides of angle RST. Point S, the common endpoint of the sides, is called the **vertex** of angle RST. A symbol for the word *angle* is \angle .

A figure formed by two rays with a common endpoint is called an *angle*.

The angle shown at the right is named $\angle XYZ$ or $\angle ZYX$. In $\angle XYZ$, is the vertex named first, second, or third? Is this also true in $\angle ZYX$? **second; Yes**

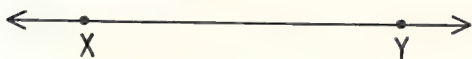
Angle XYZ separates the plane into three sets of points—those in the exterior, those on the angle, and those in the interior.



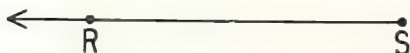
- 222** *1 The ray continues on and on in that direction.
 *2 line segment PQ
 *3 A ray has 1 endpoint and a line segment has 2 endpoints.
 *4 A ray has 1 endpoint and a line has 0 endpoints.
 *5 ray SR and ray ST

Oral Select the correct answer in each exercise below. Correct answers are underscored.

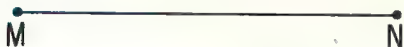
1. The figure below is a (ray, line, line segment).



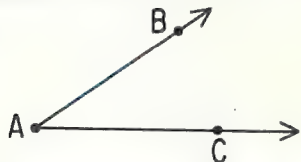
2. The figure below is a (ray, line, line segment).



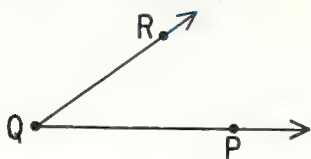
3. The figure below is a (ray, line, line segment).



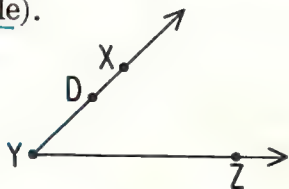
4. In the figure below, the vertex is (point A, point B, point C).



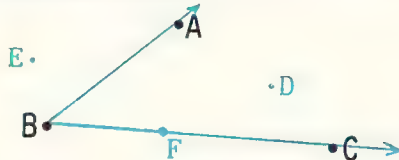
5. You may name the figure below as ($\angle QRP$, $\angle QPR$, $\angle PQR$).



6. In the figure below, point D is (in the interior, in the exterior, on the angle).



Written Draw 3 points on your paper as shown below.



1. Draw ray BA and ray BC.
See above.

2. Give two names for the angle that is formed. $\angle ABC$ and $\angle CBA$

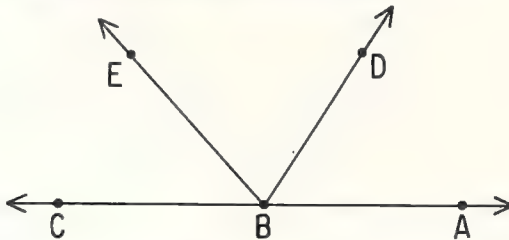
3. Which point is the vertex of the angle? What are its sides?
point B; ray BA and ray BC

4. Draw point D in the interior of $\angle ABC$. See above.

5. Draw point E in the exterior of $\angle ABC$. See above.

6. Draw point F on the angle.
See above.

Study the figure below. Then answer the following questions.



7. What two rays form $\angle EBA$?
ray BE and ray BA

8. What two rays form $\angle CBD$?
ray BC and ray BD

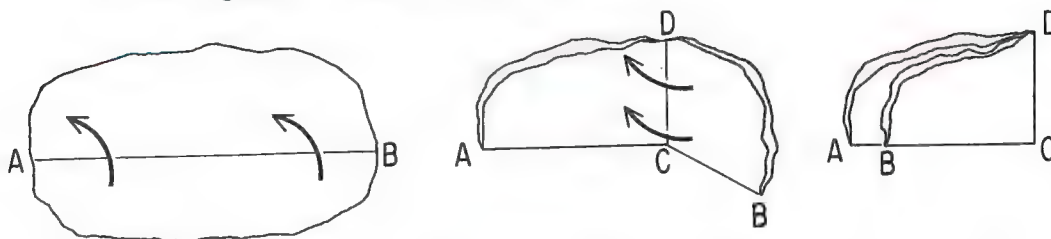
9. What are the two sides of $\angle EBD$?
ray BE and ray BD

10. Name as many angles as you can that have point B as a vertex.
See below.

Written 10. $\angle CBE$ (or $\angle EBC$); $\angle CBD$ (or $\angle DBC$); $\angle EBD$ (or $\angle DBE$); $\angle EBA$ (or $\angle ABE$); $\angle DBA$ (or $\angle ABD$) [Note: If pupils consider a straight line to be a straight angle, accept $\angle CBA$ (or $\angle ABC$) as a correct answer.]

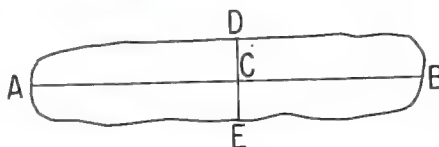
Right Angles

Fold a piece of paper as shown below.



Edges AC and CD of the folded piece of paper form a model of a **right angle**. What point is the vertex of that angle?^{*1} What are the sides of that angle?^{*2} Save that model of a right angle.

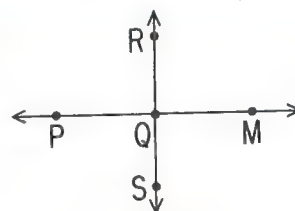
Now fold another piece of paper as shown above. Then unfold the paper as shown below.



Draw along the creases that were made by the folding of the paper. At what point do line segments AB and DE intersect?^{*1} Use your model of a right angle and check to see if $\angle BCD$ is a right angle. Do the same for $\angle DCA$, $\angle ACE$, and $\angle ECB$. What are your findings? **They are all right angles.**

When two line segments intersect so that four right angles are formed, we say that the line segments are **perpendicular** to each other.

What two lines intersect at point Q in the figure at the right?^{*3} Suppose you know that lines RS and PM are perpendicular to each other, then what can you say about angles MQR, RQP, PQS, and SQM? **They are all right angles.**

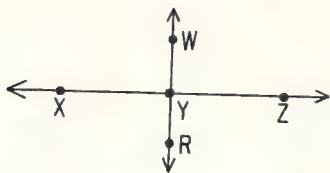


If two lines are perpendicular to each other, then four right angles are formed at their intersection.

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- *1 point C
- *2 ray CD and ray CB (or CA)
- *3 line RS and line PM

Oral Use the figure below to help you answer the following.



1. What two lines intersect at point Y? **line WR and line XZ**

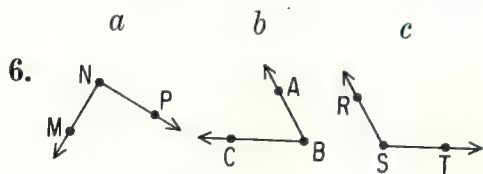
2. Use your paper model of a right angle and check to see whether $\angle ZYW$ is a right angle.

3. Do the same for $\angle WYX$. For $\angle XYR$. For $\angle RYZ$. What are your findings? **They are all right angles.**

4. Are lines WR and XZ perpendicular to each other? Why or why not? **See below.**

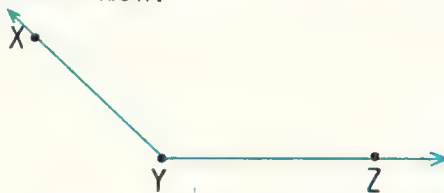
5. If you know that two lines are perpendicular to each other, what can you say about the four angles formed at the point of intersection? **They are all right angles.**

Give two names for each angle below. Then tell whether each angle is greater than, is less than, or is a right angle. Use your model of a right angle, if necessary. **See below.**



Written Do the following. Then answer each question.

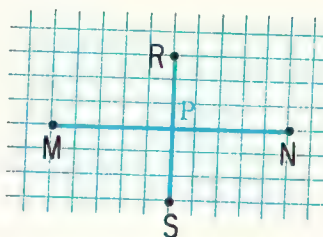
1. Draw 3 points on your paper as shown below.



2. Draw ray YX and ray YZ.
See above.

3. Give two names for the angle formed. **$\angle XYZ$ or $\angle ZYX$**

4. Is $\angle XYZ$ greater or less than a right angle? How can you tell? **greater; by comparing it with a right angle**
Draw 4 points on graph paper as shown below.



5. Draw lines RS and MN.
See above.

6. Label the point of intersection with the letter P. **See above.**

7. Check to see if angles NPR, RPM, MPS, and SPN are right angles. What are your findings? **They are right angles.**

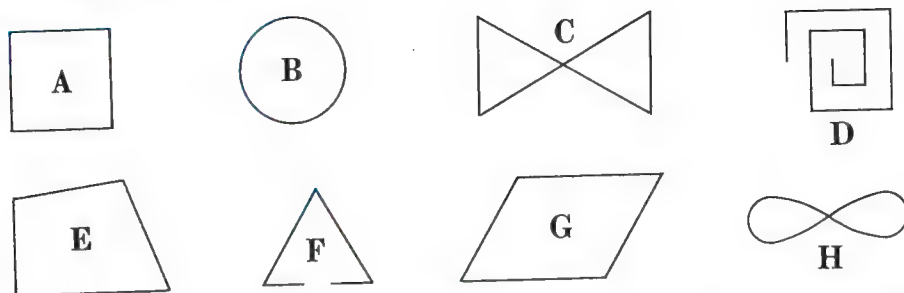
8. Are lines RS and MN perpendicular to each other? Why or why not? **Yes; The two lines intersect so that 4 right angles are formed.**

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Oral 4. Yes; The two lines intersect so that 4 right angles are formed.

6. a: $\angle MNP$ or $\angle PNM$ is a right angle; b: $\angle CBA$ or $\angle ABC$ is less than a right angle; c: $\angle RST$ or $\angle TSR$ is greater than a right angle.

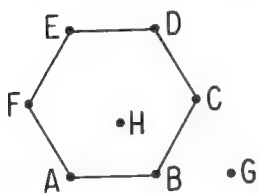
Simple Closed Figures



Which of the figures above could you draw by starting at some point on the figure, never lift your pencil from the paper, and end at the starting point? Such figures are called **closed figures**.

A, B, C, E, G, H

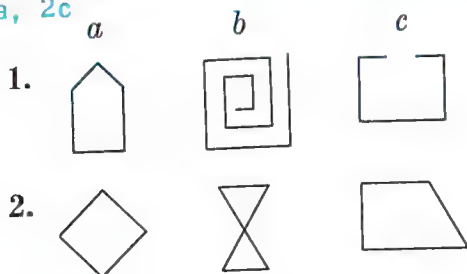
Which of the closed figures above can you draw without having the figure cross itself? Such figures as A, B, E, and G are called **simple closed figures**.



Is the figure at the left a simple closed figure? Why or why not? Is point F located in the interior, in the exterior, or on the figure? Is point G located in the interior, in the exterior, or on the figure? **Yes; It's a closed figure that**

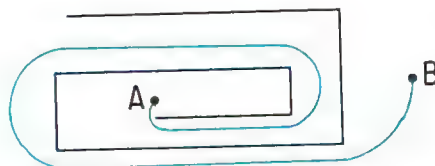
can be drawn without having the figure cross itself; on the figure; in the exterior

Oral Which of the figures below are closed figures? Which are simple closed figures? **1a, 2a, 2b, 2c; 1a, 2a, 2c**



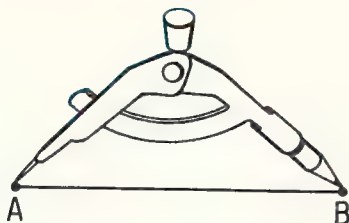
Written Do the following.

Draw a figure like the one below.



Trace from point A to point B without crossing the figure. Is the figure open or closed? **open**

Congruent Line Segments



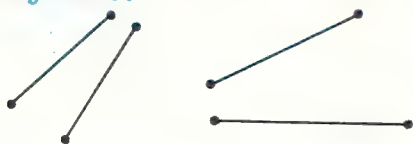
Can you tell without measuring whether line segment AB is longer than, shorter than, or the same length as line segment CD? **No** Trace line segments AB and CD.

You can open your compass as shown for line segment AB above. Place the steel tip at one endpoint and the pencil tip at the other endpoint. Now place the steel tip on point C. Without changing the opening of the compass, can the pencil tip be made to touch point D? Do the line segments have the same length? How do you know? **Yes; Yes; by comparing the length of one with the length of the other**

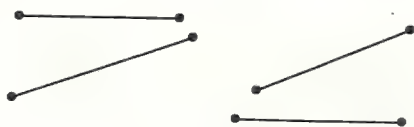
Line segments that have the same length are called **congruent line segments**.

Oral How could you use a compass to decide whether the line segments in each pair below are congruent to each other? **See explanations above. The pairs in 1a and in 2b are congruent.**

1.



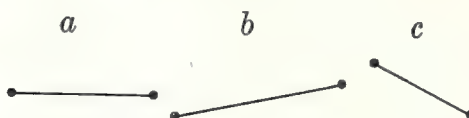
2.



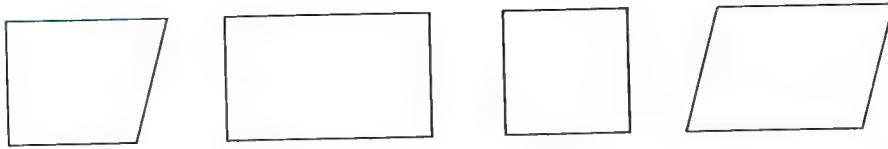
Written Do the following.

1. Draw line segment EF on your paper. Draw another line segment on your paper that is congruent to line segment EF. Label its endpoints X and Y. **Drawings will vary.**

2. Draw line segments on your paper that are congruent to those that follow.

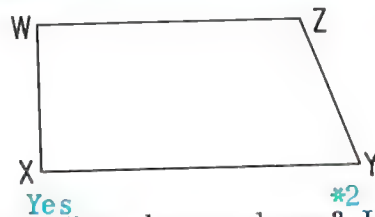


Rectangles



Why are all of the figures above called simple closed figures? ^{*1} How many line segments are there in each figure? ⁴ How many interiors does each figure have? ¹ Such figures are called **quadrilaterals**.

A *quadrilateral* is a simple closed figure formed by four line segments.



Are these figures quadrilaterals? ^{Yes} How do you know? ^{*2} How many angles are there in each figure? ⁴ With a model of a right angle, such as a folded piece of paper, see if all the interior angles in the two figures above are right angles. What are your findings? ^{*3} A quadrilateral like RSTU is called a **rectangle**.

A *rectangle* is a quadrilateral that has four right angles.

Why is figure XYZW not called a rectangle? ^{It does not have four right angles.}

Determine whether side RS and side UT in rectangle RSTU are congruent to each other. What are your findings? ^{*4} Do the same for sides RU and ST. What are your findings? ^{*4} Would you say that opposite sides of a rectangle have the same length? ^{Yes} Do you think they are also parallel? ^{Yes}

Opposite sides of a rectangle have the same length and are parallel.

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- *1 Each has but one interior region and can be traced without intersecting or retracing any part of the figure.
- *2 Each is a simple closed figure formed by four line segments.
- *3 All four angles in the figure on the left are right angles; only two angles in the figure on the right are right angles.
- *4 They are congruent.

For TEACHING HELPS, see page T228.

Oral Answer the following.

1. How many sides does a quadrilateral have? 4

2. How many interiors does a quadrilateral have? 1

3. How many angles does a quadrilateral have? 4

4. How many sides does a rectangle have? 4

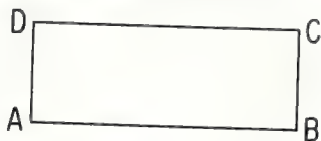
5. How many angles does a rectangle have? 4

6. What kind of angles are the angles of a rectangle? right angles

7. Which of the following figures looks like a rectangle? c



Use the rectangle below to help you answer the following.



8. Name the sides of rectangle ABCD. sides AB (or BA), BC (or CB), CD (or DC), and DA (or AD)

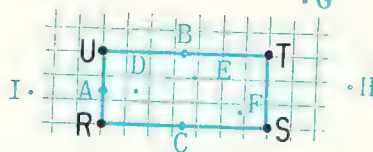
9. Give two names for each angle of rectangle ABCD. See below.

10. Which side is opposite side AB? Opposite side AD? Do you think that opposite sides have the same length? Are parallel? side DC; side BC; Yes; Yes

Oral 9. $\angle DAB$ or $\angle BAD$, $\angle ABC$ or $\angle CBA$, $\angle BCD$ or $\angle DCB$, $\angle CDA$ or $\angle ADC$
Written 2. rectangle RSTU (or STUR or TURS or URST or UTSR or RUTS or SRUT or TSUR)

Written Do the following.

1. Draw points R, S, T, and U on graph paper as shown below.



2. Connect these four points to form a rectangle. Write a name for this rectangle. See below.

3. Draw points A, B, and C on the rectangle. See above.

4. Draw points D, E, and F in the interior of the rectangle. See above.

5. Draw points G, H, and I in the exterior of the rectangle. See above.

Can you do this? Draw a quadrilateral whose opposite sides have the same length but whose angles are not right angles. Accept any parallelogram without right angles.

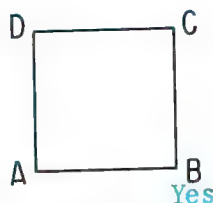
Tell why The figure below is a quadrilateral but not a rectangle. Why? It does not have four right angles.



Tell how The figure below is a rectangle. How can you tell? It is a quadrilateral with four right angles.



Squares



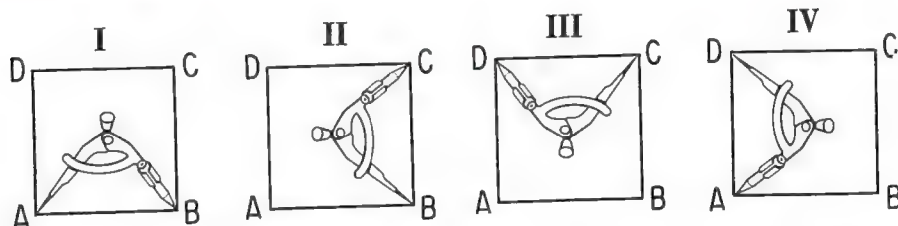
Yes

*1

Is the figure above a quadrilateral? How do you know? Name the four line segments that form figure ABCD. line segments AB (or BA), BC (or CB), CD (or DC), and DA (or AD)

How many angles are there in the figure? 4 Name the angles in this figure. Each of the angles in this figure is a right angle. Check this using a model of a right angle. Would you call the figure shown above a rectangle? Why? Yes; It is a quadrilateral that has four right angles.

Trace figure ABCD. You can open your compass as shown in I for side AB.



Place the steel tip at one endpoint of side AB and the pencil tip at the other endpoint. Now place the steel tip on point B as shown in II above. Without changing the opening of the compass can the pencil tip be made to touch point C? Do sides AB and BC have the same length? Yes; Yes

Without changing the opening of the compass, do the same for sides CD and DA as shown in III and IV above.

Are the four sides of figure ABCD congruent? Yes How do you know? *3 Are the angles right angles? Yes Such a figure is called a square.

A square is a rectangle that has four congruent line segments as its sides.

- *1 It is a simple closed figure formed by four line segments.
- *2 $\angle ABC$ (or $\angle CBA$), $\angle BCD$ (or $\angle DCB$), $\angle CDA$ (or $\angle ADC$), $\angle DAB$ (or $\angle BAD$)
- *3 The length of each side is the same.

For TEACHING HELPS, see page T229.

Oral Answer the following.

1. How many sides does a square have? **4**

2. Would you say that the sides of a square have the same length? **Yes**

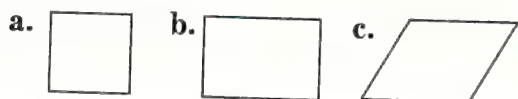
3. How many angles does a square have? **4**

4. What kind of angles does a square have? **right angles**

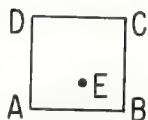
5. Are all squares rectangles? Why or why not? **See below.**

6. Are all rectangles squares? Why or why not? **See below.**

7. Which of the following figures is a square? **a**



Use the square below to help you answer the following.



8. Name the sides of square ABCD. **See below.**

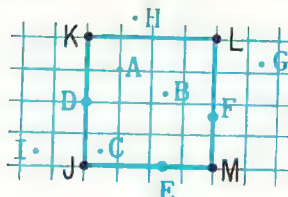
9. Name the angles of square ABCD in two ways. What kind of angles are they? **See below.**

10. Is point E located in the interior, in the exterior, or on the square? **in the interior**

- Oral**
5. Yes; A square is a quadrilateral with four right angles. **231**
 6. No; Rectangles that do not have all four sides congruent are not squares.
 8. sides AB (or BA), BC (or CB), CD (or DC), and DA (or AD)
 9. $\angle ABC$ (or $\angle CBA$), $\angle BCD$ (or $\angle DCB$), $\angle CDA$ (or $\angle ADC$), $\angle DAB$ (or $\angle BAD$); right angles

Written Do the following.

1. Draw points J, K, L, and M on graph paper as shown below.



2. Connect these four points to form a square. Write a name for this square. **See page T229.**

3. Draw points A, B, and C in the interior of the square. **See above.**

4. Draw points D, E, and F on the square. **See above.**

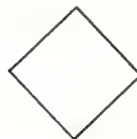
5. Draw points G, H, and I in the exterior of the square. **See above.**

Can you do this? Draw a quadrilateral that has four congruent sides so that the angles are not right angles. **Accept any parallelogram with congruent sides and no right angles.**

Tell why Answer the following.

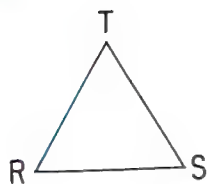
1. Why can a figure be both a rectangle and a square? **because all squares are rectangles**

2. Bill said the figure below is a square.

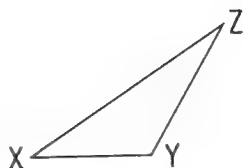


Is he right or wrong? Tell why. **right; definition of square**

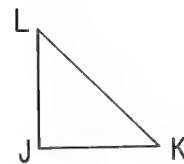
Triangles



Triangle RST



Triangle XYZ



Triangle JKL

Are the figures above simple closed figures? Why? How many line segments are there in each figure? ^{*1} Name the line segments that form the figure labeled with the letters R, S, and T. ^{*2} With the letters X, Y, and Z. ^{*3} With the letters J, K, and L. ^{*4}

How many angles are there in each figure? ^{*5} Name the angles in the figure labeled with the letters R, S, and T. ^{*6} With the letters X, Y, and Z. ^{*7} With the letters J, K, and L. ^{*8}

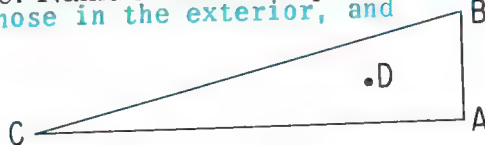
Each figure above is called a **triangle**. The prefix *tri* means *three*. Thus the word *triangle* means *three angles*.

A *triangle* is a simple closed figure formed by three line segments as its sides.

Study the figure above labeled with the letters J, K, and L. It is named *triangle JKL*. How would you name the other two triangles shown above? Notice the angle in triangle JKL whose vertex is labeled point J. This angle is a right angle. Triangle JKL is called a **right triangle**.

A *right triangle* is a triangle that has one right angle.

Into how many sets of points does the triangle below separate a plane? Name these sets of points. ³; those in the interior, those in the exterior, and ^B those on the triangle



Is point D located in the interior, in the exterior, or on the triangle? *in the interior*

- *1 Each has but one interior region and can be traced without intersecting or retracing any part of the figure.
- *2 line segments RS (or SR), ST (or TS), and TR (or RT)
- *3 line segments XY (or YX), YZ (or ZY), and ZX (or XZ)
- *4 line segments JK (or KJ), KL (or LK), and LJ (or JL)
- *5 $\angle RST$ (or $\angle TSR$), $\angle STR$ (or $\angle RTS$), and $\angle TRS$ (or $\angle SRT$)

Oral Answer the following.

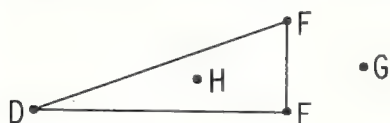
1. What do you call a simple closed figure formed by three line segments? **triangle**

2. How many angles does a triangle have? How many sides does a triangle have? **3; 3**

3. Which of the following is a triangle? **c**



Use the figure below to help you answer the following.



4. Is point D located in the interior, in the exterior, or on the triangle? **on the triangle**

5. Is point G located in the interior, in the exterior, or on the triangle? **in the exterior**

6. Is point H located in the interior, in the exterior, or on the triangle? **in the interior**

7. One of the angles in the triangle is a right angle. Is it $\angle FDE$, $\angle DEF$, or $\angle EFD$? **$\angle DEF$**

8. Give three names for the triangle above. **triangle DEF, EFD, FDE (or FED, DFE, EDF)**

*6 $\angle XYZ$ (or $\angle ZYX$), $\angle YZX$ (or $\angle XZY$), and $\angle ZXY$ (or $\angle YXZ$)

*7 $\angle JKL$ (or $\angle LKJ$), $\angle KLJ$ (or $\angle JLK$), and $\angle LJK$ (or $\angle KJL$)

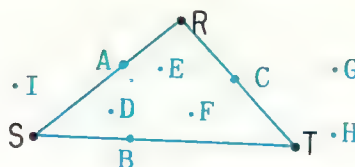
*8 triangle XYZ (or YZX or XZY or ZYX or XZY or YXZ)

triangle JKL (or KLJ or LJK or LKJ or JLK or KJL)

Written 5. sides ST (or TS), TR (or RT), and RS (or SR)

$\angle RST$ (or $\angle TSR$), $\angle STR$ (or $\angle RTS$), and $\angle TRS$ (or $\angle SRT$)

Written Draw points R, S, and T on your paper as shown below.



1. Connect these three points to form a triangle. Write a name for this triangle. **triangle STR (or TRS or RST or RTS or SRT or TSR)**

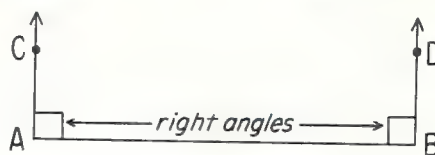
2. Draw points A, B, and C on the triangle, but not two on the same side. **See above.**

3. Draw points D, E, and F in the interior of the triangle. **See above.**

4. Draw points G, H, and I in the exterior of the triangle. **See above.**

5. Triangle STR has how many sides? **3** How many angles? **3** Name the sides and the angles. **See below.**

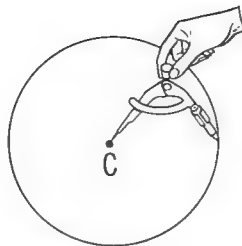
Tell why Study the figure below.



Do you think rays AC and BD will ever meet? Do you think a triangle could have more than one right angle? Why or why not? **No; No; It would not be a closed figure.**

Tell how How can you tell whether a triangle is a right triangle? **A right triangle has one right angle.**

Circles

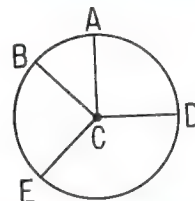


The simple closed figures shown on the screen above are called **circles**. To draw a circle you can use a **compass** as shown. Where do you place the steel point of the compass? That point is called the **center** of the circle. Hold the paper with one hand and swing the pencil leg of the compass around to draw a circle.

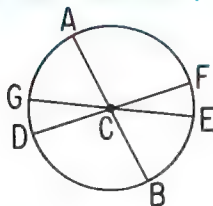
Is a circle a set of points? Are all of these points the same distance from the center? How do you know? Yes; Yes; The opening of the compass is always the same.

A *circle* is the set of all points in a plane that are the same distance from a point called the center of the circle.

What point is the center of the circle at the right? One endpoint of line segment CA is located at the center of the circle. Where is the other endpoint located? Such a line segment is called a **radius** of the circle. Name 3 other radii



(plural of radius) of the circle. Can more radii be drawn? Are all of them congruent line segments? CD (or DC), CE (or EC), and CB (or BC); Yes; Yes



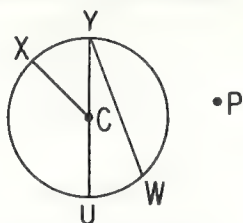
Which point is the center of the circle at the left? Notice line segment AB. Where are its endpoints located? Does line segment AB pass through the center of the circle? Such a line segment is called a **diameter** of the circle. Name 2

other diameters of the circle. Can more diameters be drawn in the circle? Are all of them congruent line segments? GE (or EG) and DF (or FD); Yes; Yes

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- *1 on point C
- *2 on the circle
- *3 Yes

Oral Use the figure below to help you answer each exercise.



1. Would you call line segment CU a radius or a diameter? **a radius**
2. Would you call line segment YU a radius or a diameter? **a diameter**
3. Would you call line segment YW a diameter? Why or why not? **No; It does not pass through point C.**
4. Name all the radii shown. Name all the diameters shown. **CX (or XC), CY (or YC), and CU (or UC); YU (or UY)**
5. Is the center of a circle in the interior, in the exterior, or on the circle? **in the interior**
6. Is point X in the interior, in the exterior, or on the circle? **on the circle**
7. Is point P in the interior, in the exterior, or on the circle? **in the exterior**
8. What can you say about the lengths of all the radii of the circle shown above? **They are the same.**
9. What can you say about the lengths of all the diameters of the circle shown above? **They are the same.**
10. Which is longer, a radius or a diameter of a circle? How are their lengths related? **a diameter; A diameter is twice as long as a radius of a circle.**

Written Do the following.

1. Open a compass to a distance of 2 inches. Then draw a circle having a radius of 2 inches. Label the center with the letter C.

2. Draw 4 diameters of the circle you drew for *Written* 1. Without measuring, what is the length of each diameter? How many radii are drawn in the circle? **4 inches; 8**

3. Open a compass to a distance of $1\frac{1}{2}$ inches. Then draw a circle having a radius of $1\frac{1}{2}$ inches. Label the center with the letter C. Draw 3 radii of the circle.

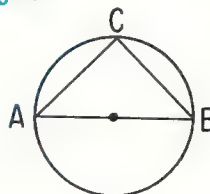
4. Draw a circle having a diameter of 4 inches.

Answer each of the following.

5. A radius of a circle is 5 inches long. How long is a diameter of the circle? **10 inches**

6. A diameter of a circle is 1 foot long. How long is a radius of the circle? **$\frac{1}{2}$ of a foot or 6 inches**

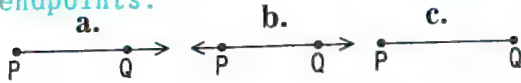
Can you do this? Tell what kind of angle is $\angle ACB$ in the figure below. **right angle**



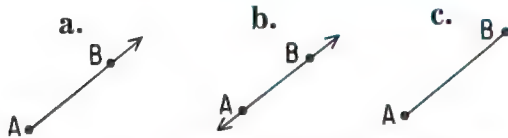
Review and Practice

Part 1 Choose an answer and tell why you chose it.

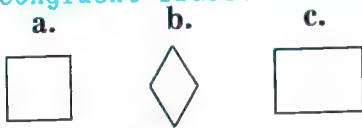
1. Which figure below is a picture of line segment PQ? **c; It has two endpoints.**



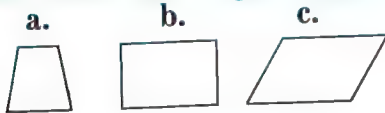
2. Which figure below is a picture of ray AB? **a; It has one endpoint.**



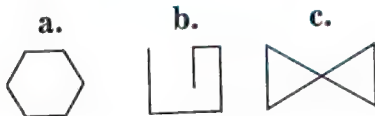
3. Which figure below is a picture of a square? **a; It is a rectangle with congruent sides.**



4. Which figure below is a picture of a rectangle? **b; It is a quadrilateral with four right angles.**



5. Which picture shows a simple closed figure? **See below.**



Part 2 Answer the following.

1. How do you name points?
with capital letters

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- Part 1** 5. **a; It has but one interior region and can be traced without intersecting or retracing any part of the figure.**
- Part 2** 3. **Accept any answer that implies that a circle has an infinite number of radii and diameters.**
4. **Accept any answer that implies an infinite number; 1**

2. A line segment has how many endpoints? A ray has how many endpoints? A line has how many endpoints? **2; 1; 0**

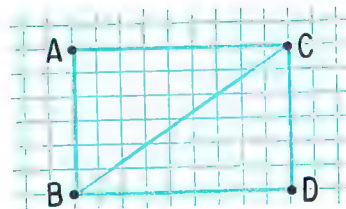
3. A circle has how many radii? A circle has how many diameters?
See below.

4. How many line segments can pass through a point? How many line segments can connect 2 points?
See below.

Part 3 Do the following.

1. On your paper draw 3 points, A, B, and C, not on the same straight line. Use rays to draw $\angle BAC$. Is $\angle BAC$ a simple closed figure? Why or why not? **Drawings will vary; No; It is not a closed figure.**

2. On graph paper draw 4 points as shown below.



Connect these four points to form a rectangle. What would you name this rectangle? Draw line segment BC. Into how many parts does line segment BC separate the interior of rectangle BDCA? Would you call the figure labeled with the letters B, A, and C a right triangle? Why or why not? **rectangle ABDC (or BDCA or DCAB or CABD or CDBA or ACDB or BACD or DBAC); 2; Yes; It has a right angle.**

Checkup Time

The numerals in () tell the pages where you can turn for help.

Important Ideas

1. You may think of a point as a location. (217)
2. Parallel lines are lines in a plane that do not intersect each other no matter how far they are extended. (221)
3. An angle is a figure formed by two rays with a common endpoint. (222)
4. Line segments that have the same length are called congruent line segments. (227)
5. A rectangle is a quadrilateral that has four right angles. (228)
6. A square is a rectangle that has four congruent line segments as its sides. (230)
7. A circle is the set of all points in a plane that are the same distance from a point called the center. (234)
4. Intersect, parallel lines (221)
5. Ray, angle, vertex (222)
6. Right angle, is perpendicular to (224)
7. Closed figure, simple closed figure (226)
8. Congruent line segments (227)
9. Quadrilateral, rectangle (228)
10. Square (230)
11. Triangle, right triangle (232)
12. Circle, center, radius, diameter (234)

Words to Know

1. Point, line segment, endpoint (217)
2. Line (218)
3. Plane (220)

Questions to Discuss

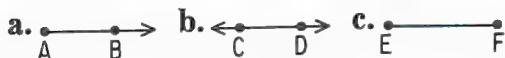
See page T237.

1. What is different about a line and a line segment? About a line and a ray? About a line segment and a ray? (217-222)
2. Into how many sets of points does a simple closed figure separate a plane? Name these sets of points. (226)
3. How would you determine that two line segments are congruent? (227)
4. Are all rectangles squares? Why or why not? (230)

Self-Evaluation

Part 1 Select the correct answer for each exercise below.

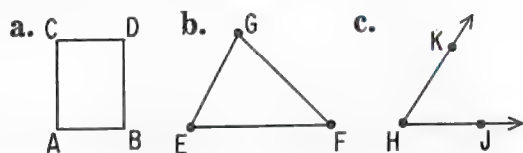
1. Which of the following is a picture of a line segment? **c**



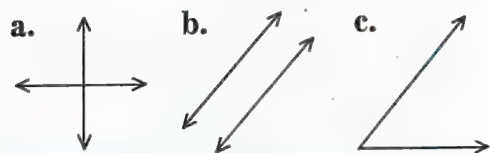
2. Which of the following is a picture of a right angle? **b**



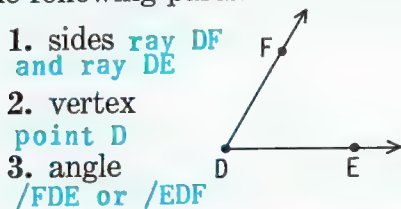
3. Which of the following is a picture of a triangle? **b**



4. Which of the following shows a pair of parallel lines? **b**



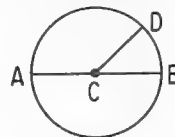
Part 2 Using the figure below, name the following parts.



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- Part 4**
- rectangle ABCD (or BCDA or CDAB or DABC or ADCB or DCBA or CBAD or BADC)
 - sides AB (or BA), BC (or CB), CD (or DC), and DA (or AD)

Part 3 Use the figure below to help you answer the following.



1. Which phrase would best describe point C? **b**

- point on the circle
- center of the circle
- point outside the circle

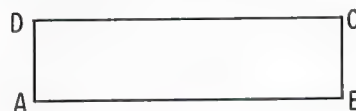
2. Which word would best name line segment CD? **a**

- radius
- diameter

3. Which word would best name line segment AB? **b**

- radius
- diameter

Part 4 Use the figure below to help you answer the following.



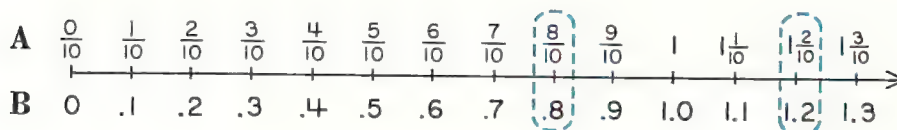
- Name the figure. **See below.**
- Name the sides of the figure. **See below.**
- Which side is opposite side AB? Side AD? **side DC (or CD); side BC (or CB)**
- Name the angles in the figure in two ways. **$\angle DAB$ (or $\angle BAD$), $\angle ABC$ (or $\angle CBA$), $\angle BCD$ (or $\angle DCB$), $\angle CDA$ (or $\angle ADC$)**

Chapter 11

DECIMALS AND MEASUREMENT

Tenths

There is an easy way to express fractional numbers that have a denominator of 10. Study the number line below.



Which numeral in **B** names the same number as $\frac{8}{10}$? Which numeral in **B** names the same number as $1\frac{2}{10}$? **1.2**

You call .8 and 1.2 **decimal numerals** or simply **decimals**. The dot in .8 and 1.2 is called a **decimal point**. You can read .8 and 1.2 in two ways as shown below.

.8 is read $\begin{cases} \text{eight tenths} \\ \text{or} \\ \text{point eight.} \end{cases}$

1.2 is read $\begin{cases} \text{one and two tenths} \\ \text{or} \\ \text{one point two.} \end{cases}$

Oral Read each decimal below in two ways. See examples above.

	a	b	c	d
1.	.3	.4	1.7	1.3
2.	.9	.6	2.6	0.8
3.	.2	.1	3.7	4.9
4.	.5	.7	4.3	6.5

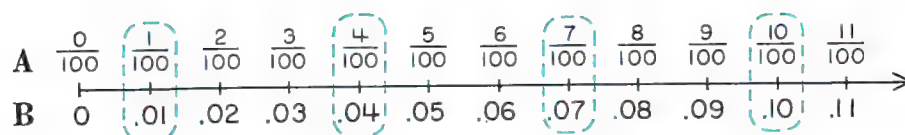
Written Copy. Change the fractions or mixed numerals to decimals and change the decimals to fractions or mixed numerals.

	a	b	c	d
1.	$\frac{7}{10}$.7	$1\frac{1}{10}$ 1.1	$.3\frac{3}{10}$	$2.92\frac{9}{10}$
2.	$\frac{4}{10}$.4	$1\frac{6}{10}$ 1.6	$.7\frac{7}{10}$	$9.19\frac{1}{10}$
3.	$\frac{5}{10}$.5	$2\frac{3}{10}$ 2.3	$.9\frac{9}{10}$	$6.76\frac{7}{10}$

Hundredths

When you express $\frac{3}{10}$ as .3, how many digits are there to the right of the decimal point in .3? How many 0's are there in the denominator of $\frac{3}{10}$? **one; one**

When a fractional number has a denominator of 100, it may also be expressed as a decimal. Study the number line below.



Which decimal in **B** names the same number as $\frac{1}{100}$? As $\frac{4}{100}$? As $\frac{7}{100}$? As $\frac{10}{100}$? What is the denominator of each of these fractions? How many 0's are in the denominator of $\frac{1}{100}$? Of $\frac{4}{100}$? Of $\frac{7}{100}$? Of $\frac{10}{100}$? How many digits are there to the right of the decimal point in .01? In .04? In .07? In .10? **.01, .04, .07, .10; 100; two, two, two, two; two, two, two, two**
You can read .01 and .10 as shown below.

.01 is read $\begin{cases} \text{one hundredth} \\ \text{or} \\ \text{point zero one.} \end{cases}$.10 is read $\begin{cases} \text{ten hundredths} \\ \text{or} \\ \text{point one zero.} \end{cases}$

Oral Read each decimal below in two ways. **See examples above.**

	a	b	c	d
1.	.02	.05	.09	.11
2.	.03	.12	.20	.31
3.	.19	.29	.41	.82
4.	.99	.86	.75	.50

Written Copy. Change the fractions to decimals and change the decimals to fractions.

	a	b	c	d
1.	$\frac{5}{100}$.05	$\frac{31}{100}$.31	.87 $\frac{87}{100}$.33 $\frac{33}{100}$
2.	$\frac{25}{100}$.25	$\frac{20}{100}$.20	.09 $\frac{9}{100}$.51 $\frac{51}{100}$

Tell how How would you show that .3 and .30 name the same number? **See below.**

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Tell how $.30 = \frac{30}{100} = \frac{3 \times 10}{10 \times 10} = \frac{3}{10} \times \frac{10}{10} = \frac{3}{10} \times 1 = \frac{3}{10} = .3$
 $.3 = \frac{3}{10} = \frac{3}{10} \times 1 = \frac{3}{10} \times \frac{10}{10} = \frac{3 \times 10}{10 \times 10} = \frac{30}{100} = .30$

Common Fractional Equivalents

You can change *some* fractions which do not have a denominator of 10 or 100 to decimals as shown below.

$$\frac{4}{5} = \frac{4}{5} \times 1 = \frac{4}{5} \times \frac{2}{2} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10} = .8$$

Why is 1 renamed as $\frac{2}{2}$ instead of $\frac{3}{3}$, $\frac{4}{4}$, or $\frac{5}{5}$? How is $\frac{4}{5} \times \frac{2}{2}$ changed to .8? How do you read .8? **to obtain a denominator of 10; see explanation above; eight tenths or point eight**
You can also change $\frac{4}{5}$ to a decimal which is read as hundredths.

$$\frac{4}{5} = \frac{4}{5} \times 1 = \frac{4}{5} \times \frac{20}{20} = \frac{4 \times 20}{5 \times 20} = \frac{80}{100} = .80$$

Why is 1 renamed as $\frac{20}{20}$ instead of $\frac{30}{30}$, $\frac{40}{40}$, or $\frac{50}{50}$? How is $\frac{4}{5} \times \frac{20}{20}$ changed to .80? How do you read .80? **to obtain a denominator of 100; see explanation above; eighty hundredths or**

You can change a decimal to a fraction in simplest form **point eight zero** as follows.

$$.80 = \frac{80}{100} = \frac{4 \times 20}{5 \times 20} = \frac{4}{5} \times \frac{20}{20} = \frac{4}{5} \times 1 = \frac{4}{5}$$

How is .80 changed to $\frac{4}{5}$? **See explanation above.**

Oral Study how $\frac{1}{2}$ is changed to .5. Then answer the following.

$$\frac{1}{2} = \frac{1}{2} \times 1 = \frac{1}{2} \times \frac{5}{5} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10} = .5$$

1. Why is 1 renamed as $\frac{5}{5}$ instead of $\frac{3}{3}$, $\frac{4}{4}$, or $\frac{6}{6}$? **to obtain a denominator of 10**

2. How is $\frac{1}{2} \times \frac{5}{5}$ changed to .5? **See explanation above.**

3. In the example above, suppose you wanted to change $\frac{1}{2}$ to .50, why would you rename 1 as $\frac{50}{50}$ instead of $\frac{5}{5}$? How would you change $\frac{1}{2} \times \frac{50}{50}$ to .50? **See below.**

Written Change each fraction to a decimal read as hundredths.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
1.	$\frac{2}{5}$.40	$\frac{1}{4}$.25	$\frac{1}{25}$.04	$\frac{3}{5}$.60	$\frac{7}{25}$.28

2.	$\frac{1}{5}$.20	$\frac{2}{25}$.08	$\frac{2}{4}$.50	$\frac{3}{25}$.12	$\frac{3}{50}$.06
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Change each of the following to a fraction in simplest form.

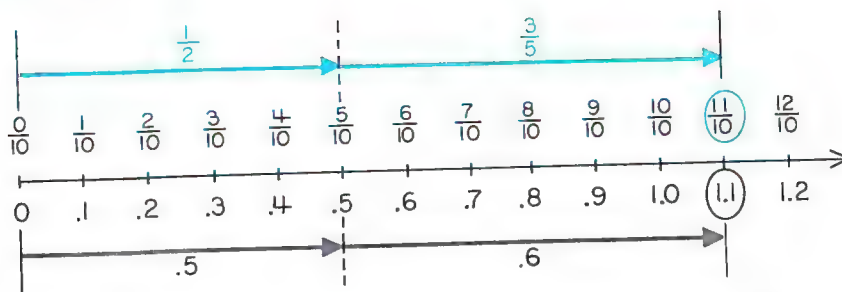
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
3.	.4 $\frac{2}{5}$.5 $\frac{1}{2}$.8 $\frac{4}{5}$.30 $\frac{3}{10}$.25 $\frac{1}{4}$
4.	.75 $\frac{3}{4}$.08 $\frac{2}{25}$.20 $\frac{1}{5}$.12 $\frac{3}{25}$.80 $\frac{4}{5}$

M P
O R
R A
E C
T I
C E
PAGE 328

Oral 3. in order to obtain a denominator of 100

$$\frac{1}{2} \times \frac{50}{50} = \frac{1 \times 50}{2 \times 50} = \frac{50}{100} = .50$$

Decimals in Addition



What closed addition sentence is suggested by the blue arrows above the number line? By the gray arrows below the number line? $\frac{1}{2} + \frac{3}{5} = \frac{11}{10}$; $.5 + .6 = 1.1$

Do $\frac{1}{2} + \frac{3}{5}$ and $.5 + .6$ both name the same number? How do you know? ^{Yes} A convenient way to show that $\frac{1}{2} + \frac{3}{5}$ and $.5 + .6$ both name the same number follows.

A	B	C
$\begin{array}{r} \frac{1}{2} \\ + \frac{3}{5} \\ \hline \end{array} \rightarrow \begin{array}{r} \frac{5}{10} \\ \frac{6}{10} \\ + \frac{0}{10} \\ \hline \frac{11}{10} \text{ or } 1\frac{1}{10} \end{array}$	$\begin{array}{r} \frac{1}{2} \\ + \frac{3}{5} \\ \hline \end{array} \rightarrow \begin{array}{r} 0 \text{ Ts} \\ 1 \\ \hline .5 \\ + .6 \\ \hline 1.1 \end{array}$ <div style="margin-top: 10px;"> <div style="border-left: 1px solid black; border-right: 1px solid black; height: 10px; width: 100%;"></div> <div style="display: flex; justify-content: space-between; padding: 0 5px;"> Add the Ts. Add the Ones. </div> </div>	$\begin{array}{r} .5 \\ + .6 \\ \hline 1.1 \text{ or } 1\frac{1}{10} \end{array}$

The example in **B** shows that you can add fractional numbers just as you add whole numbers after the fractions are changed to decimals.

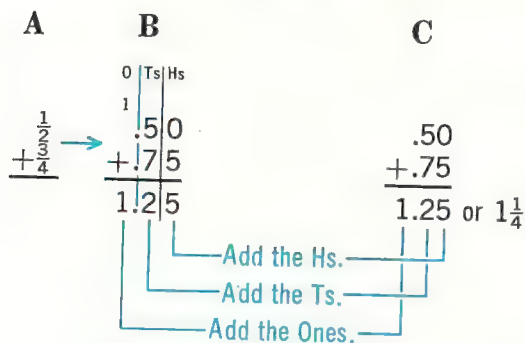
How do you change $\frac{1}{2}$ to $.5$? How do you change $\frac{3}{5}$ to $.6$? Are the decimal points in **B** kept in a straight line? How is the sum of the tenths recorded? $\frac{3}{5} = \frac{3}{5} \times 1 = \frac{3}{5} \times \frac{2}{2} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10} = .6$; Yes; 1 one and 1 Ts

You can think as shown in **B** but write only place-value numerals as shown in **C**. How is the sum of $.5$ and $.6$ expressed as a decimal? As a mixed numeral? 1.1 ; $1\frac{1}{10}$

242 *1 $\frac{1}{2}$ and $.5$ name the same number and $\frac{3}{5}$ and $.6$ name the same number.

*2 $\frac{1}{2} = \frac{1}{2} \times 1 = \frac{1}{2} \times \frac{5}{5} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10} = .5$

Oral Use the example below to help you answer the following.



1. How do you change $\frac{1}{2}$ to .50?
See below.
2. How do you change $\frac{3}{4}$ to .75?
See below.
3. In **B** the 5 of .50 is recorded in the Ts or tenths place. In which place is the 7 of .75 recorded?
tenths place
4. The 0 of .50 is recorded in the Hs or hundredths place. In which place is the 5 of .75 recorded?
hundredths place
5. How does the addition in **B** and **C** resemble addition of whole numbers? It is the same except for the decimal point.
6. What is the sum of .50 and .75?
1.25
7. How is 1.25 recorded in **B** or **C**?
1 one, 2 tenths, 5 hundredths
8. What is the sum of $\frac{1}{2}$ and $\frac{3}{4}$? $1\frac{1}{4}$

Tell how you would find the sum in each example below.

a	b	c
$\begin{array}{r} .7 \\ +.4 \\ \hline 1.1 \end{array}$	$\begin{array}{r} .38 \\ +.27 \\ \hline .65 \end{array}$	$\begin{array}{r} .27 \\ +.81 \\ \hline 1.08 \end{array}$

Oral

1. $\frac{1}{2} = \frac{1}{2} \times 1 = \frac{1}{2} \times \frac{50}{50} = \frac{1 \times 50}{2 \times 50} = \frac{50}{100} = .50$
2. $\frac{3}{4} = \frac{3}{4} \times 1 = \frac{3}{4} \times \frac{25}{25} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = .75$

Written Use the method below to solve each of the following.

$$\frac{1}{4} \rightarrow \begin{array}{r} .25 \\ +.04 \\ \hline .29 \text{ or } \frac{29}{100} \end{array}$$

$\begin{array}{l} a \\ \frac{1}{2} + \frac{1}{25} = a \end{array} \begin{array}{r} .54 \\ \hline \frac{54}{100} \end{array}$ $\begin{array}{l} b \\ \frac{3}{5} + \frac{1}{4} = b \end{array} \begin{array}{r} .85 \\ \hline \frac{85}{100} \end{array}$ $\begin{array}{l} c \\ \frac{4}{5} + \frac{1}{2} = c \end{array} \begin{array}{r} 1.30 \\ \hline \frac{130}{100} \end{array}$ $\begin{array}{l} d \\ \frac{7}{25} + \frac{3}{4} = d \end{array} \begin{array}{r} 1.03 \\ \hline \frac{103}{100} \end{array}$ $\begin{array}{l} e \\ \frac{1}{4} + \frac{4}{5} = e \end{array} \begin{array}{r} 1.05 \\ \hline \frac{105}{100} \end{array}$	$\begin{array}{l} f \\ \frac{1}{4} + \frac{1}{5} = f \end{array} \begin{array}{r} .45 \\ \hline \frac{45}{100} \end{array}$ $\begin{array}{l} g \\ \frac{1}{5} + \frac{2}{25} = g \end{array} \begin{array}{r} .28 \\ \hline \frac{28}{100} \end{array}$ $\begin{array}{l} h \\ \frac{3}{4} + \frac{1}{5} = h \end{array} \begin{array}{r} .95 \\ \hline \frac{95}{100} \end{array}$ $\begin{array}{l} j \\ \frac{6}{25} + \frac{4}{5} = j \end{array} \begin{array}{r} 1.04 \\ \hline \frac{104}{100} \end{array}$ $\begin{array}{l} k \\ \frac{1}{2} + \frac{1}{4} = k \end{array} \begin{array}{r} .75 \\ \hline \frac{75}{100} \end{array}$
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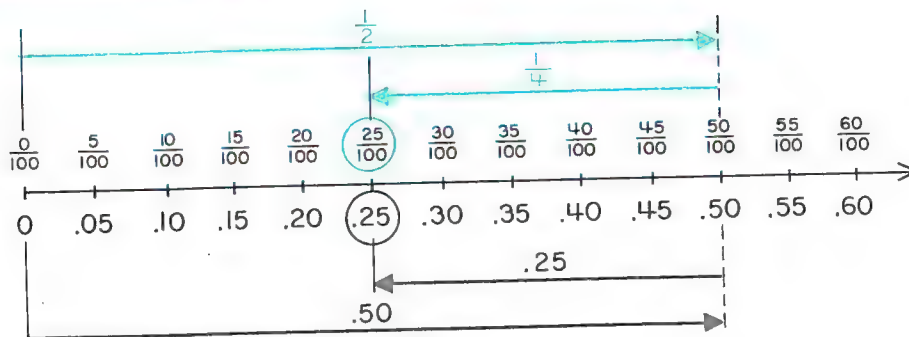
Copy. Find each sum.

	a	b	c	d
6.	$\begin{array}{r} .8 \\ +.3 \\ \hline 1.1 \end{array}$	$\begin{array}{r} .12 \\ +.04 \\ \hline .16 \end{array}$	$\begin{array}{r} 2.7 \\ +.6 \\ \hline 3.3 \end{array}$	$\begin{array}{r} 13.82 \\ +4.14 \\ \hline 17.96 \end{array}$
7.	$\begin{array}{r} .43 \\ +.39 \\ \hline .82 \end{array}$	$\begin{array}{r} 4.06 \\ +1.34 \\ \hline 5.40 \end{array}$	$\begin{array}{r} 2.1 \\ +.8 \\ \hline 2.9 \end{array}$	$\begin{array}{r} 131.8 \\ +21.9 \\ \hline 153.7 \end{array}$
8.	$\begin{array}{r} .18 \\ +.02 \\ \hline .20 \end{array}$	$\begin{array}{r} 18.4 \\ +9.0 \\ \hline 27.4 \end{array}$	$\begin{array}{r} 6.08 \\ +.91 \\ \hline 6.99 \end{array}$	$\begin{array}{r} 12.06 \\ +1.26 \\ \hline 13.32 \end{array}$
9.	$\begin{array}{r} .7 \\ +1.2 \\ \hline 1.9 \end{array}$	$\begin{array}{r} 6.30 \\ +2.81 \\ \hline 9.11 \end{array}$	$\begin{array}{r} 7.15 \\ +3.06 \\ \hline 10.21 \end{array}$	$\begin{array}{r} 141.7 \\ +68.3 \\ \hline 210.0 \end{array}$

Can you do this? Name the next three decimals in each of the following.

1. .6, .7, .8, .9, 1.0, 1.1
2. .60, .70, .80, .90, 1.00, 1.10
3. .06, .07, .08, .09, .10, .11

Decimals in Subtraction



What closed subtraction sentence is suggested by the blue arrows above the number line? By the gray arrows below the number line? $\frac{1}{2} - \frac{1}{4} = \frac{25}{100}$; $.50 - .25 = .25$ Yes

Do $\frac{1}{2} - \frac{1}{4}$ and $.50 - .25$ both name the same number? How do you know? *1 A convenient way to show that $\frac{1}{2} - \frac{1}{4}$ and $.50 - .25$ both name the same number follows.

A	B	C															
$\begin{array}{r} \frac{1}{2} \\ - \frac{1}{4} \\ \hline \end{array}$ \longrightarrow $\begin{array}{r} \frac{2}{4} \\ - \frac{1}{4} \\ \hline \frac{1}{4} \end{array}$	$\begin{array}{r} \frac{1}{2} \\ - \frac{1}{4} \\ \hline \end{array}$ \longrightarrow <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">0</td> <td style="padding: 0 5px;">Ts</td> <td style="border-left: 1px solid black; padding: 0 5px;">Hs</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">4</td> <td style="padding: 0 5px;">10</td> <td style="border-left: 1px solid black; padding: 0 5px;">0</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">5</td> <td style="padding: 0 5px;">0</td> <td style="border-left: 1px solid black; padding: 0 5px;">0</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">2</td> <td style="padding: 0 5px;">5</td> <td style="border-left: 1px solid black; padding: 0 5px;">5</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">2</td> <td style="padding: 0 5px;">5</td> <td style="border-left: 1px solid black; padding: 0 5px;">5</td> </tr> </table>	0	Ts	Hs	4	10	0	5	0	0	2	5	5	2	5	5	$\begin{array}{r} .50 \\ - .25 \\ \hline .25 \end{array}$ <p style="text-align: right;">or $\frac{1}{4}$</p>
0	Ts	Hs															
4	10	0															
5	0	0															
2	5	5															
2	5	5															

Subtract the Hs.
Subtract the Ts.

Does the example in **B** show that you change $\frac{1}{2}$ to .50 and $\frac{1}{4}$ to .25 and then subtract? How do you change $\frac{1}{2}$ to .50? $\frac{1}{4}$ to .25? Are the decimal points in **B** kept in a straight line? Where is the difference recorded when subtracting the Hs? When subtracting the Ts? Yes; hundredths place; tenths place

You can think as shown in **B** but write only place-value numerals as shown in **C**. How is the difference $.50 - .25$ expressed as a decimal? As a fraction? $.25$; $\frac{1}{4}$

244 *1 $\frac{1}{2}$ and .50 name the same number and $\frac{1}{4}$ and .25 name the same number.

*2 $\frac{1}{2} = \frac{1}{2} \times 1 = \frac{1}{2} \times \frac{50}{50} = \frac{1 \times 50}{2 \times 50} = \frac{50}{100} = .50$

*3 $\frac{1}{4} = \frac{1}{4} \times 1 = \frac{1}{4} \times \frac{25}{25} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100} = .25$

Oral Use the example below to help you answer the following.

A	B	C										
$\begin{array}{r} \frac{9}{10} \\ - \frac{1}{4} \\ \hline \end{array}$	<table border="0"> <tr> <td style="text-align: right;">0 Ts Hs</td> <td></td> </tr> <tr> <td style="text-align: right;">8 10</td> <td></td> </tr> <tr> <td style="text-align: right;">.90</td> <td></td> </tr> <tr> <td style="text-align: right;">- .25</td> <td></td> </tr> <tr> <td style="text-align: right;">.65</td> <td></td> </tr> </table>	0 Ts Hs		8 10		.90		- .25		.65		$\begin{array}{r} .90 \\ - .25 \\ \hline .65 \text{ or } \frac{13}{20} \end{array}$
0 Ts Hs												
8 10												
.90												
- .25												
.65												
	<div style="border-left: 1px solid black; border-right: 1px solid black; padding: 0 10px;"> Subtract the Hs. Subtract the Ts. </div>											

- How do you change $\frac{9}{10}$ to .90?
See below.
- How do you change $\frac{1}{4}$ to .25?
See below.
- Can you subtract in the Hs or hundredths column? Why or why not?
See below.
- How do the subtraction examples in B and C resemble the whole number subtraction shown below? They are the same except for the decimal points.

$$\begin{array}{r} 90 \\ - 25 \\ \hline \end{array}$$

- What is the simplest numeral for .90 - .25? How is it obtained?
.65; see example B.
- How is .65 recorded in B and C?
6 Ts, 5 Hs
- What is the simplest fraction for $\frac{9}{10} - \frac{1}{4}$? $\frac{13}{20}$

Tell how you would find the difference in each example below. See typical explanation for Oral 5.

a	b	c
8. $\begin{array}{r} .9 \\ - .4 \\ \hline .5 \end{array}$	$\begin{array}{r} .82 \\ - .61 \\ \hline .21 \end{array}$	$\begin{array}{r} .74 \\ - .38 \\ \hline .36 \end{array}$
9. $\begin{array}{r} .8 \\ - .2 \\ \hline .6 \end{array}$	$\begin{array}{r} .63 \\ - .19 \\ \hline .44 \end{array}$	$\begin{array}{r} .91 \\ - .37 \\ \hline .54 \end{array}$

Oral 1. $\frac{9}{10} = \frac{9}{10} \times 1 = \frac{9}{10} \times \frac{10}{10} = \frac{9 \times 10}{10 \times 10} = \frac{90}{100} = .90$

2. $\frac{1}{4} = \frac{1}{4} \times 1 = \frac{1}{4} \times \frac{25}{25} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100} = .25$

3. Not until .90 is renamed as .8 + .10 because .05 > 0.

Written Use the method below to solve each of the following open sentences.

$$\frac{\frac{3}{4}}{-\frac{2}{25}} \rightarrow \frac{.75}{-.08} \text{ or } \frac{67}{100}$$

- | | |
|--|--|
| $\begin{array}{l} a \\ 1. \frac{1}{2} - \frac{1}{25} = a \end{array}$ $\begin{array}{l} b \\ 2. \frac{3}{5} - \frac{1}{4} = b \end{array}$ $\begin{array}{l} c \\ 3. \frac{4}{5} - \frac{1}{2} = c \end{array}$ $\begin{array}{l} d \\ 4. \frac{3}{4} - \frac{7}{25} = d \end{array}$ $\begin{array}{l} e \\ 5. \frac{4}{5} - \frac{1}{4} = e \end{array}$ | $\begin{array}{l} f \\ 1. \frac{1}{4} - \frac{1}{5} = f \end{array}$ $\begin{array}{l} g \\ 2. \frac{1}{5} - \frac{2}{25} = g \end{array}$ $\begin{array}{l} h \\ 3. \frac{3}{4} - \frac{1}{5} = h \end{array}$ $\begin{array}{l} i \\ 4. \frac{4}{5} - \frac{6}{25} = i \end{array}$ $\begin{array}{l} j \\ 5. \frac{1}{2} - \frac{1}{4} = j \end{array}$ |
|--|--|

Copy. Find each difference.

	a	b	c	d
6.	$\begin{array}{r} .8 \\ - .3 \\ \hline .5 \end{array}$	$\begin{array}{r} .12 \\ - .04 \\ \hline .08 \end{array}$	$\begin{array}{r} .7 \\ - .6 \\ \hline .1 \end{array}$	$\begin{array}{r} .82 \\ - .14 \\ \hline .68 \end{array}$
7.	$\begin{array}{r} .43 \\ - .39 \\ \hline .04 \end{array}$	$\begin{array}{r} .66 \\ - .34 \\ \hline .32 \end{array}$	$\begin{array}{r} .9 \\ - .8 \\ \hline .1 \end{array}$	$\begin{array}{r} .31 \\ - .19 \\ \hline .12 \end{array}$
8.	$\begin{array}{r} .18 \\ - .02 \\ \hline .16 \end{array}$	$\begin{array}{r} 1.8 \\ - .6 \\ \hline 1.2 \end{array}$	$\begin{array}{r} 6.08 \\ - .91 \\ \hline 5.17 \end{array}$	$\begin{array}{r} 12.06 \\ - 1.26 \\ \hline 10.80 \end{array}$
9.	$\begin{array}{r} .7 \\ - .2 \\ \hline .5 \end{array}$	$\begin{array}{r} 6.30 \\ - .41 \\ \hline 5.89 \end{array}$	$\begin{array}{r} 7.15 \\ - 1.08 \\ \hline 6.07 \end{array}$	$\begin{array}{r} 141.6 \\ - 68.3 \\ \hline 73.3 \end{array}$

Can you do this? Name the next three decimals in the sequence below.

7.21, 6.14, 5.07, 4.00, 2.93, 1.86, .79

Tell how How would you show that $3 - .49 = 2.51$? $3.00 - .49 = 2.51$

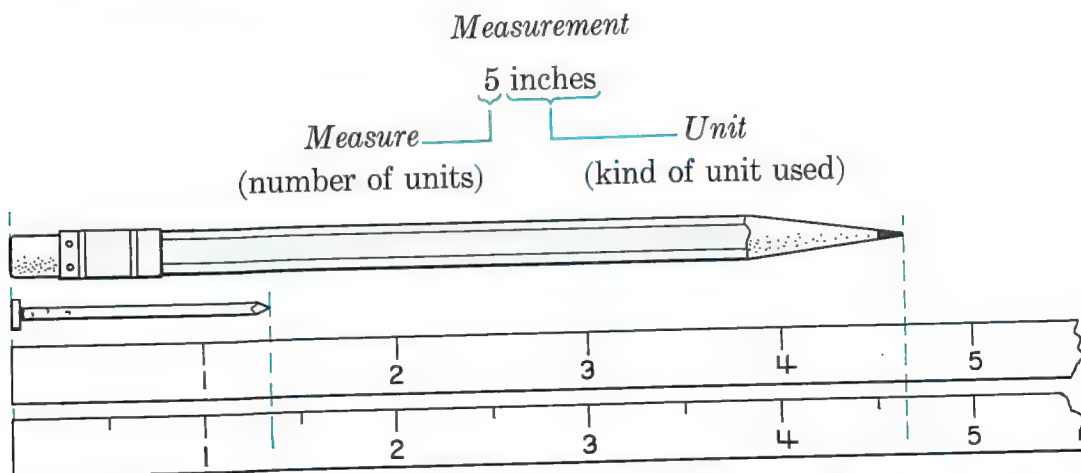
MORE PRACTICE

PAGE 329

Measurement

The distance from one mark to the next on the first ruler below is called an **inch**. An *inch* is called a **unit of measure**. To find the length of the pencil below, you could count the number of units needed to reach from one end of the pencil to the other. That number of units is called the **measure**.

Is the length of the pencil an exact number of units? ^{No} Why or why not? ^{*1} Is the inch measure of the pencil nearer to 4 or to 5? ⁵ You can say that the length of the pencil is 5 inches, to the *nearest inch*. The expression 5 inches is called a **measurement**.



The unit on the second ruler is $\frac{1}{2}$ inch. How can you tell? ^{*2} Is the length of the nail nearer to 2 or to 3 units? ^{*3} Is the half-inch measure of the nail nearer to 2 or to 3? ^{*3} Instead of giving the measurement as 3 half-inches, we usually give the measurement in inches.

$$3 \times \frac{1}{2} = \frac{3}{2} = 1\frac{1}{2}$$

The length of the nail is $1\frac{1}{2}$ inches, to the nearest $\frac{1}{2}$ inch.

Neither of the measurements 5 inches or $1\frac{1}{2}$ inches is exact, only to the nearest unit. All measurements are approximate.

*1 The tip of the lead does not end on a unit mark.

*2 It is marked in units of half inches.

Oral Use the drawing on page 246 to answer the following.

Written Using a ruler, measure each of the following line segments to the nearest inch. Then measure each of them to the nearest $\frac{1}{2}$ inch. Record each measurement.

1. To the nearest half inch, what is the length of the pencil? How did you decide on your answer? $4\frac{1}{2}$ inches; nearer to $4\frac{1}{2}$ than to 5 units

2. Which measurement better describes the length of the pencil, 5 inches or $4\frac{1}{2}$ inches? Why? $4\frac{1}{2}$ inches; nearer to $4\frac{1}{2}$ than to 5 inches

3. To the nearest inch, what is the length of the nail? 1 inch

4. Which measurement better describes the length of the nail, 1 inch or $1\frac{1}{2}$ inches? Why? $1\frac{1}{2}$ inches; nearer to $1\frac{1}{2}$ than 1 unit

5. In the measurement 5 inches, what does the 5 mean? The 5 means "the number of units."

6. Which of the following does the 5 in 5 inches name? a

- The measure
- The unit of measure
- The measurement

7. Which of the following does the word *inches* in 5 inches name? b

- The measure
- The unit of measure
- The measurement

8. Which of the following does the expression 5 inches name? c

- The measure
- The unit of measure
- The measurement

1. 2 inches; 2 inches

2. 2 inches; 2 inches

3. 1 inch; 1 inch

4. 2 inches; $1\frac{1}{2}$ inch

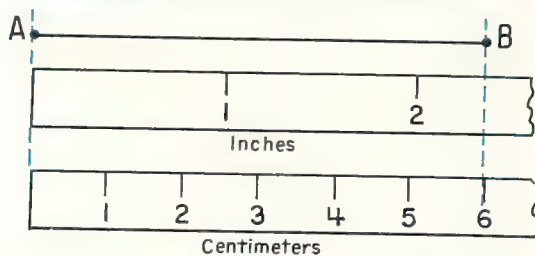
5. 1 inch; $1\frac{1}{2}$ inch

Using a ruler, draw a line segment that is described by each of the following measurements.

	a	b	c
6.	2 inches	$1\frac{1}{2}$ inches	$3\frac{1}{2}$ inches

7.	$\frac{1}{2}$ inch	1 inch	4 inches
----	--------------------	--------	----------

Tell why Study the picture below.



Yes

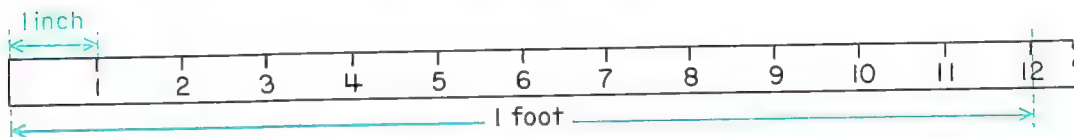
Is 1 cm. smaller than 1 in.? If two measurements are given for a line segment, the measurement made with a smaller unit is more *precise*. Which measurement of line segment AB is more precise? Why? 6 centimeters; it is made with the smaller unit.

Units of Length

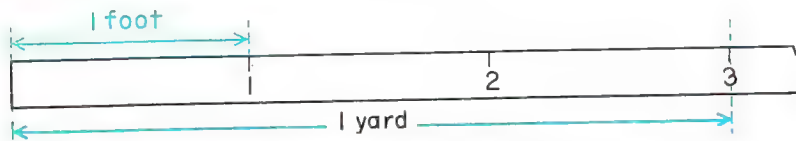
People have agreed that a segment like _____ has a length of **1 inch**. The actual length of the unit is not important. What is important is that we all agree to mean the *same* length when we say *1 inch*.

Would it be easy or convenient to find the length of a telephone pole in inches? ^{*1}Why or why not? ^{*2}To make it easier or more convenient to find such lengths, people have agreed to use other units of length.

If 12 segments, each 1 inch long, were placed end-to-end, the new unit is called **1 foot**. There is not room enough to show a unit of 1 foot on this page, so the drawing below shows only that 1 foot is 12 times as long as 1 inch.



If 3 segments, each 1 foot long, were placed end-to-end, the new unit is called **1 yard**. The drawing below shows only that 1 yard is 3 times as long as 1 foot.



You will often see expressions like $12 \text{ inches} = 1 \text{ foot}$. This does not mean that a length of 12 inches, to the nearest inch, is the same as a length of 1 foot, to the nearest foot. It means that 1 foot is 12 times as long as 1 inch. In other words, *if* we could measure exactly, then 12 inches and 1 foot would be the same length.

$12 \text{ inches (in.)} = 1 \text{ foot (ft.)}$
$3 \text{ feet (ft.)} = 1 \text{ yard (yd.)}$
$36 \text{ inches (in.)} = 1 \text{ yard (yd.)}$
$5280 \text{ feet (ft.)} = 1 \text{ mile (mi.)}$

Oral Is the *inch*, the *foot*, or the *yard*, most convenient for measuring each of the following?

1. The width of a sheet of paper **inch** ^{2 inches}
2. The length of a pencil **inch**
3. The height of a boy **inch or foot**
4. The height of a tree **foot or yard** ^{1 foot}
5. The length of a football field **yard** ^{2 inches}
6. The length of a city block **yard** ^{3 inches}
7. The length of a soda straw **inch**

Which of the following words makes each sentence below most reasonable? (inches, feet, yards, or miles)

8. Some mathematics books are about 8 ____ long. **inches**
9. Ed ran 100 ____ in 14 seconds. **yards**
10. Most classrooms are at least 20 ____ wide. **feet**
11. Some driveways are about 12 ____ wide. **feet**
12. Some pocket knives are about 3 ____ long. **inches**
13. It is about 12 ____ from Brownsville to Uniontown. **miles**
14. Some houses are about 30 ____ tall. **feet**
15. A football field is about 57 ____ wide. **yards**

Answer the following.

16. Which is longer: 1 foot or 14 inches? How much longer? **14 inches;** ^{2 inches}
17. Which is longer: 2 feet or 1 yard? How much longer? **1 yard;** ^{1 foot}
18. Which is longer: 10 inches or 1 foot? How much longer? **1 foot;** ^{2 inches}
19. Which is longer: 1 yard or 39 inches? How much longer? **39 inches;** ^{3 inches}

Written Use a ruler to measure the objects listed below. Make all measurements to the nearest inch. Record each measurement.

1. The width of your mathematics book **Accept either 7 or 8 inches depending upon the pupil's judgment.**
2. The length of a sheet of writing paper **Answers may vary.**

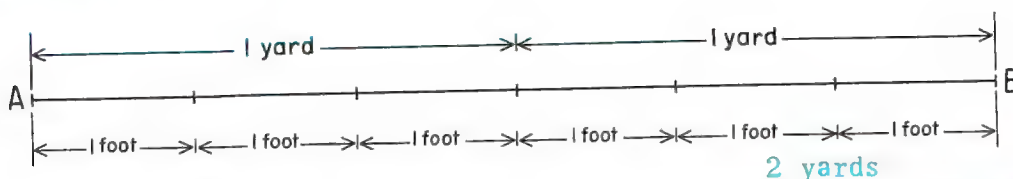
3. The length of a new pencil **Answers may vary.**

Replace each ____ with a numeral to make each sentence true.

- | <i>a</i> | <i>b</i> |
|---|--------------------------------------|
| 4. 1 ft. 1 in. = ¹³ ____ in. | 1 ft. 3 in. = ¹⁵ ____ in. |
| 5. 1 yd. 1 ft. = ⁴ ____ ft. | 1 yd. 2 ft. = ⁵ ____ ft. |
| 6. 1 ft. 6 in. = ¹⁸ ____ in. | 1 ft. 9 in. = ²¹ ____ in. |
| 7. 1 yd. 3 ft. = ⁶ ____ ft. | 2 yd. 1 ft. = ⁷ ____ ft. |
| 8. 1 ft. 5 in. = ¹⁷ ____ in. | 1 ft. 7 in. = ¹⁹ ____ in. |
| 9. 2 yd. 2 ft. = ⁸ ____ ft. | 2 ft. 2 in. = ²⁶ ____ in. |

For TEACHING HELPS, see page T249.

Renaming a Length



What is the length of line segment AB in yards? In feet? **6 feet**
This is often expressed as follows.

$$2 \text{ yards} = 6 \text{ feet}$$

Does this mean that a length of 2 yards, to the nearest yard, is the same as 6 feet, to the nearest foot? What does it mean? **No; if we could measure exactly, then 2 yards and 6 feet would be the same length.**

You can rename a length without drawing a picture. Look for the pattern in the table below.

yards \rightarrow feet		feet \rightarrow yards
\downarrow		\downarrow
$1 \times 3 = 3$	1 yard = 3 feet	$3 \div 3 = 1$
$2 \times 3 = 6$	2 yards = 6 feet	$6 \div 3 = 2$
$3 \times 3 = 9$	3 yards = 9 feet	$9 \div 3 = 3$
$4 \times 3 = 12$	4 yards = 12 feet	$12 \div 3 = 4$

How would you change 5 yards to a number of feet? How would you change 18 feet to a number of yards? **multiply 3 by 5; divide 18 by 3**

To change a number of yards to a number of feet *multiply* the number of yards by 3.

To change a number of feet to a number of yards *divide* the number of feet by 3.

You know that 1 foot = 12 inches. How would you change 2 feet to a number of inches? How would you change 48 inches to a number of feet? **multiply 12 by 2; divide 48 by 12**

You know that 1 yard = 36 inches. How would you change 2 yards to a number of inches? How would you change 108 inches to a number of yards? **multiply 36 by 2; divide 108 by 36**

Oral Tell the numeral that is missing in each of the sentences below.

- | <i>a</i> | <i>b</i> |
|----------------------------|-------------------------|
| 1. 12 in. = <u>1</u> ft. | 1 ft. = <u>12</u> in. |
| 2. 1 yd. = <u>3</u> ft. | 3 ft. = <u>1</u> yd. |
| 3. 36 in. = <u>1</u> yd. | 1 yd. = <u>36</u> in. |
| 4. 5280 ft. = <u>1</u> mi. | 1 mi. = <u>5280</u> ft. |

Answer the following.

5. In renaming a number of yards as a number of feet, do you multiply or divide by 3? **multiply**

6. In renaming a number of feet as a number of yards, you divide the number of feet by what number? **3**

7. In renaming a number of inches as a number of feet, do you multiply or divide by 12? **divide**

8. In renaming a number of feet as a number of inches, you multiply by what number? **12**

9. In renaming a number of yards as a number of inches, do you multiply or divide by 36? **multiply**

10. In renaming a number of inches as a number of yards, you divide by what number? **36**

11. You know that 1 mile = 5280 feet. How would you change 2 miles to a number of feet? **multiply 5280 by 2**

Written Copy. Find the numeral that would make each sentence true.

- | <i>a</i> | <i>b</i> |
|------------------------------|------------------------|
| 1. 5 ft. = <u>60</u> in. | 24 in. = <u>2</u> ft. |
| 2. 3 ft. = <u>36</u> in. | 15 ft. = <u>5</u> yd. |
| 3. 4 yd. = <u>12</u> ft. | 60 in. = <u>5</u> ft. |
| 4. 4 yd. = <u>144</u> in. | 72 in. = <u>2</u> yd. |
| 5. 2 yd. = <u>72</u> in. | 21 ft. = <u>7</u> yd. |
| 6. 5 yd. = <u>15</u> ft. | 36 in. = <u>3</u> ft. |
| 7. 3 mi. = <u>15,840</u> ft. | 24 ft. = <u>8</u> yd. |
| 8. 5 mi. = <u>26,400</u> ft. | 144 in. = <u>4</u> yd. |
| 9. 6 ft. = <u>72</u> in. | 12 ft. = <u>4</u> yd. |
| 10. 100 yd. = <u>300</u> ft. | 180 in. = <u>5</u> yd. |

Can you do this? Find and record the width of your classroom to the nearest foot. **Answers will vary.**

Find and record the width of your classroom to the nearest yard. **Answers will vary.**

Which of the measurements better describes the width of your classroom? Which measurement is more precise? Why? **See below.**

Tell why You know that 12 inches = 1 foot. Does 1 inch = $\frac{1}{12}$ foot? Why? Does 6 inches = $\frac{1}{2}$ foot? Why? **Yes; $1 \div 12 = \frac{1}{12}$; Yes; $6 \div 12 = \frac{6}{12}$ or $\frac{1}{2}$**

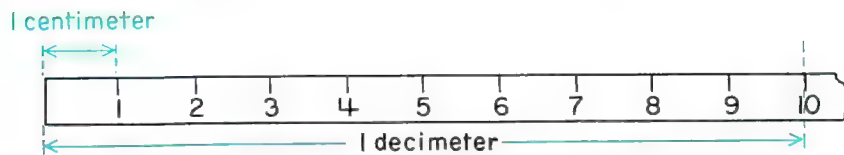
Can you do this to the nearest foot; measurement in feet; it uses the smaller unit.

Metric Units of Length

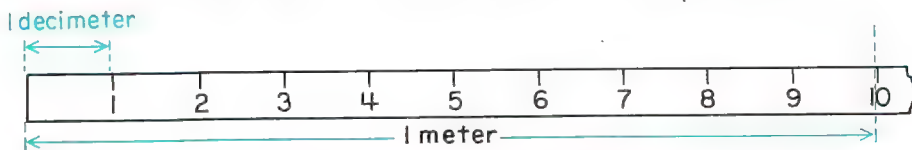
In our system of measurement, you use the inch, the foot, and the yard as units of length. Most people in the world use a different system of measurement. It is called the **metric system**. In the metric system, each unit is ten times as large as some other unit. How is the metric system like place value in decimal numerals?

People have agreed that a segment like _____ has a length of **1 centimeter**. It is one of the metric units of length.

If 10 segments, each 1 centimeter long, were placed end-to-end, the new unit is called **1 decimeter**. The drawing below shows that 1 decimeter is ten times as long as 1 centimeter.



If 10 segments, each 1 decimeter long, were placed end-to-end, the new unit is called **1 meter**. There is not room enough to show a unit of 1 meter on this page, so the drawing below shows only that 1 meter is ten times as long as 1 decimeter.



10 centimeters (cm.) = 1 decimeter (dm.)
10 decimeters (dm.) = 1 meter (m.)
100 centimeters (cm.) = 1 meter (m.)

You know that 10 cm. = 1 dm. How would you change 20 decimeters to a number of centimeters? How would you change 30 centimeters to a number of decimeters? **multiply 10 by 20; divide 30 by 10**

Oral Is the centimeter or the meter more convenient for measuring each of the following?

1. The length of a sheet of paper
centimeter
2. The length of a football field
meter
3. The length of a crayon
centimeter
4. The length of a telephone pole
meter

Which of the following words would make each sentence reasonable? (centimeters, decimeters, or meters)

5. Some paper clips are about 3 ___ long. **centimeters**
6. A new pencil is about 18 ___ long. **centimeters**
7. Some mathematics books are about $2\frac{1}{2}$ ___ long. **decimeters**

8. Some city blocks are about 400 ___ long. **meters**

Tell the numeral that is missing in each of the sentences below.

- | <i>a</i> | <i>b</i> |
|-----------------------------|-----------------------|
| 9. 100 cm. = <u>1</u> m. | 2 m. = <u>200</u> cm. |
| 10. 1 dm. = <u>10</u> cm. | 2 dm. = <u>20</u> cm. |
| 11. 1 m. = <u>10</u> dm. | 2 m. = <u>20</u> dm. |
| 12. 3 m. = <u>300</u> cm. | 3 m. = <u>30</u> dm. |
| 13. 200 cm. = <u>20</u> dm. | 4 m. = <u>400</u> cm. |

Answer the following.

14. In renaming a number of decimeters as a number of meters, do you multiply or divide by 10?
divide

15. In renaming a number of meters as a number of decimeters, you multiply the number of meters by what number? **10**

16. In renaming a number of centimeters as a number of meters, do you multiply or divide by 100?
divide

17. In renaming a number of meters as a number of centimeters, you multiply the number of meters by what number? **100**

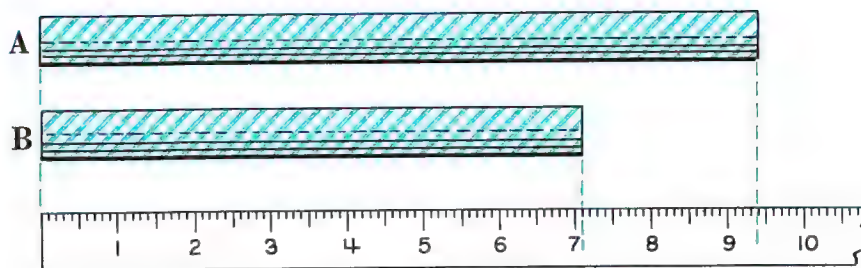
Written Copy. Find the numeral that would make each sentence true.

- | <i>a</i> | <i>b</i> |
|--------------------------|-------------------------|
| 1. 3 m. = <u>300</u> cm. | 60 dm. = <u>6</u> m. |
| 2. 3 m. = <u>30</u> dm. | 400 cm. = <u>4</u> m. |
| 3. 5 dm. = <u>50</u> cm. | 400 cm. = <u>40</u> dm. |
| 4. 5 m. = <u>50</u> dm. | 500 cm. = <u>5</u> m. |
| 5. 6 dm. = <u>60</u> cm. | 80 dm. = <u>8</u> m. |

Tell why You know that 100 cm. = 1 m. Does 1 cm. = $\frac{1}{100}$ m.? Why?
Yes; $1 \div 100 = \frac{1}{100}$

Tell how How could you discover that 1 inch and $2\frac{1}{2}$ centimeters are about the same length? **compare the units**

Adding and Subtracting Measures



Each centimeter on the ruler above is separated into how many parts of the same size? Can you use the ruler to measure the length of each piece of candy to the nearest $\frac{1}{10}$ of a centimeter? 10; Yes

Using decimals, how would you record the length of the piece of candy in A? The length of the piece of candy in B? 9.4 cm.; 7.1 cm.

Are both pieces of candy measured to the nearest $\frac{1}{10}$ of a centimeter? Yes

To find the combined length of the two pieces of candy, what open sentence would you solve? You can solve $9.4 + 7.1 = a$ as shown in C. $9.4 + 7.1 = \square$

C	D
$\begin{array}{r} 9.4 \\ +7.1 \\ \hline 16.5 \end{array}$	$\begin{array}{r} 9.4 \\ -7.1 \\ \hline 2.3 \end{array}$

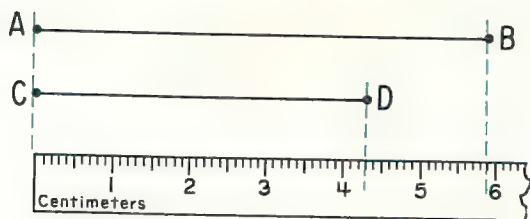
To find how much longer one piece of candy is than the other, you can subtract as shown in D.

To add or subtract measures, the measurements should all be made by using the same unit of measure.

Are 9.4 and 7.1 measures? Are both measures obtained using the same unit of measure? Which unit of measure is that? Yes; Yes; centimeter

What is the combined length of the two pieces of candy? The piece of candy in A is how much longer than the piece of candy in B? 16.5 cm.; 2.3 cm.

Oral Use the picture below to help you answer the following.



1. Using decimals, how would you record the length of line segment AB to the nearest tenth of a centimeter? **5.9 cm.**

2. Using decimals, how would you record the length of line segment CD to the nearest tenth of a centimeter? **4.3 cm.**

3. In recording the measurements in Oral 1-2, which unit of measure did you use? **centimeters**

4. What open sentence would you solve to find the combined length of line segments AB and CD?
 $5.9 + 4.3 = \square$

5. What is the combined length of the two line segments? **10.2 cm.**

6. What open sentence would you solve to find the difference in their lengths? **$5.9 - 4.3 = \square$**

7. How much longer is line segment AB than line segment CD?
1.6 cm.

8. To add or subtract measures, what must you remember when making the measurements? **Use the same unit of measurement.**

Written Find the combined length of two line segments whose lengths are given in each pair of measurements below. Then find the difference between their lengths.

1. 13.3 inches 5.2 inches
18.5 in.; 8.1 in.
2. 18.5 miles 9.6 miles
28.1 mi.; 8.9 mi.
3. 4.05 feet 2.14 feet
6.19 ft.; 1.91 ft.
4. 9.29 yards 6.35 yards
15.64 yds.; 2.94 yds.
5. 6.8 inches 2.9 inches
9.7 in.; 3.9 in.
6. 32.5 miles 16.4 miles
48.9 mi.; 16.1 mi.
7. 124.2 feet 85.7 feet
209.9 ft.; 38.5 ft.
8. 6.8 yards 2.7 yards
9.5 yds.; 4.1 yds.
9. 8.75 inches 4.20 inches
12.95 in.; 4.55 in.
10. 11.72 inches 8.32 inches
20.04 in.; 3.40 in.
11. 12.4 meters 3.8 meters
16.2 m.; 8.6 m.

Can you do this? Add in each row. Add in each column. Add along the blue line. Add along the gray line.

4.00	2.75	3.00
2.25	3.25	4.25
3.50	3.75	2.50

What did you discover about all eight sums? Why is this array called a magic square? **Each sum is 9.75; because each sum is the same**

Money



How much money is pictured above? Three ways to express this are shown below.

314¢

\$3.14

3.14 dollars

314 cents 3 dollars and 14 cents 3 and 14 hundredths dollars

Are 14 cents and 14 hundredths dollars two different names for the same amount of money? Would you say that the following sentence is true or false? **Yes; True**

One cent is equivalent to $\frac{1}{100}$ or .01 dollar.

What you have learned about decimals can be used to solve problems involving money.

Oral Tell two other ways to express each of the following. **See below for typical answers**

Written Find each sum or difference below.

	a	b	c
1.	16¢	\$4.10	8.24 dollars
2.	25¢	\$3.32	.35 dollars
3.	8¢	\$5.59	2.50 dollars
4.	62¢	\$9.95	4.82 dollars
5.	5¢	\$6.25	.95 dollars
6.	90¢	\$2.15	1.49 dollars

	a	b	c	d
1.	\$2.98 +1.42 <u>\$4.40</u>	\$3.06 +2.95 <u>\$6.01</u>	\$6.00 -5.32 <u>\$.68</u>	\$10.00 -5.98 <u>\$4.02</u>
2.	\$4.32 +1.91 <u>\$6.23</u>	\$6.81 +3.49 <u>\$10.30</u>	\$5.00 -1.49 <u>\$3.51</u>	\$3.00 -2.21 <u>\$.79</u>

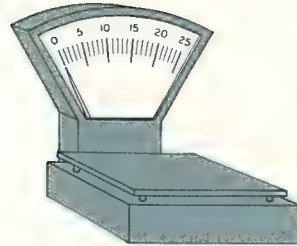
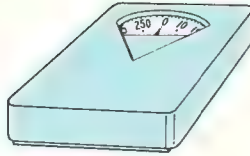
Can you do this? A set of coins is worth \$1.00. It contains some pennies, nickels, dimes, and quarters. What is the least number of coins that this set could contain? **11 (3 quarters, 1 dime, 2 nickels, and 5 pennies)**

256

Oral

	a	b	c
1.	\$.16, .16 dollars	410¢, 4.10 dollars	824¢, \$8.24
2.	\$.25, .25 dollars	332¢, 3.32 dollars	35¢, \$.35
3.	\$.08, .08 dollars	559¢, 5.59 dollars	250¢, \$2.50

Weight



One of the properties that people and objects have is called **weight**. *Weight* is caused by the pull of gravity on an object. Certain instruments, like those shown above, have been invented to measure weight. These instruments make use of certain units of weight which have been agreed upon by the people.

Some of these units of weight are shown below.

16 ounces (oz.) = 1 pound (lb.)
2000 pounds (lb.) = 1 ton (T.)

Name the different units of weight shown in the table.

You know that 1 lb. = 16 oz. How would you change 2 pounds to a number of ounces? How would you change 48 ounces to a number of pounds? **multiply 16 by 2; divide 48 by 16**

Oral Which of the units (ounces, pounds, or tons) would you use to describe the weight of each of the following?

1. A person **pounds**
2. A truck load of coal **tons**
3. A slice of bread **ounces**
4. A bag of apples **pounds**

Written Copy. Replace each with a numeral that makes each sentence true.

- | | |
|--|---|
| <p style="text-align: center;"><i>a</i></p> <ol style="list-style-type: none"> 1. 4 lb. = ⁶⁴ oz. 2. 3 T. = ⁶⁰⁰⁰ lb. 3. $\frac{1}{2}$ lb. = ⁸ oz. 4. 5 T. = ^{10,000} lb. | <p style="text-align: center;"><i>b</i></p> <ol style="list-style-type: none"> 32 oz. = ² lb. 4000 lb. = ² T. 4 oz. = ^{$\frac{1}{4}$} lb. 1000 lb. = ^{$\frac{1}{2}$} T. |
|--|---|

Capacity

When you find the amount of liquid or grain in a certain container you are finding capacity. To do this you will use certain units of measure. You will use one set of units for liquids and a different set of units for grain. Some of these units are shown in the tables below.

Liquid Measure	Dry Measure
16 fluid ounces (fl. oz.) = 1 pint (pt.)	2 pints (pt.) = 1 quart (qt.)
2 cups (c.) = 1 pint (pt.)	8 quarts (qt.) = 1 peck (pk.)
2 pints (pt.) = 1 quart (qt.)	4 pecks (pk.) = 1 bushel (bu.)
4 quarts (qt.) = 1 gallon (gal.)	

Name the different units of measure that you can use to measure an amount of liquid; that you can use to measure an amount of grain.

Oral Which unit of liquid measure (fluid ounce, cup, pint, quart, or gallon) would you use to measure each of the following?

1. The gasoline in a tank **gallon**
2. The liquid in a glass **fluid ounce**
3. The oil in an engine **quart**

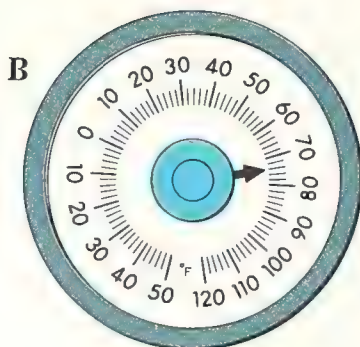
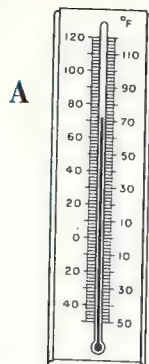
Tell the missing numeral in each of the following.

4. 2 pecks = 16 quarts
5. 32 quarts = 4 pecks
6. 3 quarts = 6 pints
7. 8 pecks = 2 bushels
8. 16 quarts = 2 pecks

Written Copy. Replace each with a numeral that makes each sentence true.

- | | |
|------------------------------|---------------------------|
| a | b |
| 1. 2 pt. = <u>32</u> fl. oz. | 48 fl. oz. = <u>3</u> pt. |
| 2. 3 pt. = <u>6</u> c. | 8 c. = <u>4</u> pt. |
| 3. 4 qt. = <u>8</u> pt. | 4 pt. = <u>2</u> qt. |
| 4. 2 gal. = <u>8</u> qt. | 12 qt. = <u>3</u> gal. |
| 5. 3 pk. = <u>24</u> qt. | 4 qt. = <u>1/2</u> pk. |
| 6. 3 bu. = <u>12</u> pk. | 12 pk. = <u>3</u> bu. |
| 7. 5 gal. = <u>20</u> qt. | 16 qt. = <u>4</u> gal. |
| 8. 6 pt. = <u>96</u> fl. oz. | 64 fl. oz. = <u>4</u> pt. |

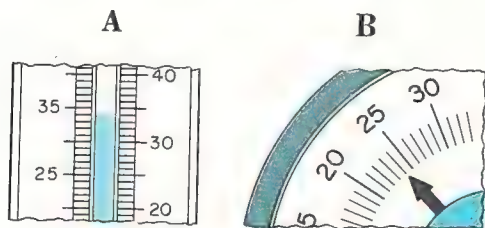
Temperature



Temperature is measured with an instrument called a thermometer. In the thermometer shown in **A**, a column of mercury or some other liquid rises and falls with changing temperature.

In the thermometer shown in **B**, strips of two metals are joined together to make but one strip. As the temperature changes, one metal expands faster than the other and causes the strip to bend, which in turn moves a pointer on the thermometer dial. This type is used in some desk or wall thermometers and in thermostats. How would you record the temperature shown in **B**? 74°

Oral Use the thermometers shown below to answer the following.

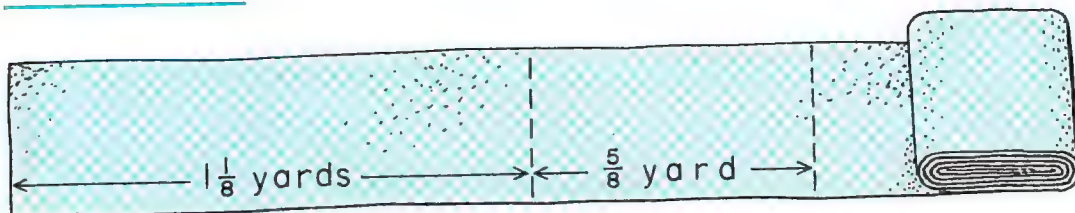


What temperature is shown on the thermometer in **A**? In **B**? Which thermometer shows the higher temperature? How much higher? 34° ; 24° ; **A**; 10°

Written Solve each problem below.

1. The temperature at 7 o'clock in the morning was 19° . By noon the temperature went up 13° . What was the temperature at noon? $19+13=\square$; 32°
2. The high temperature on Friday was 83° . The low temperature was 54° . By how many degrees did the temperature vary that day? $83-54=\square$; 29°
3. The high and low temperatures were 54° and 35° . By how many degrees did the temperature vary? $54-35=\square$; 19°

Problem Solving



Use either fractions or decimals to write an open sentence for each problem below. Solve each open sentence. Answer the problem.

1. Fran is ready to cut the material shown above along the dotted lines. After making the cuts, how long a piece of material will she have used? $1\frac{1}{8} + \frac{5}{8} = \square$; $1\frac{3}{4}$ yds.

2. Ann had a piece of material that was $\frac{3}{4}$ yard long. She used $\frac{1}{2}$ yard of it. How much material does she still have left? $\frac{3}{4} - \frac{1}{2} = \square$ or $.75 - .50 = \square$; $\frac{1}{4}$ yd. or $.25$ yd.

3. Tim walked .8 mile to the store and then walked .4 mile to the school. How far did Tim walk?
See below.

4. Julie walked $\frac{1}{2}$ of a block to Jay's house. Then together they walked $1\frac{1}{2}$ blocks to Jill's house. In all, how far did Julie walk? $\frac{1}{2} + 1\frac{1}{2} = \square$ or $.5 + 1.5 = \square$; 2 blocks

5. Tom has $\frac{1}{2}$ yard of rope. If he uses $\frac{1}{4}$ yards of it, how much rope will he have left? $\frac{1}{2} - \frac{1}{4} = \square$ or $.50 - .25 = \square$; $\frac{1}{4}$ yd. or $.25$ yd.

6. Jean had $2\frac{1}{4}$ yards of material. She bought $\frac{3}{4}$ yards more. In all, how much material does she have?
260 $2\frac{1}{4} + \frac{3}{4} = \square$ or $2.25 + .75 = \square$; 3 yds.

3. $.8 + .4 = \square$ or $\frac{8}{10} + \frac{4}{10} = \square$; 1.2 mi. or $1\frac{1}{5}$ mi.

Can you do this $0.60 = 0 + \frac{60}{100} = \frac{60}{100} = .60$

7. Jim needs $\frac{1}{2}$ yard of rope for his boat and $20\frac{1}{2}$ yards of rope for his tent. How many yards of rope does he need in all? $\frac{1}{2} + 20\frac{1}{2} = \square$ or $.5 + 20.5 = \square$; 21 yds.

8. Mary has $6\frac{1}{2}$ feet of white ribbon and $\frac{1}{2}$ foot of red ribbon. In all, Mary has how much ribbon?
 $6\frac{1}{2} + \frac{1}{2} = \square$ or $6.5 + .5 = \square$; 7 ft.

9. Jane took 1 hour to read a chapter in a book. Sally took $\frac{3}{4}$ hour to read the same chapter. How much longer did Jane take to read the chapter? $1 - \frac{3}{4} = \square$ or $1 - .75 = \square$; $\frac{1}{4}$ hour or $.25$ hour

10. One pine tree is 8.5 feet tall. Another is 6.2 feet tall. One tree is how much shorter than the other?
 $8.5 - 6.2 = \square$; 2.3 ft.

11. Bill rode his bicycle .7 mile to Sally's house, then back home again. How far did he ride his bicycle in making the round trip? $.7 + .7 = \square$ or $\frac{7}{10} + \frac{7}{10} = \square$; 1.4 mi. or $1\frac{2}{5}$ mi.

Can you do this? Show that the fractional number named by .60 is the same as that named by 0.60. See below.

Tell how Tell how you would find the unnamed minuend below.

$$\begin{aligned} n - 8.24 &= 1.91 \\ n &= 1.91 + 8.24 \\ n &= 10.15 \end{aligned}$$

Checkup Time

The numerals in () tell the pages where you can turn for help.

Important Ideas

1. When a fractional number has a denominator of 10 it can be expressed as a decimal. (239)
2. When a fractional number has a denominator of 100 it can be expressed as a decimal. (240)
3. To add numbers named by decimals like .52 and .39, add them as you would whole numbers and place the decimal point. (242)
4. To subtract numbers named by decimals like .48 and .37, subtract them as you would whole numbers and place the decimal point. (244)
5. The important thing about a unit of measure is that everyone means the same thing when they use it. (248)
6. In the metric system, each unit of measure is ten times as large as the next smaller unit of measure. (252)
4. Centimeter, decimeter, meter (252)
5. Weight (257)

Questions to Discuss

See page T261.

1. How would you change $\frac{3}{10}$ to a decimal? (239)
2. How would you change $\frac{7}{100}$ to a decimal? (240)
3. How would you change $\frac{3}{5}$ to a decimal having only one digit to the right of the decimal point? (241)
4. How would you change .60 to a fraction in simplest form? (241)
5. How would you add .42 and .35? (242)
6. How would you subtract .19 from .38? (244)
7. You know that 1 ft.=12 in. How would you change 3 feet to a number of inches? (250)

Words to Know

1. Decimal numerals, decimals, decimal points (239)
2. Unit of measure, measure, measurement (246)
3. Inch, foot, yard (248)

Written Practice

Find each sum or difference in decimal form.

<i>a</i>	<i>b</i>	<i>c</i>
$\begin{array}{r} .44 \\ +.38 \\ \hline .82 \end{array}$	$\begin{array}{r} .9 \\ -.6 \\ \hline .3 \end{array}$	$\begin{array}{r} .82 \\ -.39 \\ \hline .43 \end{array}$

Self-Evaluation

Part 1 Copy. Write the numeral that would make each sentence true.

- | a | b |
|-----------------------------------|-----------------------------|
| 1. 2 yd. = $\frac{6}{1}$ ft. | 9 ft. = $\frac{3}{1}$ yd. |
| 2. 6 in. = $\frac{1}{2}$ ft. | 4 pt. = $\frac{2}{1}$ qt. |
| 3. 32 fl. oz. = $\frac{2}{1}$ pt. | 2000 lb. = $\frac{1}{1}$ T. |
| 4. 16 oz. = $\frac{1}{4}$ lb. | 2 lb. = $\frac{32}{1}$ oz. |

Part 2 Write each fraction or mixed numeral below as a decimal.

- | a | b | c | d |
|----------------------|----------------------|---------------------|------------------------|
| 1. $\frac{8}{10}$.8 | $\frac{4}{100}$.04 | $2\frac{1}{10}$ 2.1 | $6\frac{3}{100}$ 6.03 |
| 2. $\frac{2}{10}$.2 | $\frac{16}{100}$.16 | $3\frac{7}{10}$ 3.7 | $9\frac{89}{100}$ 9.89 |

Write each decimal below as a fraction or a mixed numeral.

- | a | b | c | d |
|------------------------|----------------------|---------------------|---------------------|
| 3. .3 $\frac{3}{10}$ | .11 $\frac{11}{100}$ | 1.1 $1\frac{1}{10}$ | 2.0 $2\frac{0}{10}$ |
| 4. .09 $\frac{9}{100}$ | .37 $\frac{37}{100}$ | 6.9 $6\frac{9}{10}$ | 7.3 $7\frac{3}{10}$ |

Part 3 Change each of the following to a decimal read as hundredths.

- | a | b | c | d |
|-----------------------|--------------------|--------------------|--------------------|
| 1. $\frac{3}{50}$.06 | $\frac{5}{25}$.20 | $\frac{3}{5}$.60 | $\frac{3}{4}$.75 |
| 2. $\frac{4}{5}$.80 | $\frac{8}{25}$.32 | $\frac{7}{50}$.14 | $\frac{1}{4}$.25 |
| 3. $\frac{1}{2}$.50 | $\frac{4}{25}$.16 | $\frac{1}{50}$.02 | $\frac{5}{4}$ 1.25 |

Change each of the following to a fraction in simplest form.

- | a | b | c | d |
|------------------------|--------------------|--------------------|--------------------|
| 4. .40 $\frac{2}{5}$ | .04 $\frac{1}{25}$ | .9 $\frac{9}{10}$ | .12 $\frac{3}{25}$ |
| 5. .85 $\frac{17}{20}$ | .6 $\frac{3}{5}$ | .08 $\frac{2}{25}$ | .25 $\frac{1}{4}$ |

Part 4 Copy. Find each sum.

- | a | b | c | d |
|--|---|--|---|
| 1. $\begin{array}{r} .9 \\ +.2 \\ \hline 1.1 \end{array}$ | $\begin{array}{r} .8 \\ +.6 \\ \hline 1.4 \end{array}$ | $\begin{array}{r} .18 \\ +.21 \\ \hline .39 \end{array}$ | $\begin{array}{r} .84 \\ +6.16 \\ \hline 7.00 \end{array}$ |
| 2. $\begin{array}{r} 1.7 \\ +.6 \\ \hline 2.3 \end{array}$ | $\begin{array}{r} 2.4 \\ +.9 \\ \hline 3.3 \end{array}$ | $\begin{array}{r} .41 \\ +.69 \\ \hline 1.10 \end{array}$ | $\begin{array}{r} 7.32 \\ +.49 \\ \hline 7.81 \end{array}$ |
| 3. $\begin{array}{r} .8 \\ +.8 \\ \hline 1.6 \end{array}$ | $\begin{array}{r} 3.6 \\ +.9 \\ \hline 4.5 \end{array}$ | $\begin{array}{r} 4.88 \\ +.32 \\ \hline 5.20 \end{array}$ | $\begin{array}{r} 2.18 \\ +1.99 \\ \hline 4.17 \end{array}$ |

Part 5 Copy. Find each difference.

- | a | b | c | d |
|---|--|--|--|
| 1. $\begin{array}{r} .9 \\ - .4 \\ \hline .5 \end{array}$ | $\begin{array}{r} 1.8 \\ - .2 \\ \hline 1.6 \end{array}$ | $\begin{array}{r} .48 \\ - .13 \\ \hline .35 \end{array}$ | $\begin{array}{r} 6.32 \\ - .09 \\ \hline 6.23 \end{array}$ |
| 2. $\begin{array}{r} 1.6 \\ - .4 \\ \hline 1.2 \end{array}$ | $\begin{array}{r} 4.3 \\ - .8 \\ \hline 3.5 \end{array}$ | $\begin{array}{r} 3.82 \\ - .91 \\ \hline 2.91 \end{array}$ | $\begin{array}{r} 4.46 \\ - .48 \\ \hline 3.98 \end{array}$ |
| 3. $\begin{array}{r} 2.1 \\ - .8 \\ \hline 1.3 \end{array}$ | $\begin{array}{r} 7.3 \\ - .8 \\ \hline 6.5 \end{array}$ | $\begin{array}{r} 4.39 \\ - 1.21 \\ \hline 3.18 \end{array}$ | $\begin{array}{r} 6.40 \\ - 1.82 \\ \hline 4.58 \end{array}$ |

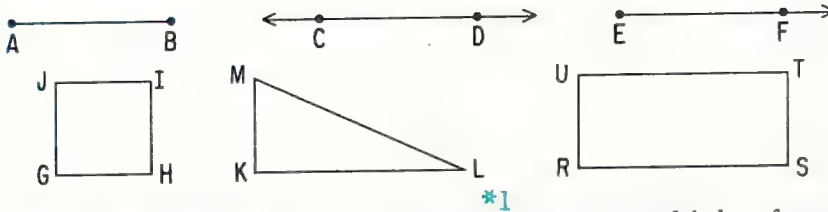
Part 6 Find each sum or difference.

- | a | b | c | d |
|---|---|--|---|
| $\begin{array}{r} \$2.48 \\ +1.21 \\ \hline \$3.69 \end{array}$ | $\begin{array}{r} \$3.62 \\ -1.81 \\ \hline \$1.81 \end{array}$ | $\begin{array}{r} \$2.08 \\ +.86 \\ \hline \$2.94 \end{array}$ | $\begin{array}{r} \$1.96 \\ - .98 \\ \hline \$.98 \end{array}$ |

Chapter 12

METRIC GEOMETRY

Geometric Figures



Name each of the figures above. Can you think of each figure as a set of points? ^{Yes} Such figures are called **geometric figures**.

Geometry is the study of sets of points or geometric figures.

Could you find the length of line segment AB above? To do that you would be finding the measure of a geometric figure. ^{Yes} Could you find the measure of the sides of square GHIJ? Of triangle KLM? Of rectangle RSTU? **Yes; Yes; Yes**

When we use measurement in the study of geometry, we say we are studying **metric geometry**. The word *metric* refers to measurement.

Metric geometry is the study of geometric figures by using measurement.

Oral Use the figures shown above to help you answer each of the following.

1. Tell why you cannot find the length of line CD pictured above.
See below.

2. Tell why you cannot find the length of ray EF pictured above. **The ray extends indefinitely in one direction.**

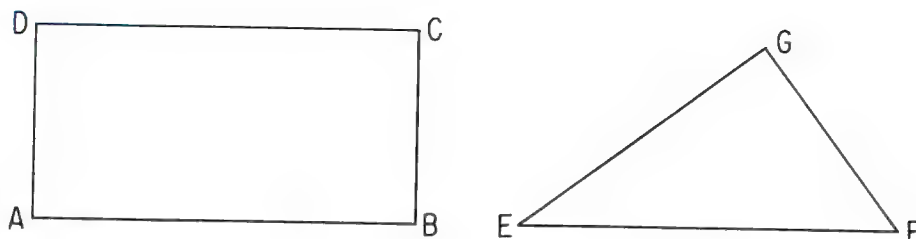
3. Do each of the following figures, square GHIJ, triangle KLM, and rectangle RSTU, separate a plane into three sets of points? **Yes**

4. Name some other geometric figures that separate a plane into three sets of points. **Answers will vary but should include rectangle, triangle, circle, and so on. 263**

*1 line segment AB, line CD, ray EF, square GHIJ, triangle KLM, rectangle RSTU

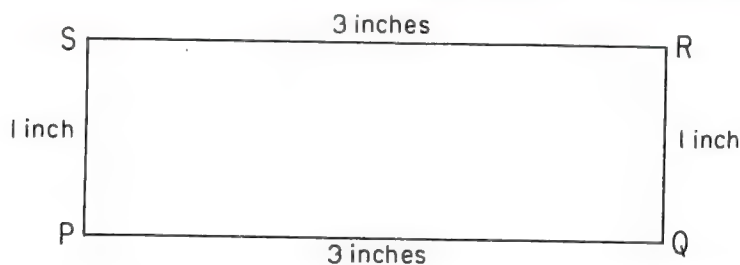
Oral 1. The line extends indefinitely in both directions.

Perimeter



With your finger, start at point A and follow along side AB to point B. Then follow along side BC to point C. Then follow along side CD to point D. Finally, follow along side DA to point A. Are you back to where you started? Did your finger trace the *distance around* rectangle ABCD? **Yes; Yes**

The *distance around* a simple closed figure is called its **perimeter**.



Let p stand for the number of inches in the perimeter of rectangle PQRS. Then you could use any of the open sentences below to find its perimeter.

$$p = 3 + 1 + 3 + 1$$

$$p = 2 \times (3 + 1)$$

$$p = (2 \times 3) + (2 \times 1)$$

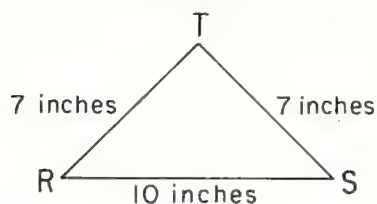
Tell how you think about the rectangle to get each open sentence above. What is its perimeter? **8 inches**

Find the length of each side of triangle EFG to the nearest centimeter.^{*1} Explain how you would find its perimeter. **$p = 5 + 3 + 4$**

To find the *perimeter* of a simple closed figure, add the measures of its sides and record the measurement. The measures of the sides should be found by using the same unit.

*1 EF=5 cm., FG=3 cm., and GE=4 cm.

Oral Use the figure below to answer the following.



1. Is figure RST a simple closed figure? Why or why not? **Yes; definition of simple closed figure**

2. Is figure RST a triangle, a square, or a rectangle? How do you know? **triangle; definition of triangle**

3. How many sides does figure RST have? Name them. **three; sides RS (or SR), ST (or TS), TR (or RT)**

4. Is the length of each side labeled on the figure? Read the measurement of side RS. Of side ST. Of side TR. **Yes; 10 inches; 7 inches; 7 inches**

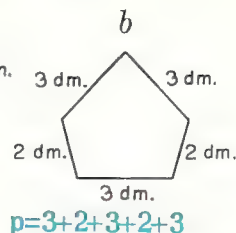
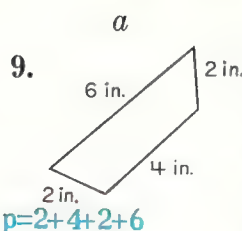
5. In recording the lengths of sides RS, ST, and TR, what is the unit of measure in each measurement? Is the unit the same for each measurement? **inch; Yes**

6. Do you add, subtract, multiply, or divide the measures to find the perimeter of a triangle? **add**

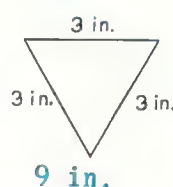
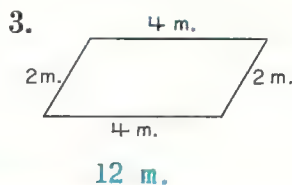
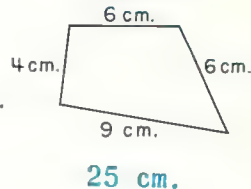
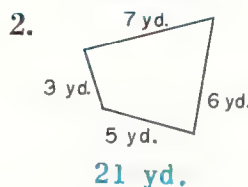
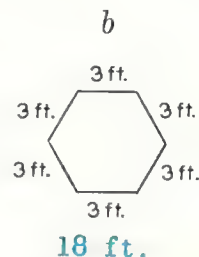
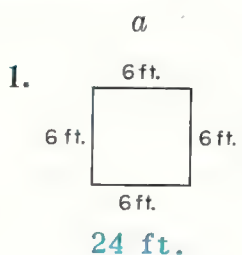
7. What open addition sentence can you use to find the perimeter of triangle RST? **$p=10+7+7$**

8. How would you record the perimeter of triangle RST? **24 inches**

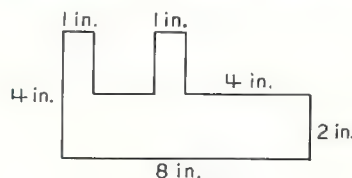
Tell how you would find the perimeter of each figure below.



Written Find the perimeter of each figure below.

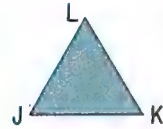
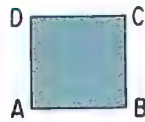


Can you do this? Find the perimeter of the figure below. **28 in.**



Area

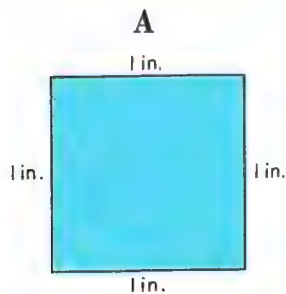
Are the figures below simple closed figures? How did you decide on your answer? **Yes; definition of a simple closed figure**



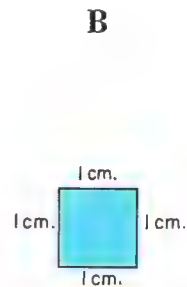
You can think of each figure as the boundary of its interior. Since the interior of any simple closed figure is only part of a plane, you can measure the interior.

The measurement of the interior of a simple closed figure is called the **area** of the figure.

To find the length of a line segment we agreed to use some other line segment as a unit of measure. In the same way, we can agree to use the interior of some simple closed figure as a unit of area measure. Two such units of area measure are shown below.



1 square inch or 1 sq. in.

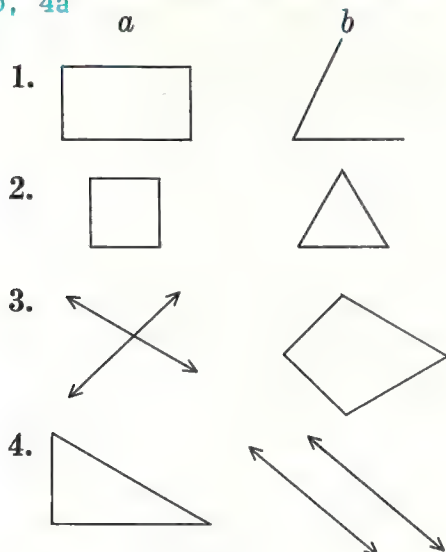


1 square centimeter or 1 sq. cm.
*1

Why do you think the unit in **A** is called a square inch? Why do you think the unit in **B** is called a square centimeter? **It is a square with sides each 1 cm. long.**

What name would you use for the unit of area formed by a square with each side 1 foot long? With each side 1 yard long? With each side 1 meter long? **square foot; square yard; square meter**

Oral Tell which of the following figures have an area. 1a, 2a, 2b, 3b, 4a



Answer the following.

5. Which of the following do we most often use as a unit of area measure? **c**

a. b. c.

6. How long is each side of a square inch? **1 inch**

7. How long is each side of a square centimeter? **1 centimeter**

8. How long is each side of a square foot? A square yard? **1 foot; 1 yard**

9. How long is each side of a square decimeter? **1 decimeter**

10. How long is each side of a square meter? **1 meter**

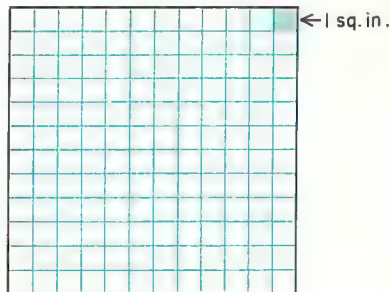
Written Do the following.

1. Draw a square having sides 1 inch long on posterboard or some stiff paper. Use a paper model of a right angle to make sure that the angles are right angles. Cut out the square. For what unit of area measure did you make a model?

square inch

2. Draw a square having sides 12 inches or 1 foot long on posterboard or some stiff paper. Use a paper model of a right angle to make sure that the four angles are all right angles. Cut out the square. For what unit of area measure did you make a model? **square foot**

3. To form the unit called 1 foot, you could place 12 segments, each 1 inch long, end-to-end. The drawing below shows how many square inches it takes to form 1 square foot.



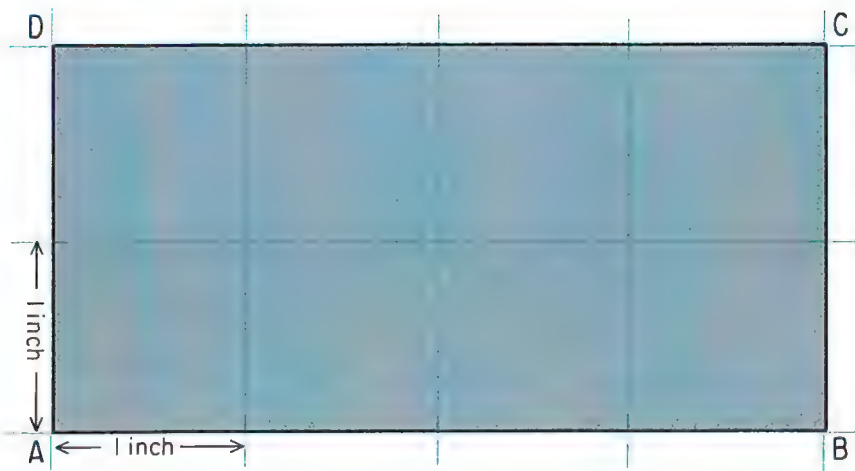
How many square inches are contained in 1 square foot? **144**

Tell how How could you determine how many square centimeters are contained in 1 square decimeter?

See below.

Tell how Repeat an activity similar to the one illustrated in **Written 3**, except begin by placing 10 segments, each 1 cm. long, end-to-end.

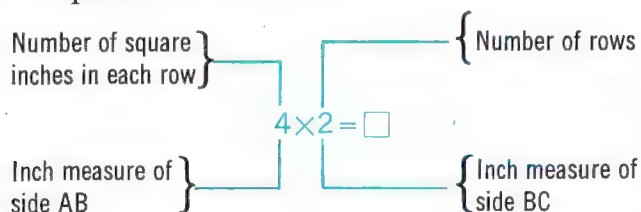
Area of a Rectangle



Each small square in the drawing above stands for which unit of area measure?^{*1} Count the small squares in rectangle ABCD. How many are there? What is the area of rectangle ABCD? 8; 8 square inches

When we say *the area of a rectangle* we mean *the area of the interior of the rectangle*.

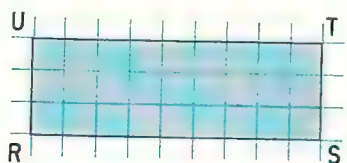
You can find the area of rectangle ABCD without drawing a picture. How many small squares are there in each row?⁴ How many rows of small squares are there?² You can express this as an open multiplication sentence.



What numeral should replace \square in $4 \times 2 = \square$? What is the area of rectangle ABCD? 8; 8 square inches

To find the *area measure* of a rectangle, multiply the measure of the length by the measure of the width. Both measures should be found by using the same unit.

Oral Use the figure below to answer the following.



1. The interior of the rectangle has been separated into small squares. Each small square stands for 1 square foot. Count the small squares in the rectangle. How many are there? **27**

2. How would you record the area of rectangle RSTU? **27 sq. ft.**

3. How many small squares are there in each row? **9**

4. How many rows of small squares are there? **3**

5. Can you say that there are 9×3 square feet in the interior of the rectangle? Why? **Yes; $27 = 9 \times 3$**

6. How long is side RS? **9 ft.**

7. How long is side ST? **3 ft.**

8. What unit of measure was used to find the length of side RS? Of side ST? **foot; foot**

9. What open multiplication sentence can you use to find the area measure of rectangle RSTU? **$9 \times 3 = \square$**

10. What is the area of rectangle RSTU? **27 sq. ft.**

Answer the following.

11. Tell how you would find the area of a rectangle whose length is 7 feet and whose width is 6 feet.

$7 \times 6 = \square$; 42 sq. ft.

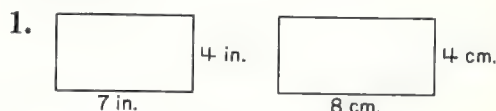
12. Tell how you would find the area of a rectangle whose length is 8 yards and whose width is 6 yards.

$8 \times 6 = \square$; 48 sq. yd.

Written Find the area of each rectangle below.

a

b



28 sq. in. 32 sq. cm.



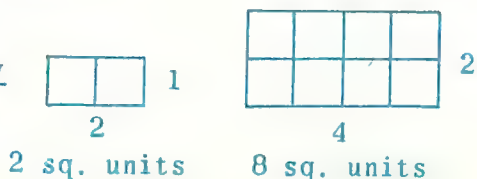
72 sq. ft. 54 sq. cm.

Find the area of each rectangle.

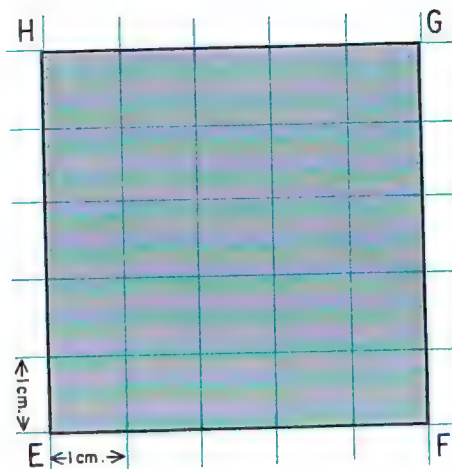
	Length	Width	Area
3.	21 yd.	12 yd.	252 sq. yd.
4.	9 cm.	12 cm.	108 sq. cm.
5.	38 in.	27 in.	1026 sq. in.
6.	21 dm.	9 dm.	189 sq. dm.
7.	148 m.	32 m.	4736 sq. m.

Tell why If you double the length and the width of a rectangle, its area measure is four times larger. Why? Draw a figure to show this. **See below.**

Tell why



Area of a Square



Each small square in the drawing above stands for which unit of area measure? ^{*1}Count the small squares inside square EFGH. How many are there? What is the area of square EFGH in square centimeters? 25; 25 square centimeters

A more convenient way to find the area of square EFGH is discussed below.

Is square EFGH a rectangle? ^{Yes} How do you know? ^{*2}How do you find the area of a rectangle? Can you find the area of square EFGH the same way? Why? ^{Yes} Multiply the measure of the length by the measure of the width; Yes; all squares are rectangles.

How many centimeters long is side EF? Side FG? What unit of measure was used to find the length of side EF? Of side FG? What open multiplication sentence can you solve to find the area measure of square EFGH? 5; 5; centimeter; centimeter; $5 \times 5 = \square$

$$5 \times 5 = \square$$

What is the simplest numeral for 5×5 ? Is 25 a measure or a measurement? Which measurement names the area of square EFGH? 25; measure; 25 sq. cm.

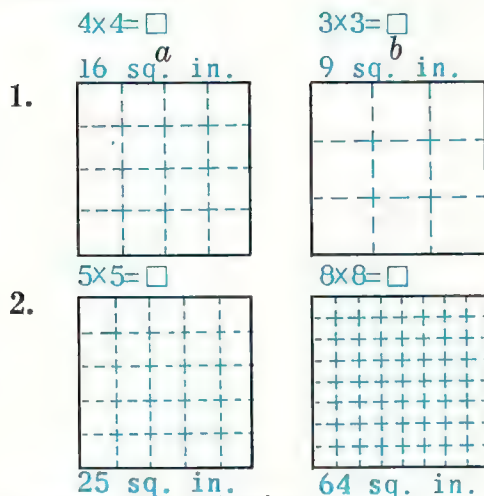
To find the *area measure* of a square, multiply the measure of one of its sides by itself.

270

*1 square centimeter

*2 It is a quadrilateral with 4 right angles.

Oral Dashed lines separate the interior of each square below into smaller squares. Each small square stands for 1 square inch. Tell an open multiplication sentence you would use to find the area measure of each square. Then tell the area of each square.



Answer the following.

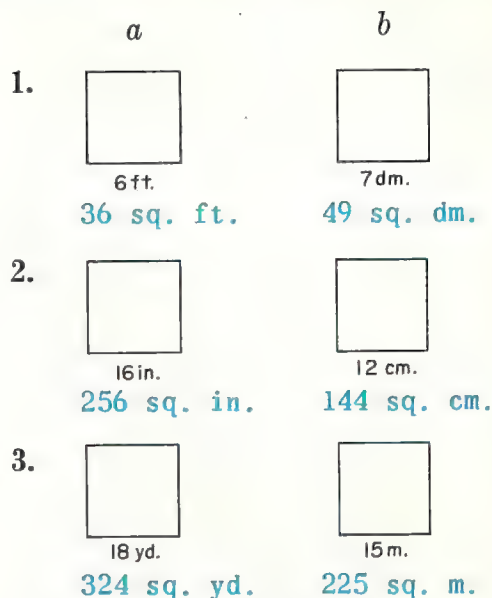
3. Can you find the area of each square in *Oral* 1 and 2 above by counting? Find the area of each square by counting. **Yes; See above.**

4. Which of the 2 methods for finding the area of a square is more convenient? Why? **open sentence method; quicker**

5. Tell how you would find the area of a square with sides 8 inches long. $8 \times 8 = \square$; **64 sq. in.**

6. Tell how you would find the area of a square with sides 24 decimeters long. $24 \times 24 = \square$; **576 sq. dm.**

Written Find the area of each square below.



Solve each of the following.

4. A lawn is in the shape of a square. One edge of the lawn is 28 feet long. What is the area of the lawn? **784 sq. ft.**

5. A living room rug is 12 feet long and 12 feet wide. What is the area of the rug? **144 sq. ft.**

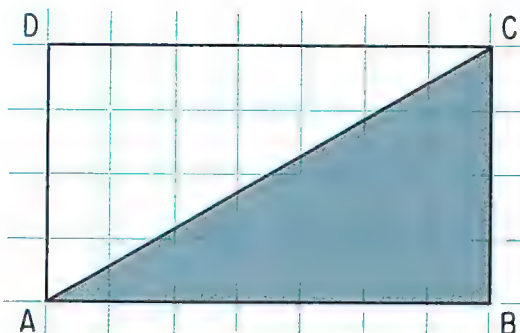
Can you do this? The area of a square is 49 square inches. How long is each side of the square? **7 inches**

Tell why Doubling the length of each side of a square makes its area measure 4 times larger. Why? Draw a figure to show this. **See below.**

Tell why



Area of a Right Triangle



The interior of rectangle ABCD is separated into small squares with each side representing 1 foot. What unit of area measure does each small square represent? **square foot**

Line segment AC and the sides of rectangle ABCD form how many triangles? Is triangle ABC a right triangle? Why? **2; Yes; It has 1 right angle.**

In a right triangle, the sides that form the right angle are called the **legs** of the right triangle.

What sides are the *legs* of triangle ABC? **side BC and side BA**

Draw and label a figure like the one shown above. Cut out triangles ABC and ACD. Lay triangle ABC on the triangle ACD so that no part overlaps. Do the triangles have the same size and shape? What is the area of rectangle ABCD? The area of triangle ABC is what part of the area of rectangle ABCD? How is this shown in the open sentences below? **Yes; 28 sq. ft.; one half; See below.**

$$\frac{1}{2} \times (7 \times 4) = \square$$

or

$$(7 \times 4) \div 2 = \square$$

$$\frac{1}{2} \times 28 = \square$$

$$28 \div 2 = \square$$

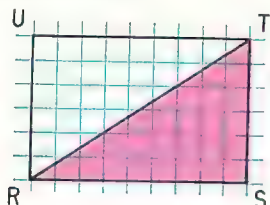
$$14 = \square$$

$$14 = \square$$

The area of triangle ABC is 14 *square feet*.

To find the *area measure* of a right triangle, find one half of the product of the measures of the two legs. Or you can divide the product of the measures of the two legs by two. The measures of the two legs should be found by using the same unit.

Oral Use the figure below to answer the following.



1. The interior of rectangle RSTU is separated into small squares with each side representing 1 inch. What unit of area measure does each small square represent? **square inch**

2. Which two triangles are formed by the sides of rectangle RSTU and line segment RT? **triangle RST and triangle RTU**

3. Are triangles RST and RTU right triangles? Why or why not? **Yes; each has 1 right angle.**

4. Which sides of rectangle RSTU are the legs of triangle RST? **side RS and side ST**

5. What is the inch measure of leg RS? Of leg ST? **9; 6**

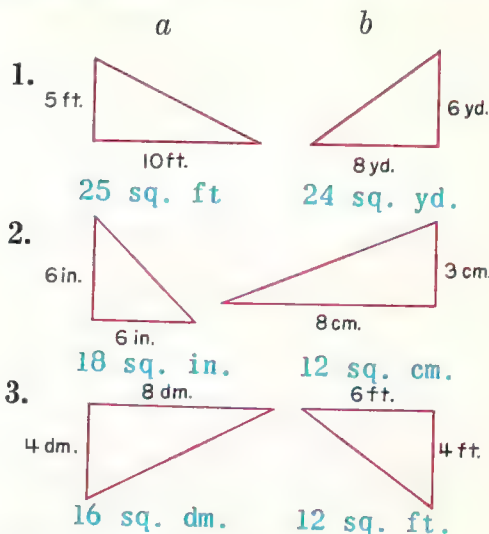
6. What open sentence could you write to find the area measure of triangle RST? $\frac{1}{2} \times (9 \times 6) = \square$ or $(9 \times 6) \div 2 = \square$

7. What is the simplest numeral for $\frac{1}{2} \times (9 \times 6)$? For $(9 \times 6) \div 2$? **27; 27**

8. What is the area of triangle RST? **27 square inches**

9. Tell how you would find the area of a right triangle whose legs are 8 inches and 4 inches long. $\frac{1}{2} \times (8 \times 4) = \square$; **16 sq. in.**

Written Find the area of each right triangle below.



Solve the following.

4. A flower garden is in the shape of a right triangle. The legs are 15 feet and 12 feet long. What is the area of the flower garden? **90 sq. ft.**

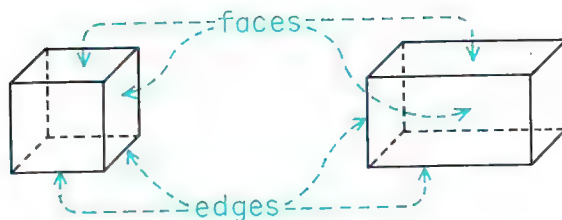
Find the area of each right triangle.

	One leg	Second leg	Area
5.	7 in.	6 in.	21 sq. in.
6.	8 ft.	7 ft.	28 sq. ft.
7.	4 yd.	9 yd.	18 sq. yd.

Can you do this? The area of a right triangle is 6 square inches. The measure of each leg is a whole number of inches. What are the lengths of the legs? (Hint: There are three possible answers.)

- 1 in. and 12 in.**
2 in. and 6 in.
3 in. and 4 in.

Volume

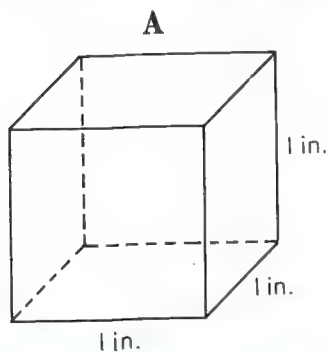


Each figure above is formed by parts of planes called **faces** of the figure. How many *faces* does each figure have? **6**

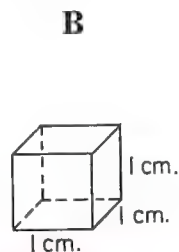
The line segment formed where two faces meet is called an **edge** of the figure. How many *edges* does each figure have? **12**

Since each figure above encloses a certain amount of space, we can measure its interior. The measurement of the interior of each figure is called the **volume** of the figure.

We agreed to use a line segment as a unit of length. We also agreed to use the interior of a simple closed figure as a unit of area measure. In the same way, we can agree to use the interior of some figure that encloses space as a unit of volume measure. It is convenient to use the interior of a cube as a unit of volume. Two such units are shown below.



1 cubic inch or 1 cu. in.

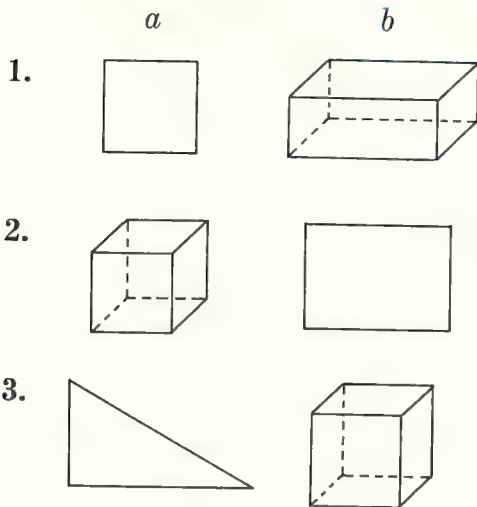


1 cubic centimeter or 1 cu. cm.

How long is each edge of the cube in A? Why do you think that unit is called a cubic inch? How long is each edge of the cube in B? Why do you think that unit is called a cubic centimeter? **1 in.; it is a cube with each edge 1 in. long; 1 cm.; it is a cube with each edge 1 cm. long.**

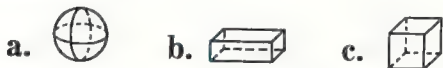
For TEACHING HELPS, see page T274.

Oral Tell which of the following figures have volume. **1b, 2a, 3b**



Answer the following.

4. Which of the following do we most often use as a unit of volume measure? **c**



5. What is the volume of a cube whose edges are all 1 inch long?
1 cubic inch

6. What is the volume of a cube whose edges are all 1 centimeter long?
1 cubic centimeter

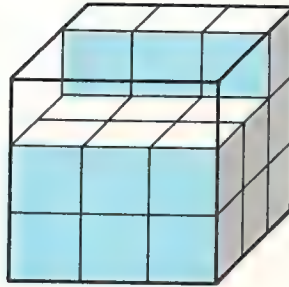
7. What is the volume of a cube whose edges are all 1 foot long?
1 cubic foot

8. What is the volume of a cube whose edges are all 1 yard long?
1 cubic yard

9. What is the volume of a cube whose edges are all 1 meter long?
1 cubic meter

10. What are you finding when you find the number of cubic units in a certain amount of space? **volume**

Written Use the figure below to answer the following.



1. Each small cube in the box stands for 1 cubic foot. How many small cubes are shown in the box? **27**

2. How many more small cubes can the box hold? **6**

3. How many small cubes can the box hold altogether? **27**

4. What is the volume of the box?
27 cubic feet

5. How long is the box in feet?
In yards? **3 ft.; 1 yd.**

6. How wide is the box in feet?
In yards? **3 ft.; 1 yd.**

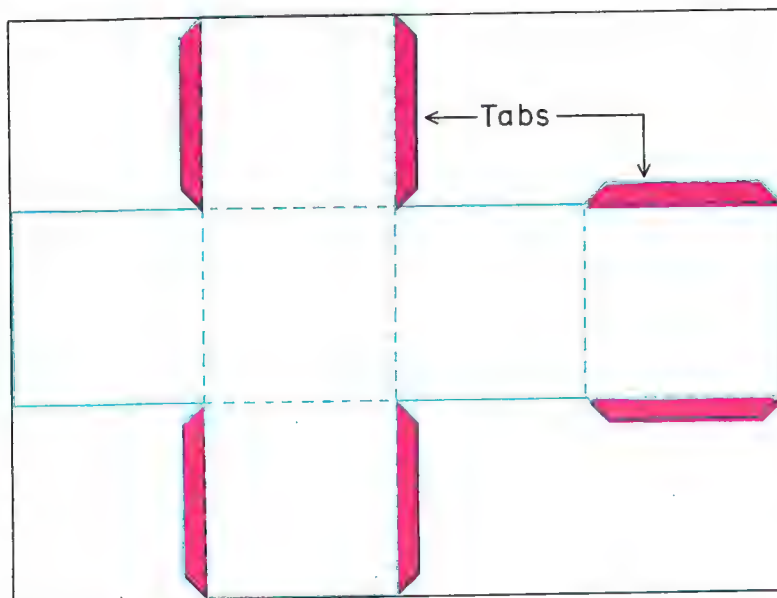
7. How high is the box in feet?
In yards? **3 ft.; 1 yd.**

8. Does the box have a volume of 1 cubic yard? **Yes**

9. How many cubic feet are there in one cubic yard? **27**

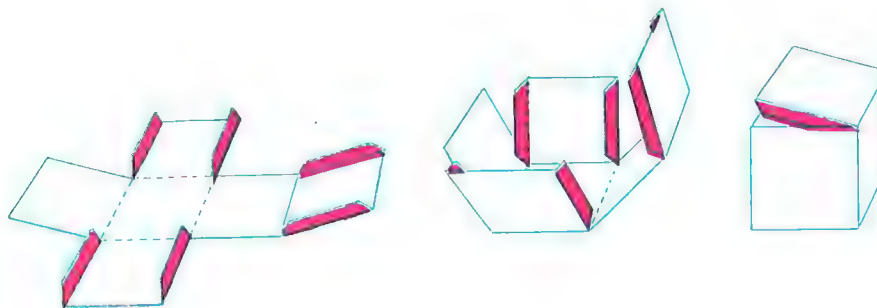
The Cubic Inch

You can make a model of a cubic inch as follows. Draw a rectangle having a length of 4 inches and a width of 3 inches on posterboard or some stiff paper as shown below.



Separate the interior into 1-inch squares. Then draw the pattern shown in blue. Cut out the pattern with the tabs.

Fold each tab up, as shown at the left below. Then fold along the dashed blue lines and form a cube. Use paste or glue on the tabs to hold the model together.



Oral Use your model of one cubic inch to help you answer the following.

1. Does the model of a cube separate space into three sets of points? Name these. **Yes; in the interior; in the exterior; and on the cube**

2. Consider the inside or interior of the cube that you constructed. Is the interior part of the cube? Why or why not? **No; The cube is formed by the faces only.**

3. How many faces does every cube have? **6**

4. Which of the following words best describes the shape of each face of a cube: *rectangle*, *triangle*, or *square*? **square**

5. What is the area of each face of the cube that you made? What is the total area of all 6 faces? **1 sq. in.; 6 sq. in.**

6. How many edges does the cube have? Do all of the edges have the same length? **12; Yes**

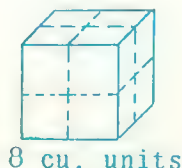
7. Is an edge of a cube a ray, a line, or a line segment? **line segment**

8. Do the edges of a cube intersect at the corners of the cube? **Yes**

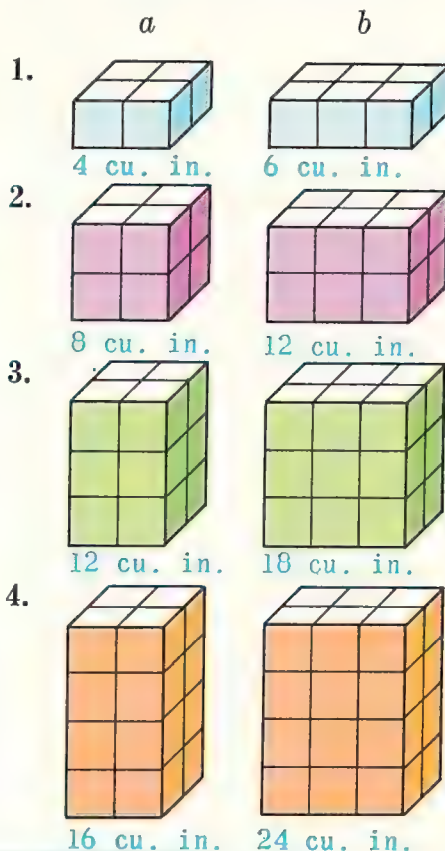
9. These corners of a cube are called *vertices* (plural of vertex). How many vertices does a cube have? **8**

10. The cubic inch is a unit of measure for which of the following: *length*, *area*, or *volume*? **volume**

Tell why

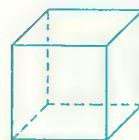


Written For each figure below, the unit of measure is a cubic inch. Find the volume of each figure.

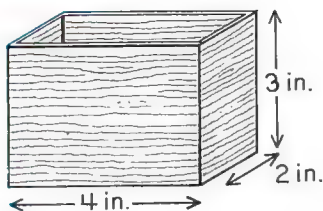


Can you do this? Make a model of one cubic decimeter.

Tell why If you double the length, the width, and the height of a cube as shown below, the volume gets 8 times larger. Why? **See below.**

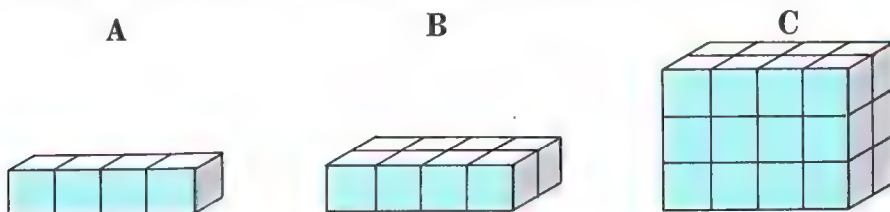


Volume of a Rectangular Solid



Each face of the box shown above has the shape of a rectangle. We call it a model of a **rectangular solid**. Just as with a cube, a rectangular solid is formed by the faces only. It has an interior that can be measured.

You can find the volume of such a box by using the models of a cubic inch that you and your classmates have made. First, place 4 models of a cubic inch side-by-side in a row as shown in **A**. Would this row fit along the side labeled 4 inches? **Yes**



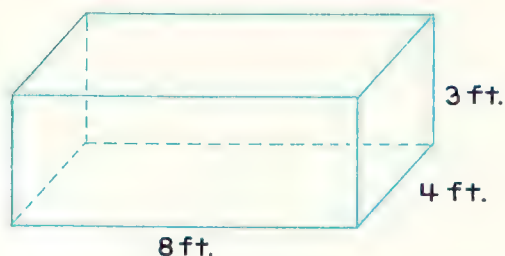
How many such rows of cubes are needed to completely cover the bottom of the box? **2** Form 2 rows as shown in **B**. How many cubes are there in the layer shown in **B**? **8**

How many layers of cubes are needed to completely fill the box? **3** Stack this number of layers as shown in **C**. How many cubes are there in **C**? How can you obtain the answer without counting? **24; $(4 \times 2) \times 3 = \square$**

Why can you use the open sentence below to find the volume of the box? **4×2 names the number in one layer; 3 names the number of layers.** $(4 \times 2) \times 3 = \square$

To find the *volume measure* of a rectangular solid, find the product of the measures of its length, width, and height. All of the measures should be found by using the same unit.

Oral Use the figure below to answer the following.



Think of filling the interior of the figure with cubes. The edges of each cube are 1 foot long.

1. How many cubes can be placed along the side labeled 8 ft.? **8**

2. How many such rows are needed to form a layer that covers the bottom? How did you decide on your answer? **4; The side is 4 ft. long.**

3. How many cubes will there be in the bottom layer? **32**

4. How many layers of cubes are needed to fill the interior of the figure? **3**

5. Since there are 32 cubic feet in one layer, how many cubic feet will there be in 3 layers? How did you decide on your answer? **96; 32×3**

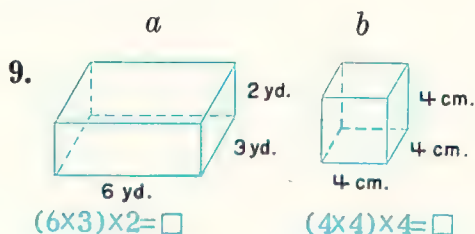
6. What open sentence can you solve to find the volume measure of this rectangular solid? **$(8 \times 4) \times 3 = \square$**

7. What is the simplest numeral for $(8 \times 4) \times 3$? **96**

8. How would you record the volume of this rectangular solid?

96 cu. ft.

Tell the open sentence you can use to find the volume measure for each rectangular solid below.



9.

$$(6 \times 3) \times 2 = \square$$

$$(4 \times 4) \times 4 = \square$$

Do the following.

10. Tell how you would find the volume measure of a rectangular solid 5 inches long, 3 inches wide, and 2 inches high. **$(5 \times 3) \times 2 = \square$; 30 cu. in.**

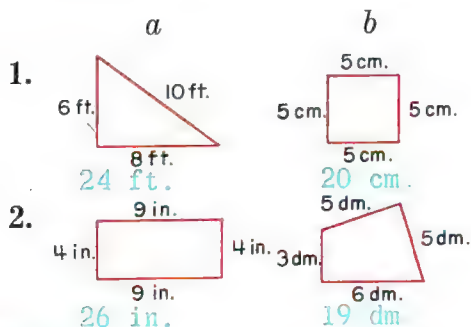
Written Find the volume of each rectangular solid whose length, width, and height are listed below.

	Length	Width	Height	Volume
1.	3 in.	2 in.	3 in.	18 cu. in.
2.	7 ft.	2 ft.	3 ft.	42 cu. ft.
3.	9 yd.	3 yd.	2 yd.	54 cu. yd.
4.	6 cm.	6 cm.	6 cm.	216 cu. cm.
5.	3 dm.	3 dm.	3 dm.	27 cu. dm.
6.	12 m.	12 m.	12 m.	1728 cu. m.

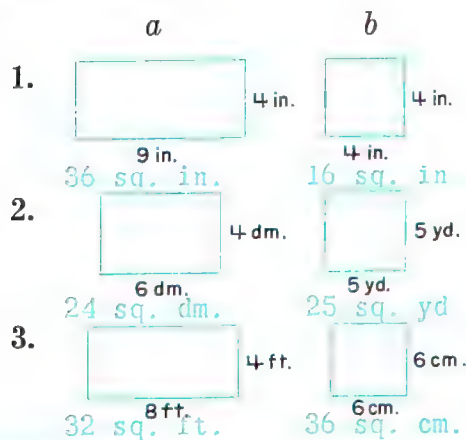
Tell how How can you double the volume measure of a rectangular solid? **double the measure of only one of its dimensions**

Review and Practice

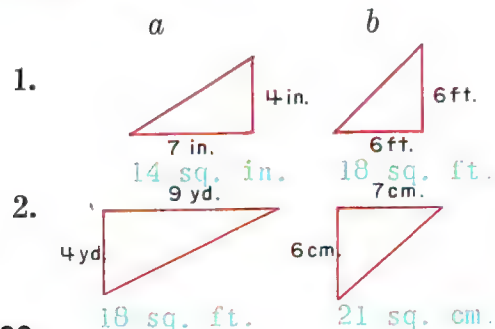
Part 1 Find the perimeter of each figure below.



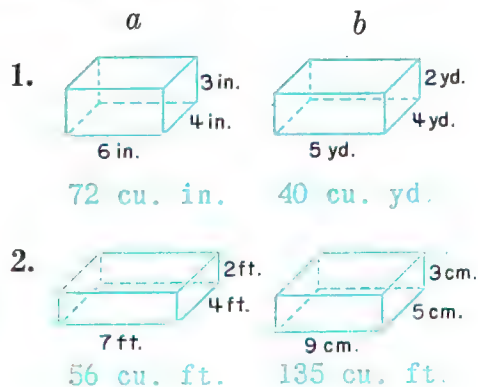
Part 2 Find the area of each rectangle below.



Part 3 Find the area of each right triangle below.



Part 4 Find the volume of each rectangular solid below.



Part 5 Solve each of the following.

1. A wall is in the shape of a rectangle. It is 18 feet long and 7 feet high. What is the area of this wall? $18 \times 7 = \square$; 126 sq. ft.

2. A box is 18 inches long, 12 inches wide, and 8 inches high. What is the volume of this box?

$(18 \times 12) \times 8 = \square$; 1728 cu. in.

3. A lawn is 21 yards long and 14 yards wide. What is the area of this lawn? $21 \times 14 = \square$; 294 sq. yd.

4. A lot is in the shape of a square with each side 65 feet long. What is the perimeter of this lot? $65 \times 4 = \square$; 260 ft.

5. A flower garden is in the shape of a right triangle. The sides that form the right angle are 8 feet and 14 feet long. What is the area of this flower garden? $\frac{1}{2} \times (8 \times 14) = \square$; 56 sq. ft.

Checkup Time

The numerals in () tell the pages where you can turn for help.

Important Ideas

1. Metric geometry is the study of geometric figures by using measurement. (263)
2. The distance around a simple closed figure is called its perimeter. (264)
3. The measurement of the interior of a simple closed figure is called the area of the figure. (266)
4. To find the area measure of a rectangle, multiply the measure of the length by the measure of the width. Both measures should be found by using the same unit. (268)
5. To find the area measure of a square, multiply the measure of one of its sides by itself. (270)
6. To find the area measure of a right triangle, find one half of the product of the measures of the two legs. The measures of the two legs should be found by using the same unit. (272)
7. To find the volume measure of a rectangular solid, find the product of its length, its width, and its height. All of the measures should be found by using the same unit. (278)

Words to Know

1. Geometric figures, metric geometry (263)
2. Perimeter (264)
3. Area, square inch (266)
4. Volume, face, cubic inch (274)
5. Rectangular solid (278)

Questions to Discuss

See page T281.

1. How would you find the perimeter of a rectangle? A triangle? (264)
2. How would you find the area of a rectangle whose length is 8 meters and whose width is 6 meters? (268)
3. How would you find the area of a square with each side 20 yards long? (270)
4. How would you find the area of a right triangle? (272)
5. How would you find the volume of a rectangular solid? (278)

Written Practice

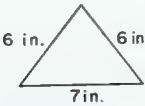
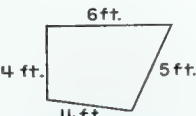
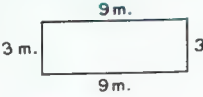
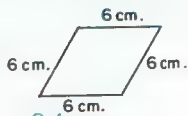
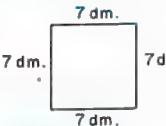
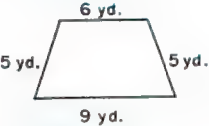
Find the area and perimeter of a rectangle that is 14 inches long and is 12 inches wide. (264, 268)

area: $14 \times 12 = \square$; 168 sq. in.

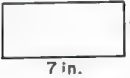
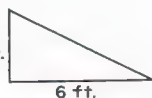

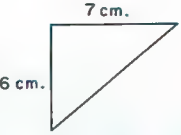
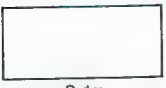

perimeter: $(2 \times 14) + (2 \times 12) = \square$; 281

Self-Evaluation

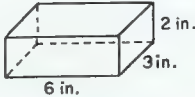
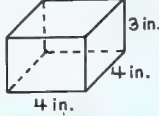
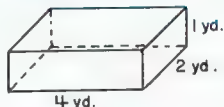
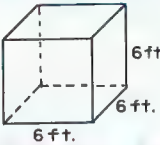
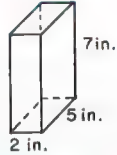
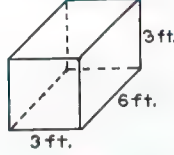
Part 1 Find the perimeter of each figure below.

- | | <i>a</i> | <i>b</i> |
|----|---|---|
| 1. | 
19 in. | 
19 ft. |
| 2. | 
24 m. | 
24 cm. |
| 3. | 
28 dm. | 
25 yd. |

Part 2 Find the area of each rectangle or right triangle below.

- | | <i>a</i> | <i>b</i> |
|----|---|---|
| 1. | 
21 sq. in. | 
9 sq. ft. |
| 2. | 
16 sq. yd. | 
21 sq. cm. |
| 3. | 
36 sq. dm. | 
81 sq. m. |

Part 3 Find the volume of each rectangular solid below.

- | | <i>a</i> | <i>b</i> |
|----|--|--|
| 1. | 
36 cu. in. | 
48 cu. in. |
| 2. | 
8 cu. yd. | 
216 cu. ft. |
| 3. | 
70 cu. in. | 
54 cu. ft. |

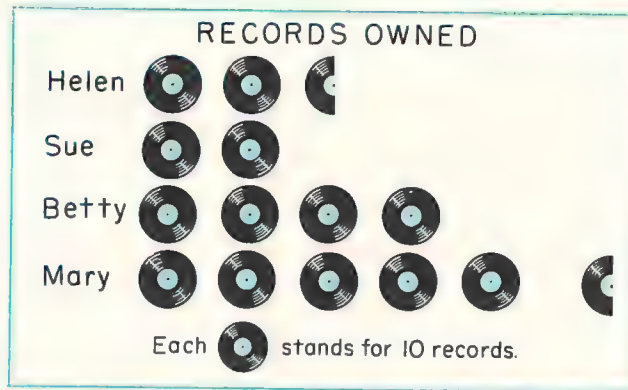
Part 4 Solve each of the following.

- A cereal box is 7 inches long, 2 inches wide, and 9 inches high. What is the volume of this box?
 $(7 \times 2) \times 9 = \square$; 126 cu. in.
- A living room is 18 feet long and 13 feet wide. What is the area of its floor? $18 \times 13 = \square$; 234 sq. ft.
- A mirror is 25 inches long and 22 inches wide. What is the area of this mirror? $25 \times 22 = \square$; 550 sq. in.
- A patio is in the shape of a square with each side 16 feet long. What is the perimeter of this patio? What is its area? $4 \times 16 = \square$, 64 ft.; $16 \times 16 = \square$, 256 sq. ft.

Chapter 13 GRAPHS



Picture Graphs

Records Owned	
Helen	25
Sue	20
Betty	40
Mary	55



The four girls named above made a table and a drawing to show how many records each of them owned. The drawing is called a **picture graph**. Each symbol stands for a certain number of things or ideas.

The title of the graph is usually above the graph. It should tell briefly what the graph is about. What is the title of the picture graph above? **Records Owned**


A picture graph should have a statement telling *how many* and *what kind* of things each symbol stands for. Why do you suppose the  is used in this picture graph? How many records does each  stand for? **because records are involved; 10**

Oral Answer these questions.

1. How many records does Betty own? How is this shown on the picture graph? **40; 4 record symbols**

2. How many records does each  stand for? **5**

3. Who owns the greatest number of records? How can you tell this on the picture graph? **See below.**

4. How would the graph change if each  stood for 5 records instead of 10 records? **See below.**

Oral 3. Mary; More record symbols are after her name than after any of the other girls' names.


4. Helen 5 record symbols
Sue 4 record symbols
Betty 8 record symbols
Mary 11 record symbols

Reading a Picture Graph

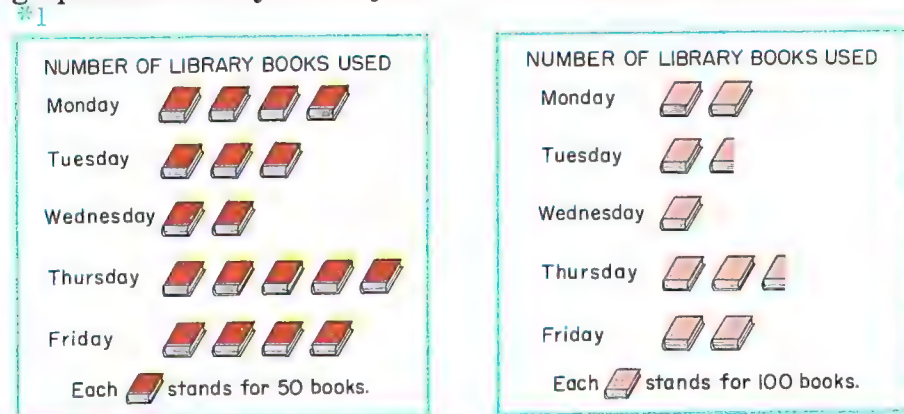
The fourth-grade pupils kept a record of how many library books were used each day of a week. Then they made the picture graph below to show this information.



On which day was the least number of books used? How many books were used that day? How did you decide? Were more books used on Thursday or on Friday? How many more? How did you decide? *100; the least number of symbols are listed after Wed.; Thurs.; 50; by comparing the number of symbols*

You can use any symbol you want when making a picture graph. You can let each symbol stand for any convenient number of things. Why is  a useful symbol for this graph? *because books are involved*

The picture graphs below show the same information as the graph above. Why do they look different than the graph above?

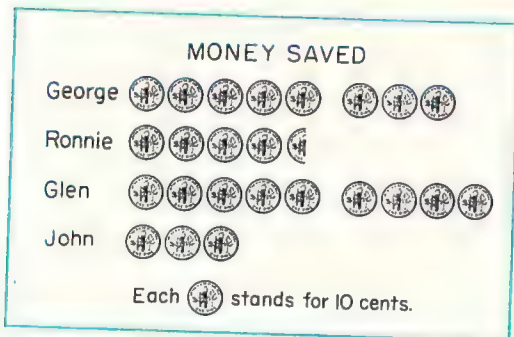




For TEACHING HELPS, see page T284.

Oral Use the graph at the top of page 284 to answer the following questions.

1. What is the title of the graph?
Number of Library Books Used
2. How many books does each symbol stand for? *25*
3. On which day was the greatest number of books used? How many books was that? *Thursday; 250*

Use the following graph to answer the questions below.



4. What does each  stand for?
10 cents
5. Who saved the most money?
How much did he save? *Glen; 90 cents*
6. Who saved the least amount of money? How much did he save?
John; 30 cents
7. What does the symbol  after Ronnie's name stand for? How much money did Ronnie save? *5 cents; 45 cents*
8. How can you find the total amount of money saved by all four boys? *$80+45+90+30=\square$*

Tell why Accept any answer which implies that

- *1 75, 45, and 60 are not multiples of 50.
- *2 75 is a multiple of 25 but 45 and 60 are not.
- *3 75, 45, and 60 are multiples of 15.

Written Copy. Complete the following table for the graph at the top of page 284.

1. <i>Number of Library Books Used</i>	
Monday	<i>200</i>
Tuesday	<i>150</i>
Wednesday	<i>100</i>
Thursday	<i>250</i>
Friday	<i>200</i>

Copy and complete the following table for the graph at the left.

2. <i>Money Saved</i>	
George	<i>80¢</i>
Ronnie	<i>45¢</i>
Glen	<i>90¢</i>
John	<i>30¢</i>

Tell why Think of making a picture graph for the following information.

<i>Students Absent</i>	
Monday	75
Tuesday	45
Wednesday	60

Why is each of the following a poor statement for the graph?

Each  stands for 50 pupils.
*See below *1.*

Each  stands for 25 pupils.
*See below *2.*

Why is the following a better statement for the graph?

Each  stands for 15 pupils.
*See below *3.*

Making a Picture Graph

In their annual cookie sale, four girl scouts had the sales given below.






Sally — 20 boxes

Terry — 45 boxes

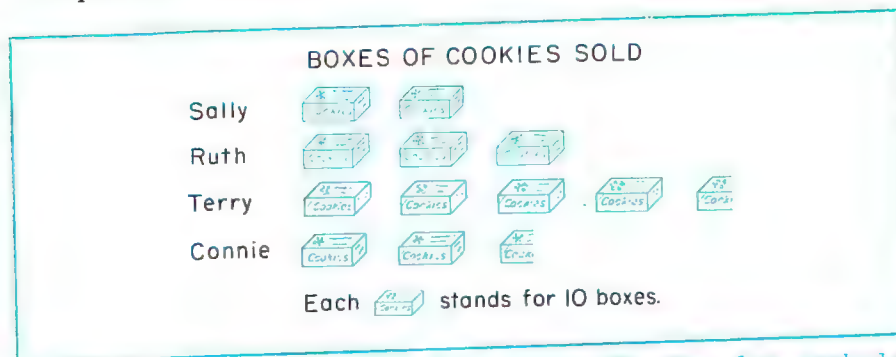
Ruth — 30 boxes

Connie — 25 boxes

To make a picture graph of this information, you can use the following steps.

- Choose a symbol that is easy to draw. Two suggestions might be a box  and a cookie . Did the girls sell single cookies or boxes of cookies? Why might the  be a better symbol? *boxes of cookies; because boxes of cookies were sold*
- Let each symbol stand for a convenient number of things. Why would it be a poor choice to let each  stand for 1 box? **1* For 25 boxes? For 50 boxes? Would it be easier to let each  stand for 5 boxes or for 10 boxes? Why? **2*
- Decide what each row of symbols stands for. To make a graph of the information above, write the name of one of the girls at the left of each row.
- Draw the correct number of symbols neatly in each row. How can you decide upon the number of symbols in each row? **3*
- Write a title above the graph. Write a statement telling what each symbol stands for. **4*

Explain how each of these steps is shown on the graph below.



- 286 *1 Too many symbols would be needed. *2 Too few symbols would be used. *3 Only parts of a symbol would be used. *4 5 boxes if only complete symbols are desired; 10 boxes if the desire is to keep the graph relatively simple. *5 Divide the number of boxes sold by that girl by the number each symbol stands for.

Oral Read the following paragraph and answer the questions below.

The principal at Shaffer Elementary School listed the number of pupils that had perfect attendance as follows.

September	80
October	110
November	60
December	50

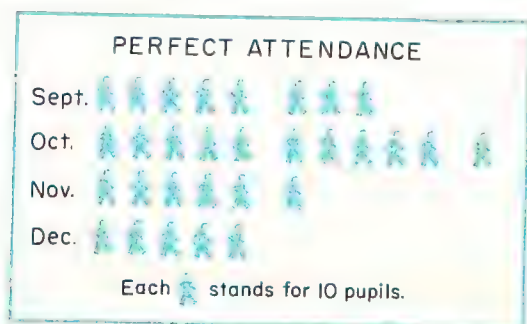
1. What is a suitable title for the picture graph? "Perfect Attendance" or something similar

2. What would you use as a symbol on this graph? What made you choose that symbol? See below.

3. What number of pupils would you let each symbol stand for? Why? See below.

4. If each symbol stands for 10 pupils, how many symbols would be needed to show the number of pupils that had perfect attendance in each of the months listed? See the next question.

5. Tell why the graph below is a good graph for this information.



Written Draw a picture graph for each exercise below.

1. Don listed the number of eggs collected for five days as follows.

Monday	60
Tuesday	90
Wednesday	45
Thursday	85
Friday	60

2. A gasoline station sold gasoline as listed below.

Friday	600 gallons
Saturday	950 gallons
Sunday	700 gallons
Monday	450 gallons
Tuesday	500 gallons

3. In a new suburb 700 homes were built. The next year 300 new homes were built. The third year 550 new homes were built. Last year 400 new homes were built.

4. During its first year of operation Concord Airport handled 800 planes. During its second year it handled 1100 planes. During its third year, it handled 1250 planes. Last year it handled 1000 planes.

Can you do this? Find a picture graph in a newspaper or a magazine. Bring it to school and explain it to the class.

Bar Graphs

Another way to picture the information about the girl scout cookie sale is shown below. Remember that the number of boxes sold by each girl was as follows.

Sally — 20 boxes

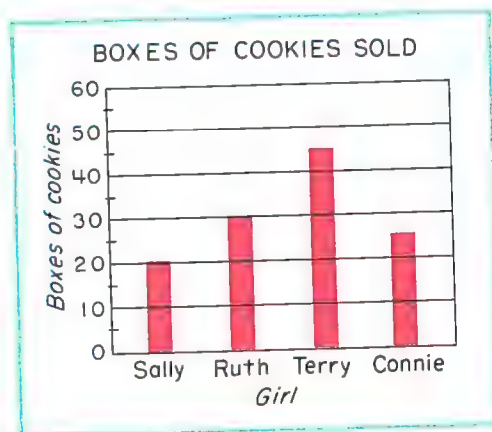
Terry — 45 boxes

Ruth — 30 boxes

Connie — 25 boxes

Instead of drawing rows of symbols, you can use a number scale and draw bars to show the number of boxes sold by each girl. Such a graph is called a **bar graph**.

In the first bar graph below, the names of the girls are given along the vertical line. The horizontal line is a number line. These two lines are interchanged to form the second bar graph below.



How many boxes of cookies are represented by each segment on the number line? Why does the bar for Connie stop between two lines on the graph? 10; She sold between 20 and 30 boxes of cookies.

The length of each segment on the number line, the width of the bars, and the space between the bars are chosen to fit the space you have.

Are the bar graphs above better looking than picture graphs? *1
Are they easier to read? Yes

Oral Use either graph on page 288 to answer the following.

1. What is the title of the graph?
Boxes of Cookies Sold
2. Does the title of the graph tell briefly what the graph is about? What is the graph about? *Yes; shows the number of boxes sold*
3. Who sold the most boxes of cookies? How many boxes of cookies did she sell? *Terry; 45*

4. Who sold the least number of boxes? How many boxes of cookies did she sell? *Sally; 20*

5. How does the bar graph show that Ruth sold 30 boxes of cookies? *Bar extends to 30.*

6. How many boxes of cookies did Connie sell? How is this shown on the graph? *25; Bar extends to 25.*

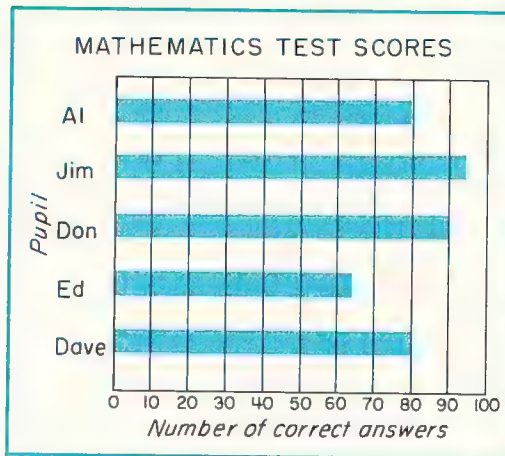
7. How does the bar graph show that Connie sold more boxes of cookies than Sally? That Connie sold less boxes of cookies than Terry? *See below.*

8. Does the amount of space between the bars change the meaning of the graph? Why or why not? *See below.*

9. How many boxes of cookies are represented by each segment on the number line? Why is this a convenient choice? *10; 20, 30, 45, and 25 can be easily represented.*

10. Which of the two bar graphs is better looking? Which of them is easier to read? *Answers will vary.*

Written Use the bar graph below to answer the following.



1. Who scored the highest on the test? What is his score? *Jim; 95*

2. Who scored the lowest on the test? What is his score? *Ed; 65*

3. Did any of the students obtain the same score? Who were they? What score did they obtain? *Yes; Al and Dave; 80*

4. What score is represented by each segment on the horizontal number line? *10*

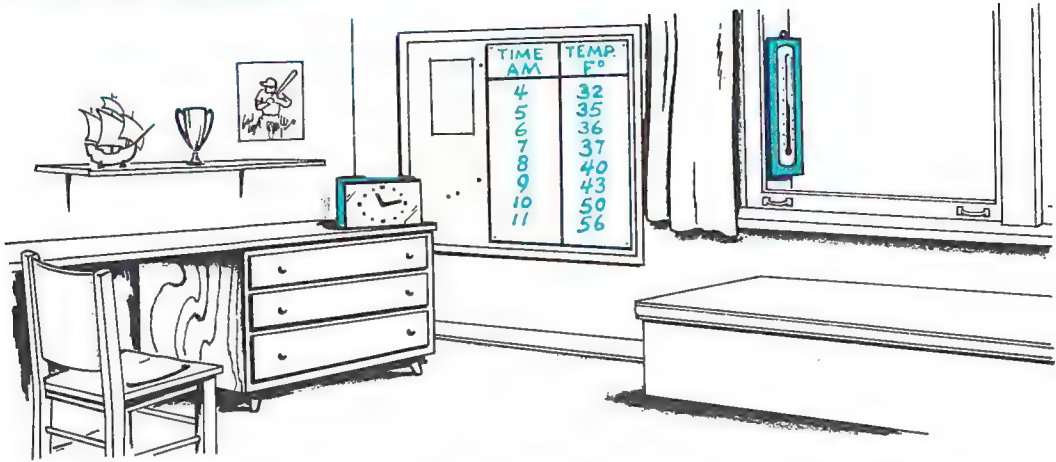
5. Copy. Complete the following table for the graph above.

<i>Mathematics Test Scores</i>	
Al	<u>80</u>
Jim	<u>95</u>
Don	<u>90</u>
Ed	<u>65</u>
Dave	<u>80</u>

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333

- Oral**
7. The bar for Connie is longer than the bar for Sally;
The bar for Connie is shorter than the bar for Terry.
 8. No; Only the length of the bars changes the meaning.

Ordered Pairs of Numbers



Two sets of measures have been recorded in the table above. What are they? **time and temperature**

How many temperature measures are recorded for each time measure? Name the temperature measure that matches each time measure in the table above. **1; See above.**

This matching of measures can be expressed as pairs of numbers as follows.

$\{(4,32), (5,35), (6,36), (7,37), (8,40), (9,43), (10,50), (11,56)\}$

From which set of numbers in the table is the first number of each pair taken? From which set of numbers is the second number of each pair taken? Is this true for each pair of numbers listed above? **time; temperature; Yes**

The two numbers of each pair are listed in a certain *order*. What is that order? **time then temperature**

When pairs of numbers are recorded as follows,

$\{(4,32), (5,35), (6,36), (7,37), (8,40), (9,43), (10,50), (11,56)\},$

we say that the numbers of each pair are *ordered*. Or we say that they are **ordered pairs of numbers**.

Oral Use the table shown below to help you answer the following questions.

Test number	1	2	3	4	5
Score	82	90	85	80	95

1. What do the numbers in the first row of the table stand for?
test number

2. What do the numbers in the bottom row of the table stand for?
score

3. How many scores match each test number? 1

4. What score matches test number 1? Test number 2? What score matches each of the other test numbers? 82; 90; 3—85; 4—80; 5—95

5. How would you list the information contained in the table above as a set of ordered pairs of numbers, taking the first number of each pair from the set of test numbers? (1,82), (2,90), (3,85), (4,80), (5,95)

6. Give a set of ordered pairs of numbers for the table below, taking the first number of each pair from the set of numbers labeled A. (1,3), (2,4), (3,5), (4,6), (5,7), (6,8)

A	1	2	3	4	5	6
B	3	4	5	6	7	8

Written List a set of ordered pairs for each table below. Take the first number of each pair from the top row. See below.

1.

C	4	5	6	7	8	9
D	56	57	54	51	45	42

2.

E	1900	1920	1940	1960
F	76	106	131	178

3.

G	21	22	23	24	25
H	35	40	43	31	20

4.

M	1963	1964	1965	1966	1967
N	410	437	474	501	514

Use each set of ordered pairs below to help you complete each table.

5. $\{(1,80), (2,90), (3,75), (4,92), (5,84), (6,96)\}$

Test number	1	2	3	4	5	6
Score	80	90	75	92	84	96

6. $\{(8,42), (10,48), (12,54), (2,60), (4,63), (6,60)\}$

Time (hr.)	8	10	12	2	4	6
Temperature (in degrees)	42	48	54	60	63	60

Written

1. (4,56), (5,57), (6,54), (7,51), (8,45), (9,42)
2. (1900,76), (1920,106), (1940,131), (1960,178)
3. (21,35), (22,40), (23,43), (24,31), (25,20)
4. (1963,410), (1964,437), (1965,474), (1966,501), (1967,514)

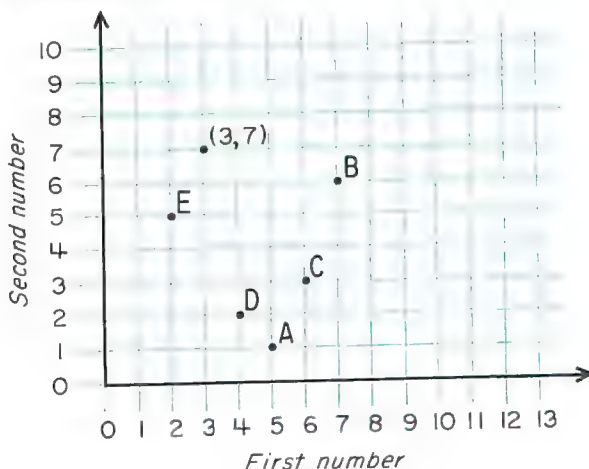
Graphing Ordered Pairs

You can represent the pairs of numbers in the following table as ordered pairs of numbers.

First number	3	5	7	6	4	2
Second number	7	1	6	3	2	5

$(3,7)$, $(5,1)$, $(7,6)$, $(6,3)$, $(4,2)$, $(2,5)$

You can also make a drawing as shown below to represent each ordered pair of numbers. Two number lines are drawn on squared paper. The set of *first numbers* is represented on the horizontal number line. The set of *second numbers* is represented on the vertical number line.



To locate the point for $(3,7)$, start at 0 and move 3 segments to the right on the horizontal number line. Then move upward 7 segments. The dot labeled $(3,7)$ is called the **graph** of that ordered pair.

How would you graph the ordered pair $(5,1)$? Which point is that on the drawing above? Move 5 segments to the right and 1 segment upward; A

What ordered pair of numbers is represented by point B? By point C? By point D? By point E?

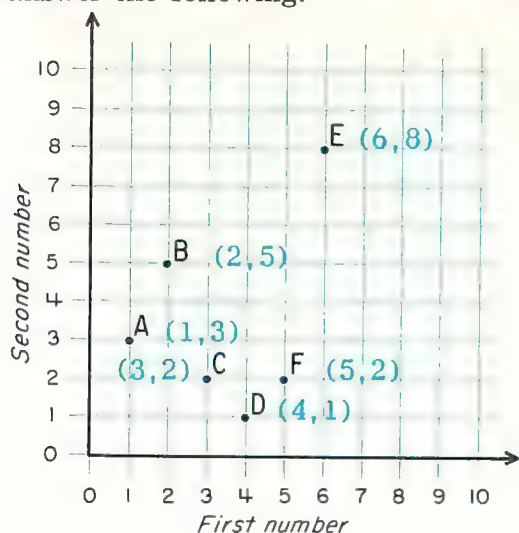
$(6,3)$

$(4,2)$

$(2,5)$

$(7,6)$

Oral Use the drawing below to answer the following.



1. What ordered pair of numbers is represented by point A? By each of the other points? **See above.**

2. How would you locate the point for (4,6)? **Move 4 segments to the right and 6 segments upward.**

3. Point B represents what ordered pair of numbers? Point F represents what ordered pair of numbers? Are (2,5) and (5,2) the same ordered pair? Why or why not? (2,5); (5,2); **No; They name different points.**

4. How would you locate the point for (0,7) on the drawing above? **Move up 7 segments from 0.**

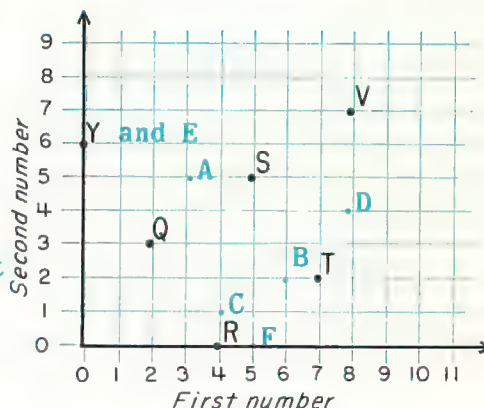
5. How would you locate the point for (5,0) on the drawing above? **Move right 5 segments from 0.**

6. How would you locate the point for (4,4) on the drawing above? **Move 4 segments right and 4 segments upward.**

Written Do the following.

1. Copy and complete the table for the drawing below.

	<i>First Number</i>	<i>Second Number</i>
Point Q	2	3
Point R	4	0
Point S	5	5
Point T	7	2
Point V	8	7
Point Y	0	6



2. Draw two number lines on squared paper as shown above. Graph the set of ordered pairs of numbers below. Label each point with the letter given before each ordered pair. **See above.**

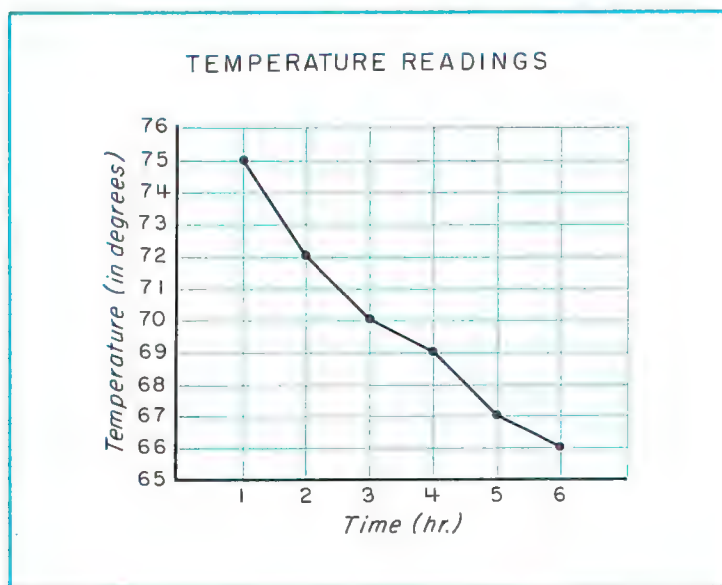
Point A (3,5)	Point D (8,4)
Point B (6,2)	Point E (0,6)
Point C (4,1)	Point F (5,0)

Line Graphs

Jim recorded the temperature at different times during the afternoon. His record is shown below.

<i>Time (hr.)</i>	1	2	3	4	5	6
<i>Temperature (in degrees)</i>	75	72	70	69	67	66

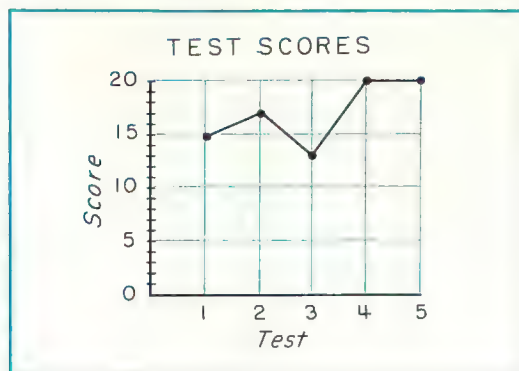
This information can also be shown on a graph. The graph of each ordered pair of numbers above is shown on the following drawing.



The dots for the ordered pairs of numbers are connected by line segments, as shown in black. Such a figure is called a **line graph**.

The line graph above shows how the temperature changed as the time changed. Did the temperature increase or decrease between 1 o'clock and 2 o'clock? Did the temperature increase any time during the afternoon? How is this shown by the graph? What is the title of the graph? **decreased; No; The line on the graph goes down as the time progresses; Temperature Readings**

Oral Use the graph below to answer the following.



1. On which test or tests was the highest score made? What is the highest score? **tests 4 and 5; 20**

2. On which test or tests was the lowest score made? What is the lowest score? **test 3; 13**

3. How does the graph show that the same score was made on both test 4 and test 5? **The line of the graph is horizontal.**

4. What is the title of the graph? Does it tell briefly what the graph is about? **Test Scores; Yes**

5. What is the graph about?

scores made on 5 different tests

6. How does the graph show an increase in score from test 3 to test 4? **The line of the graph goes up.**

7. How does the graph show a decrease from test 2 to test 3? **The line of the graph goes down.**

8. On which test was the score 17? The score 15? **test 2; test 1**

Written For each table below, write a set of ordered pairs of numbers. Then use graph paper to draw a line graph for each set of ordered pairs of numbers.

1.

Days	1	2	3	4	5	6	7
Gasoline (gallons)	400	350	500	400	800	1000	700

2.

A	1	2	3	4	5	6	7	8
B	31	34	36	38	36	30	26	24

3.

Test	1	2	3	4	5	6
Score	70	80	75	90	80	95

4.

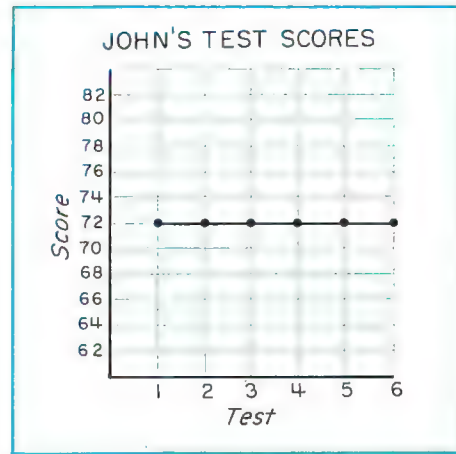
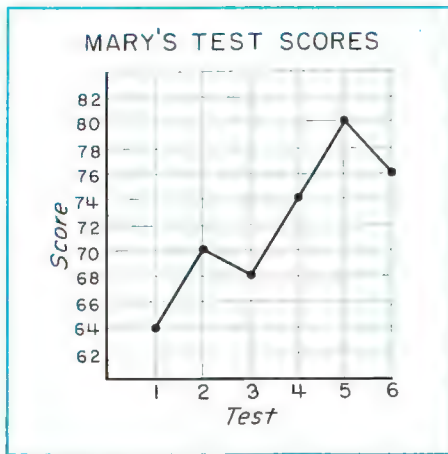
Speed (m.p.h.)	10	20	30	40	50	60
Stopping Distance (ft.)	22	45	88	125	185	255

Can you do this? Copy and complete the table below.

Side of Square	Area of Square
1 in.	1 sq. in.
2 in.	4 sq. in.
3 in.	9 sq. in.
4 in.	16 sq. in.
5 in.	25 sq. in.

Use graph paper to draw a line graph for this information.
See page T295.

Averages



Mary and John have each taken the same six mathematics tests. What did Mary score on each of the six tests? What did John score on each of the six tests? Can you tell whether Mary or John did the better work? 64, 70, 68, 74, 80, and 76; 72, 72, 72, 72, 72, and 72; No

To compare Mary's scores with John's scores you could find the **average** of each set of scores as shown below.

Mary's Average

$$\begin{array}{r}
 64 \\
 70 \\
 68 \\
 74 \\
 80 \\
 +76 \\
 \hline
 432
 \end{array}
 \qquad
 \begin{array}{r}
 72 \\
 6 \overline{)432} \\
 \underline{42} \\
 12 \\
 \underline{12} \\
 0
 \end{array}$$

John's Average

$$\begin{array}{r}
 72 \\
 72 \\
 72 \\
 72 \\
 72 \\
 +72 \\
 \hline
 432
 \end{array}
 \qquad
 \begin{array}{r}
 72 \\
 6 \overline{)432} \\
 \underline{42} \\
 12 \\
 \underline{12} \\
 0
 \end{array}$$

What is the sum of Mary's scores? Of John's scores? How many tests did Mary and John each take? What is the simplest numeral for $432 \div 6$? What is the average of Mary's scores? Of John's scores? How do Mary's scores compare with John's scores?

To find the *average* of a set of numbers, find their sum and divide this sum by the number of addends.

296

*1 72

*2 The average of her scores is the same as the average of John's scores.

Oral Study the following example of how Laura found that 17 is her average score on five spelling tests.

A	B
13	17
20	5 $\overline{)85}$
19	50
17	35
+16	35
85	0

1. How many tests did Laura take? **5**

2. What is the sum of the test scores? **85**

3. In B, what number is the sum being divided by? **5**

4. How is the divisor 5 related to the number of scores being added in A? **It is the same.**

5. What is the simplest numeral for $85 \div 5$? **17**

6. What is the average of the 5 test scores? **17**

7. Are there test scores in A that are less than 17? Which ones are they? **Yes; 13 and 16**

8. Are there test scores in A greater than 17? Which ones are they? **Yes; 20 and 19**

9. Tell how you find the average of the set of numbers below.
See below.

{24, 37, 28, 25, 26}

Written Find the average in each of the following.

1. The fourth grade sold tickets for the school play. On Monday they sold 24; on Tuesday, 20; on Wednesday, 29; and on Thursday, 15. What was the average daily sale? **22 tickets**

2. Five pupils checked the price of a pound of bacon. They obtained these prices: 72¢, 75¢, 75¢, 69¢, and 89¢. The average price was **76¢** a pound.

3. Find the average height of a group of 4 boys whose heights are 51 in., 53 in., 55 in., and 49 in. **52 in.**

4. What is the average weight of these girls: Sue, 69 lb.; Mary, 57 lb.; Jane 58 lb.; and Sally, 64 lb.? **62 lb.**

5. The following scores were made by a varsity basketball team during their first five games: 77, 45, 63, 49, and 71. Find the average score for these five games. **61**

Tell why If you are given the sum of a set of measures, you need only divide the sum of the measures by the number of measures to find the average. Why? **The addition is already done.**

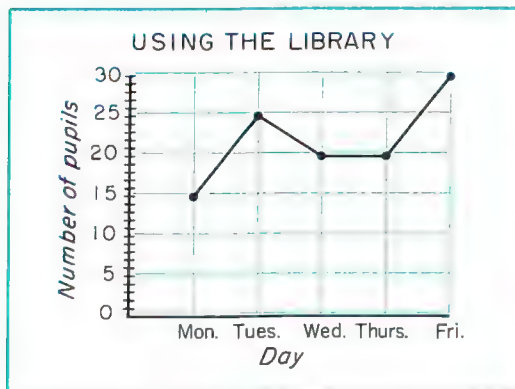
Find the average in the following problem.

Bob delivered 98 pizzas in 7 days. What was the average number of deliveries per day? **14**


Oral 9. $24+37+28+25+26=140$
 $140 \div 5=28$

Review and Practice

Part 1 Use the graph below to help you answer the following.



1. What kind of graph is it?
line graph
2. What is its title?
Using the Library
3. What is the graph about?
See below.
4. On what day was the library used by the greatest number of pupils? How does the graph show this?
Friday; highest point on graph
5. On what day was the library used by the least number of pupils? How does the graph show this?
Monday; lowest point on graph
6. On which days was the library used by the same number of pupils? How is this shown? *Wednesday and Thursday; same height on graph*
7. How many pupils used the library on Monday? On Tuesday? On the rest of the days of the week?
See below.
8. On what day did 30 pupils use the library? *Friday*

Part 2 Make a picture graph of the information given in the graph of *Part 1*. Let  stand for 5 pupils.

Part 3 Make a line graph of the information given in the following table. Use these suggestions.

Let the marks on the horizontal number line stand for the years and the segments on the vertical number line stand for the price per pound.

Price of Apples Per Pound	
1940	5¢
1945	13¢
1950	12¢
1955	15¢
1960	14½¢
1965	16¢

Part 4 Solve the following.

1. Jill made the following scores on four successive spelling tests. Find Jill's average score for the four tests. *18*

1st test	17
2nd test	20
3rd test	18
4th test	17

2. Matt made scores of 70, 84, 75, 90, and 86 on his mathematics tests. What is his average test score for these five tests? *81*

- Part 1** 3. shows the number of people who used the library on different days
7. Mon. 15, Tues. 25, Wed. 20, Thurs. 20, Fri. 30

Checkup Time

The numerals in () tell the pages where you can turn for help.

Important Ideas

1. A picture graph is a drawing which displays information by using a picture or a symbol to stand for a certain number of things. (283)

2. A bar graph is a drawing which displays numerical information by using bars. (288)

3. When pairs of numbers are recorded like (4,32), (5,35), (6,36), (7,37), (8,40), (9,43), (10,50), and (11,56), we say that they have been ordered. Or we say that they are ordered pairs of numbers. (290)

4. A line graph is a drawing which displays information by using points and line segments. (294)

5. To find the average of a set of numbers, find their sum and divide this sum by the number of addends. (296)

Words to Know

1. Picture graph (283)
2. Bar graph (288)
3. Ordered pairs of numbers (290)
4. Line graph (294)
5. Average (296)

Questions to Discuss

See page T299.

1. How would you make a picture graph for the following information? (286)

Karen, Jean, Laura, and Connie are members of the band. They are selling tickets for the concert. Karen sold 40 tickets, Jean sold 35 tickets. Laura sold 20 tickets. Connie sold 60 tickets.

2. How does a bar graph differ from a picture graph? (288)

3. How would you make a line graph for the information in the table shown below? (294)

Test	1	2	3	4	5	6
Score	90	80	85	75	90	95

4. How would you find the average of this set of numbers? (296)

{75, 90, 85, 80, 95}

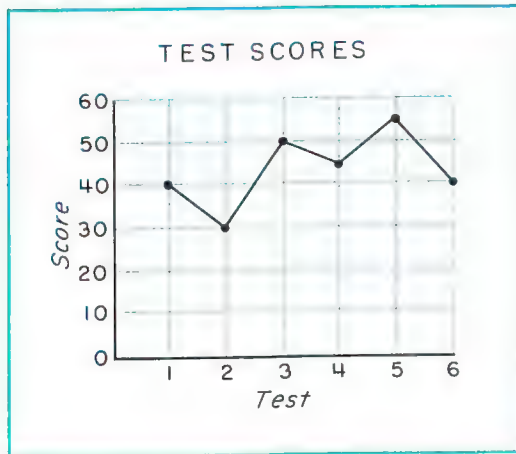
Written Practice

1. Make a picture graph using the information in *Questions to Discuss* 1. (286)

2. Make a line graph using the information in *Questions to Discuss* 3. (294)

Self-Evaluation

Part 1 Use the graph below to help you answer the following.



1. What kind of graph is it?
line graph
2. What is its title? **Test Scores**
3. What does each numeral on the horizontal number line represent?
a test
4. What does each segment on the vertical number line represent?
10 points
5. What is the graph about?
scores obtained on 6 different tests
6. On what test was the highest score made? What is that score?
test 5; 55
7. On which test was the lowest score made? What is that score?
test 2; 30
8. What score was made on the third test? The sixth test? **50; 40**
9. On which two tests was the same score made? **tests 1 and 6**

Part 2 Make a line graph of the information given in the following table.

The Stevens family were on a touring vacation. Frank recorded the miles traveled each day as shown in the table below.

Monday	350
Tuesday	250
Wednesday	300
Thursday	250
Friday	200

Part 3 Solve the following.

1. Mr. Anderson drove his car 34 miles on Monday. On Tuesday he drove it 26 miles. On Wednesday he drove it 35 miles. On Thursday he drove it 29 miles. What was the average number of miles per day that he drove his car? **31**

Find the average of each set of numbers below.

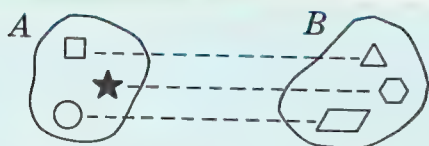
2. {55, 50, 47, 60} **53**
3. {18, 16, 17, 19, 15} **17**
4. {82, 85, 90, 78, 84, 85} **84**
5. {142, 159, 142, 141} **146**
6. {1996, 1985, 2014, 2025} **2005**

Chapter 14

REVIEW EXERCISES

This chapter contains a review of many important ideas you have used. Study the statement and the example in each colored region before doing the exercises that follow it.

Sets, Numbers, Numerals



Set A is matched one-to-one with set B . Sets A and B are called equivalent sets.

Use after page 6.

Use the following sets to complete each sentence below.



1. Set C is equivalent to set H.
2. Set F is equivalent to set E.
3. Set D is equivalent to set G .
4. Set F is equivalent to set E .
5. Set G is a subset of set C.
6. Set H is a subset of set E.

$n(A)$ is read *the number of set A*.
 $=$ is read *is equal to*.

$n(B)=7$ is read *the number of set B is equal to seven*.

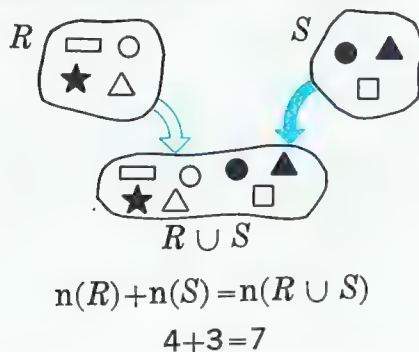
Use after page 9.

Copy and complete each row.

Set	Word	Numeral
1.	<u>five</u>	<u>5</u>
2.	<u>zero</u>	0
3.	nine	<u>9</u>
4.	<u>seven</u>	<u>7</u>
5.	<u>six</u>	6
6.	four	<u>4</u>
7.	<u>three</u>	<u>3</u>
8.	eight	<u>8</u>

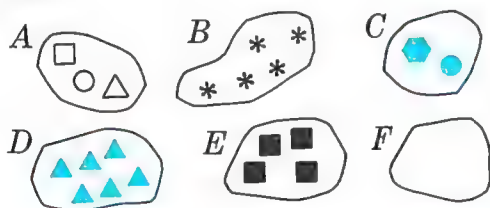
Addition and Subtraction

When two sets are joined, you think of adding their numbers.



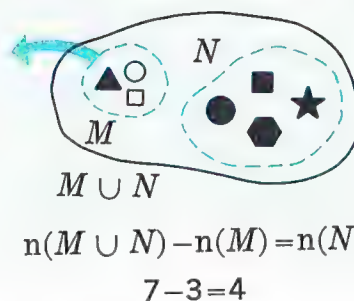
Use after page 12.

Look at the set pictures below. Then write an addition sentence by replacing $n(A)$, $n(B)$, $n(A \cup B)$, and so on, by numerals for each sentence below.



1. $n(A) + n(B) = n(A \cup B)$ $3 + 5 = 8$
2. $n(D) + n(F) = n(D \cup F)$ $6 + 0 = 6$
3. $n(E \cup C) = n(E) + n(C)$ $6 = 4 + 2$
4. $n(D \cup B) = n(D) + n(B)$ $11 = 6 + 5$
5. $n(F) + n(A) = n(F \cup A)$ $0 + 3 = 3$
6. $n(B \cup E) = n(B) + n(E)$ $9 = 5 + 4$
7. $n(A \cup D) = n(A) + n(D)$ $9 = 3 + 6$

When a subset of a set is removed, you think of subtracting numbers.



Use after page 13.

Use the sets at the left to write a subtraction sentence like $7 - 3 = 4$ for each of the following.

- | a | b |
|---------------------------------|------------------------------|
| 1. $n(B) - n(C)$
$5 - 2 = 3$ | $n(B) - n(E)$
$5 - 4 = 1$ |
| 2. $n(A) - n(F)$
$3 - 0 = 3$ | $n(E) - n(A)$
$4 - 3 = 1$ |
| 3. $n(D) - n(C)$
$6 - 2 = 4$ | $n(D) - n(B)$
$6 - 5 = 1$ |
| 4. $n(C) - n(F)$
$2 - 0 = 2$ | $n(E) - n(F)$
$4 - 0 = 4$ |
- \neq is read *is not equal to*.

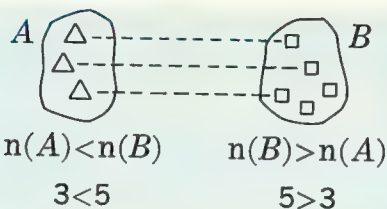
$n(M) \neq n(N)$ is read *the number of set M is not equal to the number of set N*.

Use after page 14.

Copy. Replace each \neq by either $=$ or \neq to make the sentence true.

- | $a \neq$ | $= b$ | $c \neq$ |
|-------------------|----------------|----------------|
| 1. $3 + 2 \neq 7$ | $9 = 5 + 4$ | $6 + 2 \neq 9$ |
| 2. $8 - 5 = 3$ | $4 \neq 6 - 3$ | $7 - 4 \neq 2$ |

Number Sentences



Use after page 16.

Copy. Replace each \bullet with $<$, $>$, or $=$ so each sentence is true.

- | a | b | c |
|---------------------------|------------------------|------------------------|
| 1. $7 \bullet 8-4$ | $8 \div 2 \bullet 3$ | $5+2 \bullet 9$ |
| 2. $9 \bullet 3+6$ | $1 \bullet 7 \div 7$ | $9-4 \bullet 2$ |
| 3. $4 \bullet 2 \times 3$ | $1 \bullet 7-7$ | $2 \times 4 \bullet 8$ |
| 4. $0 \bullet 7-6$ | $3 \times 4 \bullet 9$ | $9 \div 3 \bullet 5$ |

To solve an open sentence means to find its solution set.

Replacement set: $\{0,1,2,3,4\}$

Open sentence: $\square + 3 < 6$

Solution set: $\{0,1,2\}$

Use after page 19.

Copy. Use $\{0,1,2,3,4,5,6\}$ as the replacement set. Solve each open sentence.

- | a | b | c |
|-----------------------------------|--|-----------------------------------|
| 1. $\square + 3 < 6$
$0, 1, 2$ | $4 + \square < 6$
$0, 1$ | $8 - \square = 4$
4 |
| 2. $\square + 3 = 6$
3 | $9 - \square > 2$
$0, 1, 2, 3, 4, 5, 6$ | $8 - \square > 4$
$0, 1, 2, 3$ |
| 3. $\square + 3 > 6$
$4, 5, 6$ | $\square + 5 > 8$
$4, 5, 6$ | $8 - \square < 4$
$5, 6$ |

Use after page 26.

- | a | b | c | d |
|-----------|------------|------------|------------|
| 1. $50+2$ | $100+50+4$ | $700+20$ | $300+30+3$ |
| 2. $70+4$ | $300+20+1$ | $500+10+6$ | $400+70$ |
| 3. $30+2$ | $600+40+9$ | $100+7$ | $800+60+5$ |

The meaning of a decimal numeral can be shown by expanded notation.

$$357 = 300 + 50 + 7$$

Use after page 26.

Name each of the following numbers in expanded notation. See below.

- | | a | b | c | d |
|----|-----|-----|-----|-----|
| 1. | 52 | 154 | 720 | 333 |
| 2. | 74 | 321 | 516 | 470 |
| 3. | 32 | 649 | 107 | 865 |

Write the simplest name for each number named below.

- | | a | b | c |
|----|----------------|---------------------|-------------------|
| 4. | $30+4$
34 | $200+80$
280 | $900+8$
908 |
| 5. | $70+6$
76 | $500+10+7$
517 | $400+60$
460 |

Roman numerals are also used.

$I=1$ $IV=4$ $X=10$ $IX=9$
 $V=5$ $VI=6$ $XX=20$ $XI=11$

Use after page 29.

Write a Roman numeral for each decimal numeral. Write a decimal numeral for each Roman numeral.

- | | a | b | c | d |
|----|------------|---------------|------------|------------|
| 1. | 7
VII | 34
$XXXIV$ | VII
7 | XXI
21 |
| 2. | 3
III | 26
$XXVI$ | XXIV
24 | XIV
14 |
| 3. | 9
IX | 19
XIX | XIX
19 | XXXI
31 |

Properties of Addition and Subtraction

Addition is commutative. Changing the order of the addends does not change the sum.

$$7+4=4+7$$

Zero is the identity number of addition. The sum of zero and any number is that number.

Use after page 35.

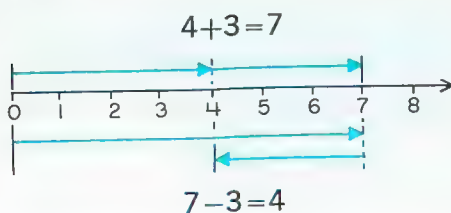
Copy. Replace each \square with the simplest numeral to make each sentence true.

- | a | b |
|--------------------|---------------|
| 1. $3+4=\square+3$ | $3+0=\square$ |
| 2. $6+5=5+\square$ | $\square+5=5$ |
| 3. $0+3=3+\square$ | $\square+0=0$ |
| 4. $\square+8=8+7$ | $7+\square=7$ |

Addition and subtraction are inverse operations.

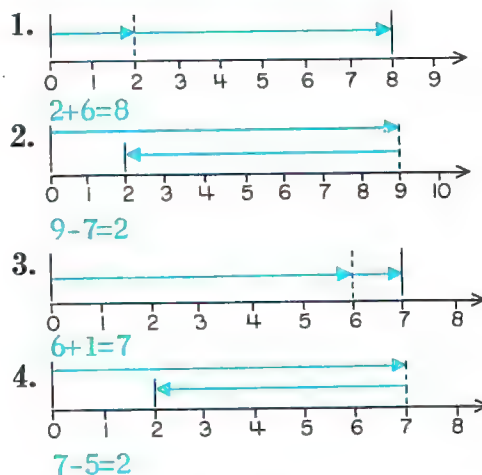
If $4+3=7$, then $7-3=4$.

This can be shown on a number line.



Use after page 39.

Write an addition or a subtraction sentence for each number line below.



Write an addition sentence for each subtraction sentence below.

- | a | b | c |
|-----------------------|----------------------|----------------------|
| 5. $9-5=4$
$4+5=9$ | $10-7=3$
$3+7=10$ | $14-6=8$
$8+6=14$ |
| 6. $6-4=2$
$2+4=6$ | $11-6=5$
$5+6=11$ | $15-9=6$
$6+9=15$ |
| 7. $8-0=8$
$8+0=8$ | $13-4=9$
$9+4=13$ | $17-8=9$
$9+8=17$ |

Write a subtraction sentence for each addition sentence below.

- | a | b | c |
|--------------------------|----------------------|----------------------|
| 8. $3+5=8$
$8-5=3$ | $7+8=15$
$15-8=7$ | $3+6=9$
$9-6=3$ |
| 9. $4+9=13$
$13-9=4$ | $9+0=9$
$9-0=9$ | $5+6=11$
$11-6=5$ |
| 10. $7+1=8$
$8-1=7$ | $5+7=12$
$12-7=5$ | $4+6=10$
$10-6=4$ |
| 11. $6+9=15$
$15-9=6$ | $3+8=11$
$11-8=3$ | $3+7=10$
$10-7=3$ |

Addition and Subtraction

Addition is associative. The way you group addends does not change the sum.

$$(3+4)+2=3+(4+2)$$

Use after page 41.

Copy. Find each sum.

- | a | b |
|----------------------------|-------------------------|
| 8 | 8 |
| 1. $(3+1)+4=\square$
13 | $3+(1+4)=\square$
13 |
| 2. $5+(5+3)=\square$
15 | $(5+5)+3=\square$
15 |
| 3. $(3+7)+5=\square$
13 | $3+(7+5)=\square$
13 |
| 4. $2+(8+3)=\square$
16 | $(2+8)+3=\square$
16 |
| 5. $(9+1)+6=\square$ | $9+(1+6)=\square$ |

To find the sum of 135 and 327, add the ones, add the tens, and add the hundreds. If the sum of the ones is greater than 9, rename it in expanded notation.

1 1 3 5 +3 2 7 — 2	1 1 3 5 +3 2 7 — 6 2	1 1 3 5 +3 2 7 — 4 6 2
--------------------------------	----------------------------------	------------------------------------

Use after page 49.

Copy. Find each sum.

- | a | b | c |
|--|---|---|
| 1. $\begin{array}{r} 36 \\ +21 \\ \hline 57 \end{array}$ | $\begin{array}{r} 36 \\ +27 \\ \hline 63 \end{array}$ | $\begin{array}{r} 57 \\ +18 \\ \hline 75 \end{array}$ |
| 2. $\begin{array}{r} 43 \\ +52 \\ \hline 95 \end{array}$ | $\begin{array}{r} 29 \\ +14 \\ \hline 43 \end{array}$ | $\begin{array}{r} 52 \\ +29 \\ \hline 81 \end{array}$ |

- | | | |
|---|--|--|
| 3. $\begin{array}{r} 124 \\ +23 \\ \hline 147 \end{array}$ | $\begin{array}{r} 254 \\ +18 \\ \hline 272 \end{array}$ | $\begin{array}{r} 367 \\ +28 \\ \hline 395 \end{array}$ |
| 4. $\begin{array}{r} 315 \\ +132 \\ \hline 447 \end{array}$ | $\begin{array}{r} 426 \\ +309 \\ \hline 735 \end{array}$ | $\begin{array}{r} 538 \\ +126 \\ \hline 664 \end{array}$ |
| 5. $\begin{array}{r} 204 \\ +372 \\ \hline 576 \end{array}$ | $\begin{array}{r} 172 \\ +519 \\ \hline 691 \end{array}$ | $\begin{array}{r} 324 \\ +538 \\ \hline 862 \end{array}$ |

To subtract 318 from 562, subtract the ones, subtract the tens, and subtract the hundreds. If you cannot subtract in any place-value position, rename 562.

5 12 5 6 2 -3 1 8 — 4	5 12 5 6 2 -3 1 8 — 4 4	5 12 5 6 2 -3 1 8 — 2 4 4
--	--	--

Use after page 51.

Copy. Find each difference.

- | a | b | c |
|---|--|--|
| 1. $\begin{array}{r} 79 \\ -54 \\ \hline 25 \end{array}$ | $\begin{array}{r} 82 \\ -35 \\ \hline 47 \end{array}$ | $\begin{array}{r} 64 \\ -29 \\ \hline 35 \end{array}$ |
| 2. $\begin{array}{r} 75 \\ -14 \\ \hline 61 \end{array}$ | $\begin{array}{r} 60 \\ -35 \\ \hline 25 \end{array}$ | $\begin{array}{r} 93 \\ -47 \\ \hline 46 \end{array}$ |
| 3. $\begin{array}{r} 157 \\ -32 \\ \hline 125 \end{array}$ | $\begin{array}{r} 248 \\ -19 \\ \hline 229 \end{array}$ | $\begin{array}{r} 526 \\ -18 \\ \hline 508 \end{array}$ |
| 4. $\begin{array}{r} 375 \\ -234 \\ \hline 141 \end{array}$ | $\begin{array}{r} 470 \\ -253 \\ \hline 217 \end{array}$ | $\begin{array}{r} 834 \\ -607 \\ \hline 227 \end{array}$ |
| 5. $\begin{array}{r} 479 \\ -368 \\ \hline 111 \end{array}$ | $\begin{array}{r} 631 \\ -512 \\ \hline 119 \end{array}$ | $\begin{array}{r} 453 \\ -218 \\ \hline 235 \end{array}$ |

Addition and Subtraction

If the sum of the tens is greater than 90, rename it in expanded notation.

$$\begin{array}{r} 564 \\ +273 \\ \hline 7 \end{array} \quad \begin{array}{r} \overset{1}{5}64 \\ +273 \\ \hline 37 \end{array} \quad \begin{array}{r} \overset{1}{5}64 \\ +273 \\ \hline 837 \end{array}$$

Use after page 53.

Copy. Find each sum.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 437 \\ +126 \\ \hline 563 \end{array}$	$\begin{array}{r} 437 \\ +180 \\ \hline 617 \end{array}$	$\begin{array}{r} 437 \\ +186 \\ \hline 623 \end{array}$
2.	$\begin{array}{r} 564 \\ +228 \\ \hline 792 \end{array}$	$\begin{array}{r} 564 \\ +271 \\ \hline 835 \end{array}$	$\begin{array}{r} 564 \\ +278 \\ \hline 842 \end{array}$
3.	$\begin{array}{r} 145 \\ +429 \\ \hline 574 \end{array}$	$\begin{array}{r} 145 \\ +392 \\ \hline 537 \end{array}$	$\begin{array}{r} 145 \\ +499 \\ \hline 644 \end{array}$
4.	$\begin{array}{r} 349 \\ +216 \\ \hline 565 \end{array}$	$\begin{array}{r} 349 \\ +480 \\ \hline 829 \end{array}$	$\begin{array}{r} 349 \\ +286 \\ \hline 635 \end{array}$
5.	$\begin{array}{r} 642 \\ +109 \\ \hline 751 \end{array}$	$\begin{array}{r} 642 \\ +263 \\ \hline 905 \end{array}$	$\begin{array}{r} 642 \\ +269 \\ \hline 911 \end{array}$
6.	$\begin{array}{r} 821 \\ +59 \\ \hline 880 \end{array}$	$\begin{array}{r} 821 \\ +92 \\ \hline 913 \end{array}$	$\begin{array}{r} 821 \\ +99 \\ \hline 920 \end{array}$
7.	$\begin{array}{r} 426 \\ +317 \\ \hline 743 \end{array}$	$\begin{array}{r} 426 \\ +192 \\ \hline 618 \end{array}$	$\begin{array}{r} 426 \\ +397 \\ \hline 823 \end{array}$
8.	$\begin{array}{r} 547 \\ +309 \\ \hline 856 \end{array}$	$\begin{array}{r} 547 \\ +180 \\ \hline 727 \end{array}$	$\begin{array}{r} 547 \\ +389 \\ \hline 936 \end{array}$
9.	$\begin{array}{r} 193 \\ +308 \\ \hline 501 \end{array}$	$\begin{array}{r} 193 \\ +380 \\ \hline 573 \end{array}$	$\begin{array}{r} 193 \\ +438 \\ \hline 631 \end{array}$

If you cannot subtract in every place-value position, rename the minuend so you can subtract in every place-value position.

$$\begin{array}{r} \overset{1}{7}\overset{14}{2}47 \\ -347 \\ \hline 7 \end{array} \quad \begin{array}{r} \overset{6}{7}\overset{11}{2}47 \\ -347 \\ \hline 77 \end{array} \quad \begin{array}{r} \overset{6}{7}\overset{11}{2}47 \\ -347 \\ \hline 377 \end{array}$$

Use after page 55.

Copy. Find each difference.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 756 \\ -428 \\ \hline 328 \end{array}$	$\begin{array}{r} 756 \\ -472 \\ \hline 284 \end{array}$	$\begin{array}{r} 756 \\ -478 \\ \hline 278 \end{array}$
2.	$\begin{array}{r} 643 \\ -108 \\ \hline 535 \end{array}$	$\begin{array}{r} 643 \\ -182 \\ \hline 461 \end{array}$	$\begin{array}{r} 643 \\ -188 \\ \hline 455 \end{array}$
3.	$\begin{array}{r} 356 \\ -227 \\ \hline 129 \end{array}$	$\begin{array}{r} 356 \\ -280 \\ \hline 76 \end{array}$	$\begin{array}{r} 356 \\ -287 \\ \hline 69 \end{array}$
4.	$\begin{array}{r} 946 \\ -439 \\ \hline 507 \end{array}$	$\begin{array}{r} 946 \\ -461 \\ \hline 485 \end{array}$	$\begin{array}{r} 946 \\ -469 \\ \hline 477 \end{array}$
5.	$\begin{array}{r} 513 \\ -307 \\ \hline 206 \end{array}$	$\begin{array}{r} 513 \\ -322 \\ \hline 191 \end{array}$	$\begin{array}{r} 513 \\ -327 \\ \hline 186 \end{array}$
6.	$\begin{array}{r} 425 \\ -116 \\ \hline 309 \end{array}$	$\begin{array}{r} 425 \\ -134 \\ \hline 291 \end{array}$	$\begin{array}{r} 425 \\ -136 \\ \hline 289 \end{array}$
7.	$\begin{array}{r} 738 \\ -409 \\ \hline 329 \end{array}$	$\begin{array}{r} 738 \\ -446 \\ \hline 292 \end{array}$	$\begin{array}{r} 738 \\ -469 \\ \hline 269 \end{array}$
8.	$\begin{array}{r} 843 \\ -515 \\ \hline 328 \end{array}$	$\begin{array}{r} 843 \\ -571 \\ \hline 272 \end{array}$	$\begin{array}{r} 843 \\ -575 \\ \hline 268 \end{array}$

Multiplication and Division

You can think of multiplication as joining equivalent sets or as repeated addition.



$$2+2+2+2+2=5\times 2=10$$

Use after page 62.

Copy. Replace each \square by the simplest numeral. Then write a multiplication sentence for each addition sentence.

$$1. \quad 7+7+7+7+7+7=\square \quad \overset{42}{6\times 7=42}$$

$$2. \quad 3+3+3+3=\square \quad \overset{12}{4\times 3=12}$$

$$3. \quad 4+4+4=\square \quad \overset{12}{3\times 4=12}$$

$$4. \quad 5+5+5+5+5=\square \quad \overset{25}{5\times 5=25}$$

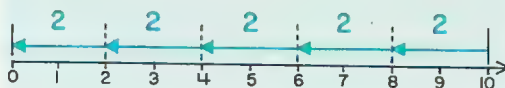
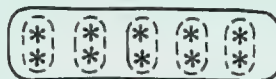
$$5. \quad 8+8+8+8+8+8=\square \quad \overset{48}{6\times 8=48}$$

$$6. \quad 9+9+9+9=\square \quad \overset{36}{4\times 9=36}$$

Copy. Replace each \square by the simplest numeral.

- | | a | b | c |
|-----|---------------------|---------------------|---------------------|
| 7. | $3\times 9=\square$ | $9\times 3=\square$ | $\square=4\times 7$ |
| 8. | $4\times 5=\square$ | $5\times 4=\square$ | $\square=5\times 8$ |
| 9. | $5\times 6=\square$ | $6\times 5=\square$ | $\square=6\times 4$ |
| 10. | $2\times 7=\square$ | $7\times 2=\square$ | $\square=3\times 7$ |
| 11. | $8\times 3=\square$ | $3\times 8=\square$ | $\square=5\times 5$ |

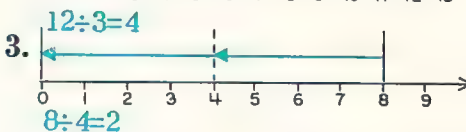
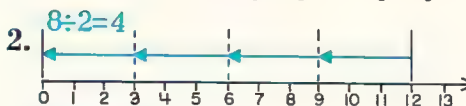
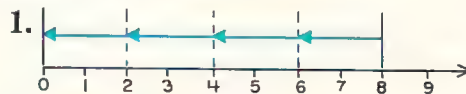
You can think of division as separating a set into equivalent subsets or as repeated subtraction.



$$10\div 2=5$$

Use after page 65.

Write a division sentence for each number line drawing.



Copy. Replace each \square by the simplest numeral.

- | | a | b |
|----|--------------------|--------------------|
| 4. | $15\div 3=\square$ | $15\div 5=\square$ |
| 5. | $12\div 6=\square$ | $12\div 2=\square$ |
| 6. | $21\div 7=\square$ | $21\div 3=\square$ |
| 7. | $12\div 4=\square$ | $12\div 3=\square$ |
| 8. | $20\div 4=\square$ | $20\div 5=\square$ |

Multiplication and Division

Multiplication is commutative. Changing the order of the factors does not change the product.

$$5 \times 6 = 6 \times 5$$

The product of any number and zero is zero.

$$3 \times 0 = 0$$

$$0 \times 9 = 0$$

Use after page 69.

Copy. Replace each \square by the simplest numeral.

a

b

1. $5 \times 7 = \square \times 5$

$8 \times 0 = \square$

2. $7 \times \square = 4 \times 7$

$1 \times 6 = \square$

3. $9 \times 8 = \square \times 9$

$\square \times 7 = 0$

4. $6 \times 3 = 3 \times \square$

$5 \times \square = 5$

5. $\square \times 5 = 5 \times 8$

$0 \times 0 = \square$

6. $6 \times \square = 9 \times 6$

$0 \times 1 = \square$

7. $\square \times 4 = 4 \times 8$

$\square \times 1 = 7$

8. $8 \times 2 = 2 \times \square$

$8 \times \square = 8$

9. $1 \times 3 = 3 \times \square$

$\square \times 4 = 4$

10. $6 \times \square = 0 \times 6$

$\square \times 4 = 0$

11. $\square \times 6 = 6 \times 7$

$\square \times 4 = 8$

12. $8 \times 1 = \square \times 8$

$\square \times 4 = 12$

There are two division sentences for every multiplication sentence, and there are two multiplication sentences for each division sentence.

$$7 \times 2 = 14 \begin{cases} 14 \div 2 = 7 \\ 14 \div 7 = 2 \end{cases}$$

$$24 \div 4 = 6 \begin{cases} 6 \times 4 = 24 \\ 4 \times 6 = 24 \end{cases}$$

Use after page 75.

Write two division sentences for each multiplication sentence.

Representative answers only.

a

b

c

1. $5 \times 6 = 30$ $4 \times 8 = 32$ $5 \times 3 = 15$
 $30 \div 6 = 5$ $30 \div 5 = 6$

2. $2 \times 9 = 18$ $7 \times 3 = 21$ $4 \times 5 = 20$
 $18 \div 9 = 2$ $18 \div 2 = 9$

3. $1 \times 7 = 7$ $7 \times 5 = 35$ $6 \times 9 = 54$
 $7 \div 7 = 1$ $7 \div 1 = 7$

4. $8 \times 7 = 56$ $9 \times 4 = 36$ $8 \times 9 = 72$
 $56 \div 7 = 8$ $56 \div 8 = 7$

5. $6 \times 7 = 42$ $8 \times 2 = 16$ $4 \times 7 = 28$
 $42 \div 7 = 6$ $42 \div 6 = 7$

Write two multiplication sentences for each division sentence. **Representative answers only.**

a

b

c

6. $48 \div 6 = 8$ $24 \div 6 = 4$ $35 \div 7 = 5$
 $8 \times 6 = 48$ $6 \times 8 = 48$

7. $72 \div 8 = 9$ $18 \div 9 = 2$ $42 \div 6 = 7$
 $9 \times 8 = 72$ $8 \times 9 = 72$

8. $28 \div 4 = 7$ $63 \div 7 = 9$ $54 \div 9 = 6$
 $7 \times 4 = 28$ $4 \times 7 = 28$

9. $15 \div 3 = 5$ $18 \div 6 = 3$ $16 \div 2 = 8$
 $5 \times 3 = 15$ $3 \times 5 = 15$

10. $45 \div 5 = 9$ $40 \div 8 = 5$ $20 \div 4 = 5$
 $9 \times 5 = 45$ $5 \times 9 = 45$

Multiplication and Division

You can express division in either way shown below.

$$15 \div 3 = 5 \qquad 3 \overline{)15} \begin{matrix} 5 \end{matrix}$$

Use after page 77.

Copy. Find each quotient.

- | <i>a</i> | <i>b</i> |
|----------------------------|-------------------------|
| 1. $18 \div 2 = \boxed{9}$ | $32 \div 4 = \boxed{8}$ |
| 2. $64 \div 8 = \boxed{8}$ | $21 \div 7 = \boxed{3}$ |
| 3. $72 \div 9 = \boxed{8}$ | $54 \div 6 = \boxed{9}$ |

Copy. Find each quotient.

- | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|---|--|--|--|
| 4. $2 \overline{)16} \begin{matrix} 8 \end{matrix}$ | $3 \overline{)24} \begin{matrix} 8 \end{matrix}$ | $4 \overline{)36} \begin{matrix} 9 \end{matrix}$ | $7 \overline{)63} \begin{matrix} 9 \end{matrix}$ |
| 5. $5 \overline{)35} \begin{matrix} 7 \end{matrix}$ | $6 \overline{)42} \begin{matrix} 7 \end{matrix}$ | $7 \overline{)56} \begin{matrix} 8 \end{matrix}$ | $8 \overline{)32} \begin{matrix} 4 \end{matrix}$ |
| 6. $8 \overline{)24} \begin{matrix} 3 \end{matrix}$ | $9 \overline{)27} \begin{matrix} 3 \end{matrix}$ | $5 \overline{)45} \begin{matrix} 9 \end{matrix}$ | $3 \overline{)18} \begin{matrix} 6 \end{matrix}$ |

Multiplication is associative.

$$(3 \times 2) \times 4 = 3 \times (2 \times 4)$$

Use after page 79.

Copy. Find each product.

- | <i>a</i> | <i>b</i> |
|---|--------------------------------------|
| 1. $(2 \times 4) \times 3 = \boxed{24}$ | $2 \times (4 \times 3) = \boxed{24}$ |
| 2. $5 \times (3 \times 2) = \boxed{30}$ | $(5 \times 3) \times 2 = \boxed{30}$ |
| 3. $(2 \times 2) \times 7 = \boxed{28}$ | $2 \times (2 \times 7) = \boxed{28}$ |
| 4. $(4 \times 2) \times 9 = \boxed{72}$ | $4 \times (2 \times 9) = \boxed{72}$ |

Knowing the products and quotients in a multiplication-division table helps you find products and quotients involving tens and hundreds.

$5 \times 3 = 15$	$8 \div 2 = 4$
$5 \times 30 = 150$	$80 \div 2 = 40$
$5 \times 300 = 1500$	$800 \div 2 = 400$

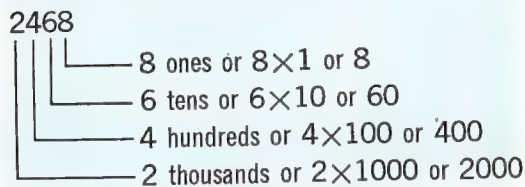
Use after page 83.

Copy. Replace each \square by the simplest numeral.

- | <i>a</i> | <i>b</i> |
|---|--|
| 1. $4 \times 30 = \square$ 120 | $4 \times 300 = \square$ 1200 |
| 2. $3 \times 20 = \square$ 60 | $3 \times 200 = \square$ 600 |
| 3. $5 \times 60 = \square$ 300 | $5 \times 600 = \square$ 3000 |
| 4. $7 \times 40 = \square$ 280 | $7 \times 400 = \square$ 2800 |
| 5. $3 \times 90 = \square$ 270 | $3 \times 900 = \square$ 2700 |
| 6. $40 \div 2 = \square$ 20 | $400 \div 2 = \square$ 200 |
| 7. $90 \div 3 = \square$ 30 | $900 \div 3 = \square$ 300 |
| 8. $40 \div 5 = \square$ 8 | $400 \div 5 = \square$ 80 |
| 9. $150 \div 5 = \square$ 30 | $1500 \div 5 = \square$ 300 |
| 10. $210 \div 7 = \square$ 30 | $2100 \div 7 = \square$ 300 |
| 11. $280 \div 4 = \square$ 70 | $2800 \div 4 = \square$ 700 |
| 12. $450 \div 9 = \square$ 50 | $4500 \div 9 = \square$ 500 |

Numeration

Each digit in a numeral has a value depending on the place it occupies in the numeral.



Use after page 89.

Write a multiplication numeral for the value of each digit in each numeral below. *See below for representative answers.*

	<i>a</i>	<i>b</i>	<i>c</i>
1.	341	2041	20,415
2.	627	6809	62,701
3.	140	8714	70,983
4.	802	9270	22,222
5.	793	4137	99,999

1,023,007,894

The numeral above is read as follows.

1 billion, 23 million, 7 thousand, 894

Use after page 91.

Write each of the following as a simplest numeral.

- 3 billion, 2 million, 480
3,002,000,480
- 7 million, 68 thousand, 324
7,068,324

310

Use after page 89.

- $(3 \times 100) + (4 \times 10) + (1 \times 1)$
- $(2 \times 1000) + (0 \times 100) + (4 \times 10) + (1 \times 1)$
- $(2 \times 10000) + (0 \times 1000) + (4 \times 100) + (1 \times 10) + (5 \times 1)$

3. 4 billion, 230 thousand, 94
4,000,230,094

4. 2 billion, 4 million, 5
2,004,000,005

5. 6 million, 87 thousand
6,087,000

6. 9 billion, 43 thousand, 892
9,000,043,892

7. 1 million, 132 thousand, 304
1,132,304

5,256 can be rounded off as follows.

To the nearest thousand 5000

To the nearest hundred 5300

To the nearest ten 5260

Use after page 95.

Estimate each sum or difference. Record your estimate. Then find the exact sum or difference. Compare your estimate with the exact answer.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 79 \ 80 \\ +68 \ +70 \\ \hline 147 \ 150 \end{array}$	$\begin{array}{r} 197 \ 200 \\ -82 \ -80 \\ \hline 115 \ 120 \end{array}$	$\begin{array}{r} 206 \ 210 \\ +195 \ +200 \\ \hline 401 \ 410 \end{array}$
2.	$\begin{array}{r} 92 \ 90 \\ -58 \ -60 \\ \hline 34 \ 30 \end{array}$	$\begin{array}{r} 32 \ 30 \\ +49 \ +50 \\ \hline 81 \ 80 \end{array}$	$\begin{array}{r} 202 \ 200 \\ -98 \ -100 \\ \hline 104 \ 100 \end{array}$
3.	$\begin{array}{r} 81 \ 80 \\ +73 \ +70 \\ \hline 154 \ 150 \end{array}$	$\begin{array}{r} 71 \ 70 \\ -43 \ -40 \\ \hline 28 \ 30 \end{array}$	$\begin{array}{r} 498 \ 500 \\ +199 \ +200 \\ \hline 697 \ 700 \end{array}$
4.	$\begin{array}{r} 73 \ 70 \\ -39 \ -40 \\ \hline 34 \ 30 \end{array}$	$\begin{array}{r} 84 \ 80 \\ +58 \ +60 \\ \hline 142 \ 140 \end{array}$	$\begin{array}{r} 598 \ 600 \\ -199 \ -200 \\ \hline 399 \ 400 \end{array}$
5.	$\begin{array}{r} 91 \ 90 \\ +87 \ +90 \\ \hline 178 \ 180 \end{array}$	$\begin{array}{r} 91 \ 90 \\ -38 \ -40 \\ \hline 53 \ 50 \end{array}$	$\begin{array}{r} 601 \ 600 \\ +289 \ +290 \\ \hline 890 \ 890 \end{array}$

Addition and Subtraction

To add 42,631 and 32,797, add the ones, add the tens, add the hundreds, and so on. Rename any of these sums that cannot be named by a single digit in that place-value position.

TTh	Th	H	T	O	
	1	1			
4	2	6	3	1	42631
+3	2	7	9	7	+32797
7	5	4	2	8	75428

Use after page 101.

Copy. Find each sum.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 4117 \\ +3682 \\ \hline 7799 \end{array}$	$\begin{array}{r} 18246 \\ +9702 \\ \hline 27948 \end{array}$	$\begin{array}{r} 96002 \\ +1978 \\ \hline 97980 \end{array}$
2.	$\begin{array}{r} 3962 \\ +1024 \\ \hline 4986 \end{array}$	$\begin{array}{r} 4390 \\ +21628 \\ \hline 26018 \end{array}$	$\begin{array}{r} 47311 \\ +12789 \\ \hline 60100 \end{array}$
3.	$\begin{array}{r} 2834 \\ +4096 \\ \hline 6930 \end{array}$	$\begin{array}{r} 20649 \\ +14871 \\ \hline 35520 \end{array}$	$\begin{array}{r} 64318 \\ +2701 \\ \hline 67019 \end{array}$
4.	$\begin{array}{r} 1987 \\ +6213 \\ \hline 8200 \end{array}$	$\begin{array}{r} 31764 \\ +10888 \\ \hline 42652 \end{array}$	$\begin{array}{r} 34276 \\ +50013 \\ \hline 84289 \end{array}$
5.	$\begin{array}{r} 2046 \\ +3007 \\ \hline 5053 \end{array}$	$\begin{array}{r} 10490 \\ +29819 \\ \hline 40309 \end{array}$	$\begin{array}{r} 62114 \\ +10688 \\ \hline 72802 \end{array}$
6.	$\begin{array}{r} 1251 \\ +980 \\ \hline 2231 \end{array}$	$\begin{array}{r} 6431 \\ +81993 \\ \hline 88424 \end{array}$	$\begin{array}{r} 84006 \\ +9896 \\ \hline 93902 \end{array}$
7.	$\begin{array}{r} 1476 \\ +2724 \\ \hline 4200 \end{array}$	$\begin{array}{r} 13419 \\ +4090 \\ \hline 17509 \end{array}$	$\begin{array}{r} 86401 \\ +2998 \\ \hline 89399 \end{array}$
8.	$\begin{array}{r} 3241 \\ +5984 \\ \hline 9225 \end{array}$	$\begin{array}{r} 24382 \\ +41718 \\ \hline 66100 \end{array}$	$\begin{array}{r} 64215 \\ +12087 \\ \hline 76302 \end{array}$

To subtract 11,083 from 60,149, subtract the ones, subtract the tens, subtract the hundreds, and so on. If you cannot subtract in every place-value position, rename the minuend so that you can subtract in every place-value position.

TTh	Th	H	T	O	
5	10	0	14		
6	0	1	4	9	60149
-1	1	0	8	3	-11083
4	9	0	6	6	49066

Use after page 105.

Copy. Find each difference.

	<i>a</i>	<i>b</i>	<i>c</i>
1.	$\begin{array}{r} 8417 \\ -2106 \\ \hline 6311 \end{array}$	$\begin{array}{r} 16934 \\ -3201 \\ \hline 13733 \end{array}$	$\begin{array}{r} 39872 \\ -11842 \\ \hline 28030 \end{array}$
2.	$\begin{array}{r} 9287 \\ -3012 \\ \hline 6275 \end{array}$	$\begin{array}{r} 61881 \\ -10821 \\ \hline 51060 \end{array}$	$\begin{array}{r} 18462 \\ -3161 \\ \hline 15301 \end{array}$
3.	$\begin{array}{r} 8645 \\ -2629 \\ \hline 6016 \end{array}$	$\begin{array}{r} 34921 \\ -11930 \\ \hline 22991 \end{array}$	$\begin{array}{r} 77304 \\ -21936 \\ \hline 55368 \end{array}$
4.	$\begin{array}{r} 9036 \\ -2286 \\ \hline 6750 \end{array}$	$\begin{array}{r} 10062 \\ -3718 \\ \hline 6344 \end{array}$	$\begin{array}{r} 24615 \\ -3148 \\ \hline 21467 \end{array}$
5.	$\begin{array}{r} 4310 \\ -1190 \\ \hline 3120 \end{array}$	$\begin{array}{r} 64002 \\ -11361 \\ \hline 52641 \end{array}$	$\begin{array}{r} 88771 \\ -29461 \\ \hline 59310 \end{array}$
6.	$\begin{array}{r} 8201 \\ -2190 \\ \hline 6011 \end{array}$	$\begin{array}{r} 49300 \\ -19165 \\ \hline 30135 \end{array}$	$\begin{array}{r} 70002 \\ -1899 \\ \hline 68103 \end{array}$
7.	$\begin{array}{r} 9426 \\ -2485 \\ \hline 6941 \end{array}$	$\begin{array}{r} 28643 \\ -9928 \\ \hline 18715 \end{array}$	$\begin{array}{r} 40102 \\ -10892 \\ \hline 29210 \end{array}$

Multiplication

Multiplication can be distributed over addition.

$$\begin{aligned} 8 \times 53 &= 8 \times (50 + 3) \\ &= (8 \times 50) + (8 \times 3) \\ &= 400 + 24 \\ &= 424 \end{aligned}$$

$$\begin{aligned} 53 \times 8 &= (50 + 3) \times 8 \\ &= (50 \times 8) + (3 \times 8) \\ &= 400 + 24 \\ &= 424 \end{aligned}$$

Use after page 111.

Copy. Find each product as shown above.

- | <i>a</i> | <i>b</i> |
|---|--|
| 1. $8 \times 12 = \square$ 96 | $15 \times 3 = \square$ 45 |
| 2. $6 \times 14 = \square$ 84 | $52 \times 9 = \square$ 468 |
| 3. $8 \times 24 = \square$ 192 | $38 \times 6 = \square$ 228 |
| 4. $3 \times 33 = \square$ 99 | $72 \times 5 = \square$ 360 |
| 5. $9 \times 41 = \square$ 369 | $86 \times 2 = \square$ 172 |
| 6. $8 \times 35 = \square$ 280 | $34 \times 9 = \square$ 306 |
| 7. $7 \times 59 = \square$ 413 | $59 \times 6 = \square$ 354 |
| 8. $4 \times 32 = \square$ 128 | $48 \times 7 = \square$ 336 |
| 9. $5 \times 18 = \square$ 90 | $19 \times 4 = \square$ 76 |
| 10. $2 \times 38 = \square$ 76 | $28 \times 3 = \square$ 84 |

Distributing multiplication over addition helps you find a product in simplest form.

$$\begin{array}{r} 8412 \\ \times 6 \\ \hline 12 \text{ ————— } 6 \times 2 \\ 60 \text{ ————— } 6 \times 10 \\ 2400 \text{ ————— } 6 \times 400 \\ 48000 \text{ ————— } 6 \times 8000 \\ \hline 50472 \end{array}$$

Use after page 117.

Copy. Find each product.

- | | <i>a</i> | <i>b</i> | <i>c</i> |
|----|--|--|--|
| 1. | $15 \times 8 = \square$ 120 | $131 \times 3 = \square$ 393 | $1354 \times 7 = \square$ 9478 |
| 2. | $49 \times 3 = \square$ 147 | $481 \times 5 = \square$ 2405 | $5031 \times 4 = \square$ 20124 |
| 3. | $38 \times 8 = \square$ 304 | $639 \times 9 = \square$ 5751 | $3692 \times 6 = \square$ 22152 |
| 4. | $42 \times 4 = \square$ 168 | $734 \times 6 = \square$ 4404 | $3714 \times 8 = \square$ 29712 |
| 5. | $27 \times 5 = \square$ 135 | $102 \times 4 = \square$ 408 | $9104 \times 9 = \square$ 81936 |
| 6. | $59 \times 7 = \square$ 413 | $625 \times 7 = \square$ 4375 | $8481 \times 5 = \square$ 42405 |
| 7. | $74 \times 9 = \square$ 666 | $265 \times 8 = \square$ 2120 | $3410 \times 3 = \square$ 10230 |

Division

A quotient may be expressed in different ways.

$12 \div 3$

$\frac{12}{3}$

$3 \overline{)12}$

Use after page 121.

Study the pattern in row 1. Complete each row so that the symbols name the same number.

$1. \quad 36 \div 9 \quad \frac{36}{9} \quad 9 \overline{)36} \quad 4$

$2. \quad 27 \div 3 \quad \frac{27}{3} \quad 3 \overline{)27} \quad 9$

$3. \quad 56 \div 8 \quad \frac{56}{8} \quad 8 \overline{)56} \quad 7$

$4. \quad 28 \div 4 \quad \frac{28}{4} \quad 4 \overline{)28} \quad 7$

$5. \quad 45 \div 5 \quad \frac{45}{5} \quad 5 \overline{)45} \quad 9$

Division can be distributed over addition when the dividend is named as a sum.

$$\begin{aligned} 96 \div 8 &= (80 + 16) \div 8 \\ &= (80 \div 8) + (16 \div 8) \\ &= 10 + 2 \\ &= 12 \end{aligned}$$

Use after page 123.

Copy. Find each quotient as shown above.

$a \quad 14 \quad b \quad 17$
 $1. \quad 84 \div 6 = \square \quad 68 \div 4 = \square$

$2. \quad 39 \div 3 = \square \quad 80 \div 5 = \square$

$3. \quad 84 \div 4 = \square \quad 55 \div 5 = \square$

$4. \quad 63 \div 3 = \square \quad 21$

$5. \quad 88 \div 4 = \square \quad 22$

$6. \quad 96 \div 6 = \square \quad 16$

$7. \quad 81 \div 3 = \square \quad 27$

$98 \div 7 = \square \quad 14$

$70 \div 5 = \square \quad 14$

$77 \div 7 = \square \quad 11$

$69 \div 3 = \square \quad 23$

You divide hundreds and thousands as you divide tens and ones.

$$\begin{array}{r} 1324 \\ 6 \overline{)7944} \\ \underline{6000} \\ 1944 \\ \underline{1800} \\ 144 \\ \underline{120} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

$$\begin{array}{r} 1217 \text{ r}2 \\ 8 \overline{)9738} \\ \underline{8000} \\ 1738 \\ \underline{1600} \\ 138 \\ \underline{80} \\ 58 \\ \underline{56} \\ 2 \end{array}$$

Use after page 131.

Find each quotient and remainder.

$a \quad 13 \quad b \quad 26 \quad c \quad 933$
 $1. \quad 6 \overline{)78} \quad 6 \overline{)156} \quad 7 \overline{)6531}$

$2. \quad 4 \overline{)68} \quad 7 \overline{)168} \quad 4 \overline{)3860}$

$3. \quad 7 \overline{)91} \quad 8 \overline{)184} \quad 2 \overline{)1560}$

$4. \quad 2 \overline{)35} \quad 5 \overline{)214} \quad 8 \overline{)6684}$

$5. \quad 3 \overline{)96} \quad 3 \overline{)198} \quad 7 \overline{)4872}$

$6. \quad 8 \overline{)97} \quad 7 \overline{)365} \quad 6 \overline{)4579}$

Multiplication

Using multiples of 10 as factors is easy if you know the multiples of numbers less than ten.

$$\begin{aligned} 60 \times 40 &= (6 \times 10) \times (4 \times 10) \\ &= (6 \times 4) \times (10 \times 10) \\ &= 24 \times 100 \\ &= 2400 \end{aligned}$$

Use after page 137.

Copy. Find each product.

- | | <i>a</i> | <i>b</i> |
|----|---|---|
| 1. | $7 \times 60 = \square$ 420 | $70 \times 60 = \square$ 4200 |
| 2. | $5 \times 80 = \square$ 400 | $50 \times 80 = \square$ 4000 |
| 3. | $6 \times 90 = \square$ 540 | $60 \times 90 = \square$ 5400 |
| 4. | $2 \times 50 = \square$ 100 | $20 \times 50 = \square$ 1000 |
| 5. | $3 \times 40 = \square$ 120 | $30 \times 40 = \square$ 1200 |
| 6. | $9 \times 40 = \square$ 360 | $90 \times 40 = \square$ 3600 |
| 7. | $8 \times 30 = \square$ 240 | $80 \times 30 = \square$ 2400 |
| 8. | $4 \times 70 = \square$ 280 | $40 \times 70 = \square$ 2800 |

You can find the simplest numeral for 382×167 as shown below.

$$\begin{array}{r} 167 \\ \times 382 \\ \hline 334 \text{ ————— } 2 \times 167 \\ 13360 \text{ ————— } 80 \times 167 \\ 50100 \text{ ————— } 300 \times 167 \\ \hline 63794 \end{array}$$

Use after page 143.

314

Use after page 145.

- | | <i>a</i> | <i>b</i> | <i>c</i> |
|----|-------------|--------------|---------------|
| 1. | <u>800</u> | <u>6000</u> | <u>450000</u> |
| 2. | <u>1600</u> | <u>11000</u> | <u>450000</u> |
| 3. | <u>2700</u> | <u>35000</u> | <u>156000</u> |
| 4. | <u>4000</u> | <u>28000</u> | <u>120000</u> |

Copy. Find each product.

- | | <i>a</i> | <i>b</i> | <i>c</i> |
|----|---|---|---|
| 1. | $\begin{array}{r} 24 \\ \times 35 \\ \hline \end{array}$ 840 | $\begin{array}{r} 149 \\ \times 14 \\ \hline \end{array}$ 2086 | $\begin{array}{r} 725 \\ \times 714 \\ \hline \end{array}$ 517650 |
| 2. | $\begin{array}{r} 15 \\ \times 42 \\ \hline \end{array}$ 630 | $\begin{array}{r} 408 \\ \times 23 \\ \hline \end{array}$ 9384 | $\begin{array}{r} 662 \\ \times 906 \\ \hline \end{array}$ 599772 |
| 3. | $\begin{array}{r} 87 \\ \times 18 \\ \hline \end{array}$ 1566 | $\begin{array}{r} 219 \\ \times 31 \\ \hline \end{array}$ 6789 | $\begin{array}{r} 275 \\ \times 135 \\ \hline \end{array}$ 37125 |
| 4. | $\begin{array}{r} 17 \\ \times 26 \\ \hline \end{array}$ 442 | $\begin{array}{r} 906 \\ \times 69 \\ \hline \end{array}$ 62514 | $\begin{array}{r} 307 \\ \times 207 \\ \hline \end{array}$ 63549 |

You can estimate a product by rounding off each factor to the nearest 10, 100, or 1000.

$\begin{array}{r} 704 \\ \times 598 \\ \hline \end{array}$	rounded to the nearest 100 rounded to the nearest 100	$\begin{array}{r} 700 \\ \times 600 \\ \hline \end{array}$
--	--	--

Use after page 145.

Record your estimate of each product. Find the exact product. See below for estimated products.

- | | <i>a</i> | <i>b</i> | <i>c</i> |
|----|---|---|---|
| 1. | $\begin{array}{r} 38 \\ \times 19 \\ \hline \end{array}$ 722 | $\begin{array}{r} 198 \\ \times 29 \\ \hline \end{array}$ 5742 | $\begin{array}{r} 902 \\ \times 501 \\ \hline \end{array}$ 451902 |
| 2. | $\begin{array}{r} 42 \\ \times 38 \\ \hline \end{array}$ 1596 | $\begin{array}{r} 109 \\ \times 98 \\ \hline \end{array}$ 10682 | $\begin{array}{r} 896 \\ \times 503 \\ \hline \end{array}$ 450688 |
| 3. | $\begin{array}{r} 29 \\ \times 91 \\ \hline \end{array}$ 2639 | $\begin{array}{r} 702 \\ \times 49 \\ \hline \end{array}$ 34398 | $\begin{array}{r} 389 \\ \times 405 \\ \hline \end{array}$ 157545 |
| 4. | $\begin{array}{r} 48 \\ \times 79 \\ \hline \end{array}$ 3792 | $\begin{array}{r} 698 \\ \times 42 \\ \hline \end{array}$ 29316 | $\begin{array}{r} 296 \\ \times 397 \\ \hline \end{array}$ 117512 |

Division and Solving Problems

In division you are really subtracting multiples of the divisor from the dividend.

$$\begin{array}{r}
 127 \\
 53 \overline{)6731} \begin{cases} 5T \times 1H < 6731 \\ 5T \times 2H > 6731 \end{cases} \\
 \underline{5300} \\
 1431 \begin{cases} 5T \times 2T < 1431 \\ 5T \times 3T > 1431 \end{cases} \\
 \underline{1060} \\
 371 \begin{cases} 5T \times 7 < 371 \\ 5T \times 8 > 371 \end{cases} \\
 \underline{371}
 \end{array}$$

Use after page 151.

Find each quotient and remainder.

a	b	c
1. $13 \overline{)91}$ <u>7</u>	$34 \overline{)982}$ <u>28r30</u>	$92 \overline{)3128}$ <u>34</u>
2. $16 \overline{)48}$ <u>3</u>	$18 \overline{)558}$ <u>31</u>	$37 \overline{)5328}$ <u>144</u>
3. $14 \overline{)42}$ <u>3</u>	$14 \overline{)350}$ <u>25</u>	$23 \overline{)2829}$ <u>123</u>
4. $19 \overline{)57}$ <u>3</u>	$41 \overline{)656}$ <u>16</u>	$21 \overline{)2245}$ <u>106r19</u>
5. $14 \overline{)70}$ <u>5</u>	$12 \overline{)216}$ <u>18</u>	$22 \overline{)5214}$ <u>237</u>
6. $21 \overline{)84}$ <u>4</u>	$15 \overline{)480}$ <u>32</u>	$31 \overline{)3813}$ <u>123</u>
7. $32 \overline{)96}$ <u>3</u>	$27 \overline{)891}$ <u>33</u>	$12 \overline{)2688}$ <u>224</u>
8. $24 \overline{)96}$ <u>4</u>	$24 \overline{)386}$ <u>16r2</u>	$15 \overline{)1665}$ <u>111</u>
9. $31 \overline{)62}$ <u>2</u>	$42 \overline{)714}$ <u>17</u>	$13 \overline{)1950}$ <u>150</u>
10. $13 \overline{)78}$ <u>6</u>	$14 \overline{)560}$ <u>40</u>	$43 \overline{)7095}$ <u>165</u>

Use the following suggestions to solve each problem below.

- Read the problem carefully.
- Decide which operation to use.
- Write an open sentence.
- Solve the open sentence.
- Answer the problem.

Use after page 153.

1. A plane travels 382 miles in 1 hour. How far could it travel in 8 hours at this speed? $8 \times 382 = \square$; **3056 miles**

2. Harry rode his bicycle 24 miles in 6 days on his paper route. How many miles does he ride on his paper route each day? $24 \div 6 = \square$; **4 miles**

3. A library has 325 fiction books, 156 science books, and 162 history books. In all, how many books does the library have? $325 + 156 + 162 = \square$; **643 books**

4. A book contains 385 pages. Jane read 296 of them. How many more pages does she still have to read to finish the book? $385 - 296 = \square$; **89 pages**

5. A box of grapes weighs 24 pounds. How many pounds would 152 boxes of these grapes weigh? $24 \times 152 = \square$; **3648 pounds**

6. There are 9 classes of 32 pupils each in a school. How many pupils are there in the school? $9 \times 32 = \square$; **288 pupils**

Least Common Multiple

You find the greatest common factor of 24 and 30 as shown below.

Factor set of 24 = {1, 2, 3, 4, 6, 8, 12, 24}

Factor set of 30 = {1, 2, 3, 5, 6, 10, 15, 30}

Set of common factors = {1, 2, 3, 6}

The greatest common factor is 6.

Use after page 155.

Write the factor set for each of the following numbers. See below for the factor set of c and d.

a	b	c	d
1. 6 {1, 2, 3, 6}	28 {1, 2, 4, 7, 14, 28}	36 {1, 2, 3, 4, 6, 12}	40 {1, 2, 4, 5, 10, 20}
2. 9 {1, 3, 9}	12 {1, 2, 3, 4, 6, 12}	16 {1, 2, 4, 8, 16}	48 {1, 2, 3, 4, 6, 8, 12, 16, 24, 48}
3. 20 {1, 2, 4, 5, 10, 20}	42 {1, 2, 3, 6, 7, 14, 21, 42}	18 {1, 2, 3, 6, 9, 18}	54 {1, 2, 3, 6, 9, 18, 27, 54}

Find the greatest common factor of each of the following pairs of numbers.

a	b
4. 8 and 24	12 and 10
5. 32 and 16	18 and 36
6. 4 and 9	6 and 16
7. 8 and 28	9 and 12
8. 9 and 15	10 and 20
9. 12 and 16	15 and 25

To find the least common multiple of two numbers, divide their product by their greatest common factor.

Factor set of 8 = {1, 2, 4, 8}

Factor set of 10 = {1, 2, 5, 10}

The greatest common factor is 2.

$$8 \times 10 = 80 \quad 80 \div 2 = 40$$

The least common multiple is 40.

Use after page 157.

Find the least common multiple of each pair of numbers listed below.

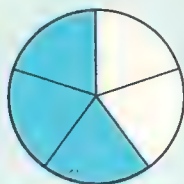
a	b
1. 4 and 9	3 and 8
2. 6 and 14	9 and 10
3. 2 and 3	2 and 5
4. 3 and 9	4 and 10
5. 7 and 9	3 and 7
6. 4 and 20	5 and 25
7. 6 and 12	6 and 18
8. 20 and 24	15 and 25
9. 30 and 36	42 and 7

316 Use after page 155.

c	d
1. {1, 2, 3, 4, 6, 9, 12, 18, 36}	{1, 2, 4, 5, 8, 10, 20, 40}
2. {1, 2, 4, 8, 16}	{1, 2, 3, 4, 6, 8, 12, 16, 24, 48}
3. {1, 2, 3, 6, 9, 18}	{1, 2, 3, 6, 9, 18, 27, 54}

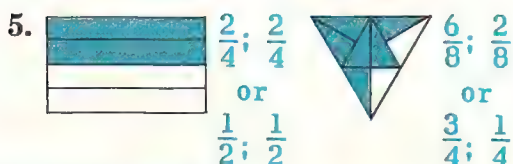
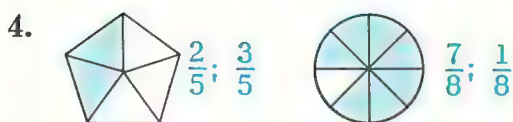
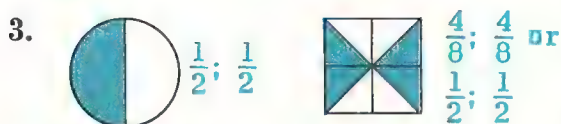
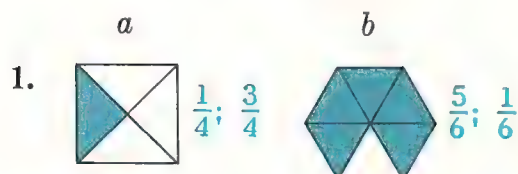
Fractions

The fraction $\frac{3}{5}$ means 3 out of 5 parts of the same size.



Use after page 163.

What fraction tells how much of the interior is colored in each figure below? What fraction tells how much of the interior is not colored?



Use after page 167.

1. $W\frac{4}{1}; F\frac{1}{4}$ $W\frac{9}{3}; F\frac{3}{9}$ $W\frac{8}{2}; F\frac{2}{8}$

2. $W\frac{10}{5}; F\frac{5}{10}$

$W\frac{5}{1}; F\frac{1}{5}$

$W\frac{18}{3}; F\frac{3}{18}$

3. $W\frac{6}{3}; F\frac{3}{6}$

$W\frac{12}{6}; F\frac{6}{12}$

$W\frac{10}{2}; F\frac{2}{10}$

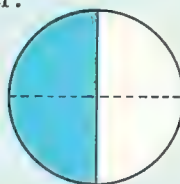
A fraction names either a whole number or a fractional number.

$\frac{2}{1}, \frac{15}{5}, \frac{8}{2}, \frac{18}{3}$

$\frac{2}{3}, \frac{5}{7}, \frac{11}{20}, \frac{4}{25}$

Whole numbers Fractional numbers

Two fractions can name the same number.



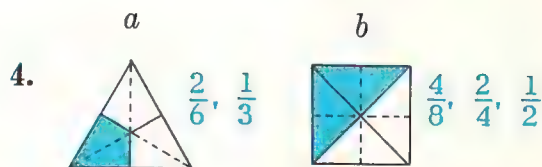
$\frac{1}{2} = \frac{2}{4}$

Use after page 167.

Use the numerals in each pair below to write two different fractions. Then tell which fraction names a whole number and which fraction names a fractional number. **See below.**

	<i>a</i>	<i>b</i>	<i>c</i>
1.	4 and 1	9 and 3	8 and 2
2.	10 and 5	5 and 1	18 and 3
3.	6 and 3	12 and 6	2 and 10

What two different fractions tell how much of the interior of each figure below is colored?



Fractional Numbers

A fraction may name a number less than one, equal to one, or greater than one.

$$\frac{2}{5} < 1 \quad \frac{5}{5} = 1 \quad \frac{7}{5} > 1$$

Division can be expressed by a fraction.

$$2 \div 3 = \frac{2}{3}$$

Use after page 169.

Tell whether each fraction below names a number less than one, greater than one, or the number one.

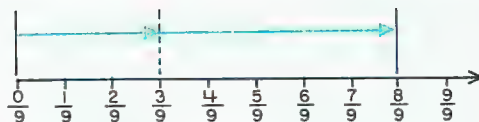
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
1.	$\frac{11}{5} > 1$	$\frac{12}{6} > 1$	$\frac{3}{8} < 1$	$\frac{10}{8} > 1$	$\frac{5}{5} = 1$
2.	$\frac{8}{10} < 1$	$\frac{5}{6} < 1$	$\frac{2}{2} = 1$	$\frac{14}{13} > 1$	$\frac{7}{7} = 1$

Write a fraction for each division numeral below.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
3.	$1 \div 5 = \frac{1}{5}$	$2 \div 7 = \frac{2}{7}$	$7 \div 8 = \frac{7}{8}$	$9 \div 3 = \frac{9}{3}$
4.	$4 \div 4 = \frac{4}{4}$	$4 \div 5 = \frac{4}{5}$	$8 \div 5 = \frac{8}{5}$	$6 \div 9 = \frac{6}{9}$
5.	$3 \div 8 = \frac{3}{8}$	$8 \div 3 = \frac{8}{3}$	$7 \div 7 = \frac{7}{7}$	$1 \div 6 = \frac{1}{6}$

Write a division numeral for each fraction below.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
6.	$\frac{1}{6} = 1 \div 6$	$\frac{3}{10} = 3 \div 10$	$\frac{4}{7} = 4 \div 7$	$\frac{12}{2} = 12 \div 2$
7.	$\frac{14}{7} = 14 \div 7$	$\frac{15}{5} = 15 \div 5$	$\frac{8}{8} = 8 \div 8$	$\frac{2}{9} = 2 \div 9$



$$\frac{3}{9} + \frac{5}{9} = \frac{3+5}{9} = \frac{8}{9}$$

Use after page 171.

Copy. Find each sum.

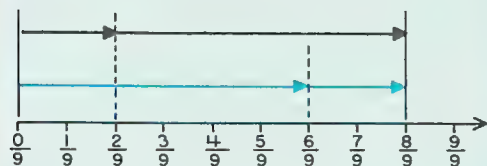
	<i>a</i>	<i>b</i>
1.	$\frac{1}{9} + \frac{4}{9} = \frac{5}{9}$	$\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$
2.	$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$	$\frac{4}{10} + \frac{3}{10} = \frac{7}{10}$
3.	$\frac{7}{12} + \frac{4}{12} = \frac{11}{12}$	$\frac{9}{14} + \frac{2}{14} = \frac{11}{14}$
4.	$\frac{16}{24} + \frac{3}{24} = \frac{19}{24}$	$\frac{8}{36} + \frac{9}{36} = \frac{17}{36}$
5.	$\frac{15}{30} + \frac{4}{30} = \frac{19}{30}$	$\frac{8}{17} + \frac{8}{17} = \frac{16}{17}$
6.	$\frac{15}{31} + \frac{15}{31} = \frac{30}{31}$	$\frac{14}{37} + \frac{9}{37} = \frac{23}{37}$

Copy. Find each sum.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
7.	$\frac{4}{14} + \frac{9}{14} = \frac{13}{14}$	$\frac{7}{15} + \frac{7}{15} = \frac{14}{15}$	$\frac{2}{16} + \frac{5}{16} = \frac{7}{16}$	$\frac{8}{20} + \frac{3}{20} = \frac{11}{20}$
8.	$\frac{7}{20} + \frac{6}{20} = \frac{13}{20}$	$\frac{8}{21} + \frac{8}{21} = \frac{16}{21}$	$\frac{6}{24} + \frac{5}{24} = \frac{11}{24}$	$\frac{15}{32} + \frac{14}{32} = \frac{29}{32}$
9.	$\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$	$\frac{2}{40} + \frac{19}{40} = \frac{21}{40}$	$\frac{7}{18} + \frac{4}{18} = \frac{11}{18}$	$\frac{6}{22} + \frac{11}{22} = \frac{17}{22}$
10.	$\frac{17}{25} + \frac{4}{25} = \frac{21}{25}$	$\frac{14}{30} + \frac{15}{30} = \frac{29}{30}$	$\frac{16}{31} + \frac{8}{31} = \frac{24}{31}$	$\frac{28}{51} + \frac{16}{51} = \frac{44}{51}$

Patterns of Addition of Fractional Numbers

For all numbers that can be named by fractions, addition is commutative.



$$\frac{2}{9} + \frac{6}{9} = \frac{6}{9} + \frac{2}{9}$$

Use after page 173.

Copy. Replace each \square by a single fraction that makes each sentence true.

1. $\frac{8}{14} + \frac{5}{14} = \frac{5}{14} + \frac{8}{14} = \square \frac{13}{14}$

2. $\frac{7}{20} + \frac{6}{20} = \frac{6}{20} + \frac{7}{20} = \square \frac{13}{20}$

3. $\frac{8}{21} + \frac{9}{21} = \frac{9}{21} + \frac{8}{21} = \square \frac{17}{21}$

4. $\frac{6}{24} + \frac{11}{24} = \frac{11}{24} + \frac{6}{24} = \square \frac{17}{24}$

5. $\frac{12}{32} + \frac{13}{32} = \frac{13}{32} + \frac{12}{32} = \square \frac{25}{32}$

6. $\frac{7}{15} + \frac{4}{15} = \frac{4}{15} + \frac{7}{15} = \square \frac{11}{15}$

7. $\frac{9}{16} + \frac{6}{16} = \frac{6}{16} + \frac{9}{16} = \square \frac{15}{16}$

8. $\frac{8}{18} + \frac{5}{18} = \frac{5}{18} + \frac{8}{18} = \square \frac{13}{18}$

9. $\frac{9}{20} + \frac{4}{20} = \frac{4}{20} + \frac{9}{20} = \square \frac{13}{20}$

10. $\frac{5}{16} + \frac{6}{16} = \frac{6}{16} + \frac{5}{16} = \square \frac{11}{16}$

11. $\frac{6}{21} + \frac{4}{21} = \frac{4}{21} + \frac{6}{21} = \square \frac{10}{21}$

Use after page 175.

1. $\frac{7}{15} + \frac{5}{15} = \frac{3}{15} + \frac{9}{15} \quad \frac{12}{15} = \frac{12}{15}$

2. $\frac{11}{20} + \frac{2}{20} = \frac{5}{20} + \frac{8}{20} \quad \frac{13}{20} = \frac{13}{20}$

3. $\frac{8}{24} + \frac{10}{24} = \frac{11}{24} + \frac{7}{24} \quad \frac{18}{24} = \frac{18}{24}$

4. $\frac{6}{30} + \frac{10}{30} = \frac{11}{30} + \frac{5}{30} \quad \frac{16}{30} = \frac{16}{30}$

5. $\frac{10}{35} + \frac{9}{35} = \frac{8}{35} + \frac{11}{35} \quad \frac{19}{35} = \frac{19}{35}$

For all numbers that can be named by fractions, addition is associative.

$$\left(\frac{4}{10} + \frac{3}{10}\right) + \frac{2}{10} = \frac{4}{10} + \left(\frac{3}{10} + \frac{2}{10}\right)$$

$$\frac{7}{10} + \frac{2}{10} = \frac{4}{10} + \frac{5}{10}$$

$$\frac{9}{10} = \frac{9}{10}$$

Use after page 175.

Use the method shown above to show that each of the following is a true sentence. See below.

1. $\left(\frac{3}{15} + \frac{4}{15}\right) + \frac{5}{15} = \frac{3}{15} + \left(\frac{4}{15} + \frac{5}{15}\right)$

2. $\left(\frac{5}{20} + \frac{6}{20}\right) + \frac{2}{20} = \frac{5}{20} + \left(\frac{6}{20} + \frac{2}{20}\right)$

3. $\frac{8}{24} + \left(\frac{3}{24} + \frac{7}{24}\right) = \left(\frac{8}{24} + \frac{3}{24}\right) + \frac{7}{24}$

4. $\frac{6}{30} + \left(\frac{5}{30} + \frac{5}{30}\right) = \left(\frac{6}{30} + \frac{5}{30}\right) + \frac{5}{30}$

5. $\left(\frac{8}{35} + \frac{2}{35}\right) + \frac{9}{35} = \frac{8}{35} + \left(\frac{2}{35} + \frac{9}{35}\right)$

Copy. Solve each open sentence by replacing n with a single fraction.

a

b

6. $\frac{8}{19} + \frac{2}{19} + \frac{9}{19} = n \quad \frac{19}{19}$

$\frac{7}{25} + \frac{3}{25} + \frac{8}{25} = n \quad \frac{18}{25}$

7. $\frac{6}{24} + \frac{4}{24} + \frac{9}{24} = n \quad \frac{19}{24}$

$\frac{2}{20} + \frac{8}{20} + \frac{6}{20} = n \quad \frac{16}{20}$

8. $\frac{14}{30} + \frac{6}{30} + \frac{5}{30} = n \quad \frac{25}{30}$

$\frac{19}{35} + \frac{11}{35} + \frac{9}{35} = n \quad \frac{39}{35}$

9. $\frac{7}{40} + \frac{3}{40} + \frac{9}{40} = n \quad \frac{19}{40}$

$\frac{17}{48} + \frac{3}{48} + \frac{19}{48} = n \quad \frac{39}{48}$

10. $\frac{7}{16} + \frac{4}{16} + \frac{6}{16} = n \quad \frac{17}{16}$

$\frac{9}{20} + \frac{1}{20} + \frac{8}{20} = n \quad \frac{18}{20}$

11. $\frac{9}{20} + \frac{8}{20} + \frac{2}{20} = n \quad \frac{19}{20}$

$\frac{7}{21} + \frac{6}{21} + \frac{4}{21} = n \quad \frac{17}{21}$

Multiplication of Fractional Numbers

You can think of multiplication of fractional numbers as repeated addition.



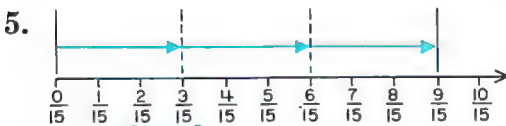
$$\frac{2}{8} + \frac{2}{8} + \frac{2}{8} + \frac{2}{8} + \frac{2}{8} = 5 \times \frac{2}{8} = \frac{5 \times 2}{8} = \frac{10}{8}$$

Use after page 183.

Copy. State each product as a repeated addition and as a single fraction. See below for representative answers of the repeated addition.

- | | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|----|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|
| 1. | $6 \times \frac{1}{2} = \frac{6}{2}$ | $5 \times \frac{5}{8} = \frac{25}{8}$ | $2 \times \frac{3}{5} = \frac{6}{5}$ | $9 \times \frac{2}{3} = \frac{18}{3}$ |
| 2. | $4 \times \frac{1}{4} = \frac{4}{4}$ | $7 \times \frac{2}{9} = \frac{14}{9}$ | $3 \times \frac{4}{9} = \frac{12}{9}$ | $8 \times \frac{5}{6} = \frac{40}{6}$ |
| 3. | $8 \times \frac{3}{8} = \frac{24}{8}$ | $3 \times \frac{1}{2} = \frac{3}{2}$ | $9 \times \frac{1}{9} = \frac{9}{9}$ | $7 \times \frac{2}{5} = \frac{14}{5}$ |
| 4. | $7 \times \frac{1}{4} = \frac{7}{4}$ | $6 \times \frac{2}{3} = \frac{12}{3}$ | $2 \times \frac{3}{4} = \frac{6}{4}$ | $6 \times \frac{3}{4} = \frac{18}{4}$ |

Write a multiplication sentence for each number line drawing.



320 $5 \times \frac{2}{10} = \frac{10}{10}$

Use after page 183.

Use after page 186.

You can multiply two fractional numbers as shown below.

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

Use after page 185.

Copy. Find the single fraction which names each product below.

- | | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|----|--|--|--|--|
| 1. | $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ | $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$ | $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$ | $\frac{4}{7} \times \frac{3}{9} = \frac{12}{63}$ |
| 2. | $\frac{3}{7} \times \frac{4}{5} = \frac{12}{35}$ | $\frac{4}{9} \times \frac{2}{3} = \frac{8}{27}$ | $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$ | $\frac{5}{6} \times \frac{1}{2} = \frac{5}{12}$ |
| 3. | $\frac{2}{3} \times \frac{4}{7} = \frac{8}{21}$ | $\frac{5}{7} \times \frac{5}{7} = \frac{25}{49}$ | $\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$ | $\frac{2}{9} \times \frac{1}{3} = \frac{2}{27}$ |
| 4. | $\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$ | $\frac{2}{7} \times \frac{1}{3} = \frac{2}{21}$ | $\frac{2}{3} \times \frac{1}{9} = \frac{2}{27}$ | $\frac{8}{9} \times \frac{2}{5} = \frac{16}{45}$ |
| 5. | $\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$ | $\frac{3}{5} \times \frac{1}{4} = \frac{3}{20}$ | $\frac{7}{9} \times \frac{2}{5} = \frac{14}{45}$ | $\frac{1}{2} \times \frac{4}{5} = \frac{4}{10}$ |

For all numbers that can be named by fractions, multiplication is commutative.

$$\frac{3}{5} \times \frac{1}{2} = \frac{1}{2} \times \frac{3}{5}$$

Use after page 186.

For each fractional number below, write a closed multiplication sentence as follows. See below for representative answers.

$$\frac{8}{15} = \frac{4}{5} \times \frac{2}{3} = \frac{2}{3} \times \frac{4}{5}$$

- | | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> | <i>e</i> |
|----|-----------------|----------------|-----------------|----------------|----------------|
| 1. | $\frac{15}{24}$ | $\frac{8}{15}$ | $\frac{12}{21}$ | $\frac{6}{12}$ | $\frac{3}{10}$ |
| 2. | $\frac{3}{30}$ | $\frac{8}{25}$ | $\frac{3}{16}$ | $\frac{5}{18}$ | $\frac{5}{12}$ |

1. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{6}{2} = 3$ $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} = \frac{25}{8}$ $\frac{3}{5} + \frac{3}{5} = \frac{6}{5}$

1. $\frac{15}{24} = \frac{5}{8} \times \frac{5}{6} = \frac{5}{6} \times \frac{5}{8}$ $\frac{8}{15} = \frac{2}{3} \times \frac{4}{5} = \frac{4}{5} \times \frac{2}{3}$ $\frac{12}{21} = \frac{2}{3} \times \frac{6}{7} = \frac{6}{7} \times \frac{2}{3}$

Fractions

A fractional number is not changed when it is multiplied by the number one.

$$\begin{aligned}\frac{2}{3} \times 1 &= \frac{2}{3} \times \frac{1}{1} \\ &= \frac{2 \times 1}{3 \times 1} \\ &= \frac{2}{3}\end{aligned}$$

$$\begin{aligned}1 \times \frac{3}{4} &= \frac{1}{1} \times \frac{3}{4} \\ &= \frac{1 \times 3}{1 \times 4} \\ &= \frac{3}{4}\end{aligned}$$

Use after page 187.

Copy. Find each product as a single fraction using the method shown above.

- | a | b | c | d |
|---------------------------|------------------------|------------------------|------------------------|
| 1. $\frac{4}{5} \times 1$ | $\frac{3}{8} \times 1$ | $1 \times \frac{1}{3}$ | $1 \times \frac{2}{3}$ |
| 2. $\frac{3}{7} \times 1$ | $\frac{4}{9} \times 1$ | $1 \times \frac{5}{6}$ | $1 \times \frac{7}{8}$ |
| 3. $\frac{7}{9} \times 1$ | $\frac{6}{7} \times 1$ | $1 \times \frac{1}{4}$ | $1 \times \frac{2}{9}$ |
| 4. $\frac{5}{8} \times 1$ | $\frac{5}{9} \times 1$ | $1 \times \frac{1}{2}$ | $1 \times \frac{3}{4}$ |

The identity number of multiplication helps you find many names for every fractional number.

$$\begin{aligned}\frac{2}{3} &= \frac{2}{3} \times 1 \\ &= \frac{2}{3} \times \frac{2}{2} \\ &= \frac{2 \times 2}{3 \times 2} \\ &= \frac{4}{6}\end{aligned}$$

$$\begin{aligned}\frac{2}{3} &= \frac{2}{3} \times 1 \\ &= \frac{2}{3} \times \frac{3}{3} \\ &= \frac{2 \times 3}{3 \times 3} \\ &= \frac{6}{9}\end{aligned}$$

Use after page 189.

Do the following. See below.

- | a | b |
|--|---|
| 1. Rename $\frac{1}{2}$ as $\frac{6}{12}$. | Rename $\frac{2}{3}$ as $\frac{8}{12}$. |
| 2. Rename $\frac{3}{4}$ as $\frac{12}{16}$. | Rename $\frac{3}{4}$ as $\frac{15}{20}$. |

Use after page 189.

$$\begin{aligned}1. \quad \frac{1}{2} &= \frac{1}{2} \times 1 = \frac{1}{2} \times \frac{a}{a} = \frac{1 \times 6}{2 \times 6} = \frac{6}{12} \\ 2. \quad \frac{3}{4} &= \frac{3}{4} \times 1 = \frac{3}{4} \times \frac{4}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}\end{aligned}$$

Copy. Write three more fractions for each fractional number. Answers may vary from those given.

- | a | b | c | d |
|------------------|--|---|---|
| 3. $\frac{3}{5}$ | $\frac{6}{10}, \frac{9}{15}, \frac{2}{6}, \frac{4}{12}, \frac{6}{18}$ | $\frac{4}{9}, \frac{8}{18}, \frac{12}{27}, \frac{1}{3}, \frac{2}{6}, \frac{3}{9}$ | |
| 4. $\frac{7}{9}$ | $\frac{14}{18}, \frac{21}{27}, \frac{5}{6}, \frac{10}{12}, \frac{15}{18}$ | $\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{7}{8}, \frac{14}{16}, \frac{21}{24}$ | |
| 5. $\frac{8}{9}$ | $\frac{16}{18}, \frac{24}{27}, \frac{7}{10}, \frac{14}{20}, \frac{21}{30}$ | $\frac{4}{8}, \frac{8}{16}, \frac{12}{24}, \frac{2}{7}, \frac{4}{14}, \frac{6}{21}$ | |
| 6. $\frac{2}{5}$ | $\frac{4}{10}, \frac{6}{15}, \frac{3}{7}, \frac{6}{14}, \frac{9}{21}$ | $\frac{5}{9}, \frac{10}{18}, \frac{15}{27}, \frac{5}{12}, \frac{10}{24}, \frac{15}{36}$ | |

You can change a fraction to simplest form by renaming the fractional number as shown below.

$$\begin{aligned}\frac{4}{6} &= \frac{2 \times 2}{3 \times 2} \\ &= \frac{2}{3} \times \frac{2}{2} \\ &= \frac{2}{3} \times 1 \\ &= \frac{2}{3}\end{aligned}$$

$$\begin{aligned}\frac{6}{9} &= \frac{2 \times 3}{3 \times 3} \\ &= \frac{2}{3} \times \frac{3}{3} \\ &= \frac{2}{3} \times 1 \\ &= \frac{2}{3}\end{aligned}$$

Use after page 191.

Copy. Change each fraction below to simplest form.

- | a | b | c | d |
|--------------------|---------------|-----------------|---------------|
| 1. $\frac{5}{10}$ | $\frac{1}{2}$ | $\frac{2}{6}$ | $\frac{1}{3}$ |
| 2. $\frac{15}{18}$ | $\frac{5}{6}$ | $\frac{15}{21}$ | $\frac{5}{7}$ |
| 3. $\frac{15}{20}$ | $\frac{3}{4}$ | $\frac{12}{15}$ | $\frac{4}{5}$ |
| 4. $\frac{8}{24}$ | $\frac{1}{3}$ | $\frac{6}{12}$ | $\frac{1}{2}$ |
| 5. $\frac{8}{12}$ | $\frac{2}{3}$ | $\frac{16}{20}$ | $\frac{4}{5}$ |
| 6. $\frac{6}{16}$ | $\frac{3}{8}$ | $\frac{20}{28}$ | $\frac{5}{7}$ |

321

$$\begin{aligned}1. \quad \frac{1}{2} &= \frac{1}{2} \times 1 = \frac{1}{2} \times \frac{a}{a} = \frac{1 \times 6}{2 \times 6} = \frac{6}{12} \\ 2. \quad \frac{3}{4} &= \frac{3}{4} \times 1 = \frac{3}{4} \times \frac{4}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}\end{aligned}$$

$$\begin{aligned}\frac{2}{3} &= \frac{2}{3} \times 1 = \frac{2}{3} \times \frac{b}{b} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \\ \frac{3}{4} &= \frac{3}{4} \times 1 = \frac{3}{4} \times \frac{5}{5} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}\end{aligned}$$

Mixed Numerals

You can change a mixed numeral to a fraction as shown in **A**. You can change $\frac{7}{2}$ to a mixed numeral as shown in **B**.

A

$$\begin{aligned} 3\frac{2}{3} &= 3 + \frac{2}{3} \\ &= \frac{9}{3} + \frac{2}{3} \\ &= \frac{9+2}{3} \\ &= \frac{11}{3} \end{aligned}$$

B

$$\begin{aligned} \frac{7}{2} &= \frac{6+1}{2} \\ &= \frac{6}{2} + \frac{1}{2} \\ &= 3 + \frac{1}{2} \\ &= 3\frac{1}{2} \end{aligned}$$

Use after page 195.

Copy. Change each mixed numeral to a fraction.

- | | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|----|----------------|----------------|----------------|----------------|
| 1. | $5\frac{2}{5}$ | $6\frac{1}{3}$ | $8\frac{3}{4}$ | $8\frac{1}{2}$ |
| 2. | $9\frac{2}{3}$ | $4\frac{1}{4}$ | $3\frac{1}{8}$ | $5\frac{5}{6}$ |
| 3. | $2\frac{3}{7}$ | $6\frac{4}{5}$ | $4\frac{4}{9}$ | $2\frac{3}{8}$ |
| 4. | $4\frac{2}{3}$ | $2\frac{7}{8}$ | $7\frac{1}{5}$ | $9\frac{5}{7}$ |
| 5. | $3\frac{1}{9}$ | $7\frac{1}{4}$ | $5\frac{1}{2}$ | $4\frac{3}{4}$ |
| 6. | $6\frac{3}{5}$ | $9\frac{3}{8}$ | $8\frac{2}{3}$ | $2\frac{1}{2}$ |

Copy. Change each fraction to a mixed numeral.

- | | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|----|----------------|----------------|----------------|----------------|
| 7. | $\frac{11}{2}$ | $\frac{22}{3}$ | $\frac{7}{5}$ | $\frac{9}{4}$ |
| 8. | $\frac{11}{3}$ | $\frac{13}{5}$ | $\frac{27}{5}$ | $\frac{13}{4}$ |

9. $\frac{16}{7}$ $2\frac{2}{7}$ $\frac{27}{8}$ $3\frac{3}{8}$ $\frac{56}{9}$ $6\frac{2}{9}$ $\frac{15}{8}$ $1\frac{7}{8}$
10. $\frac{23}{6}$ $3\frac{5}{6}$ $\frac{23}{4}$ $5\frac{3}{4}$ $\frac{17}{3}$ $5\frac{2}{3}$ $\frac{19}{2}$ $9\frac{1}{2}$
11. $\frac{36}{7}$ $5\frac{1}{7}$ $\frac{29}{8}$ $3\frac{5}{8}$ $\frac{15}{2}$ $7\frac{1}{2}$ $\frac{29}{4}$ $7\frac{1}{4}$

A mixed numeral is in simplest form if the fraction is in simplest form and the fraction names a number less than one. A mixed numeral can be changed to simplest form as shown below.

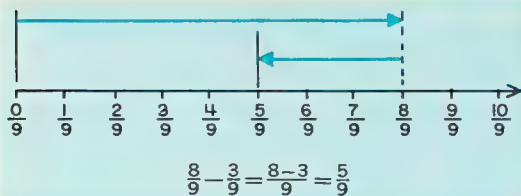
$$\begin{aligned} 2\frac{3}{6} &= 2 + \frac{3}{6} \\ &= 2 + (\frac{1 \times 3}{2 \times 3}) \\ &= 2 + (\frac{1}{2} \times \frac{3}{3}) \\ &= 2 + (\frac{1}{2} \times 1) \\ &= 2 + \frac{1}{2} \\ &= 2\frac{1}{2} \end{aligned}$$

Use after page 197.

Copy. Change each mixed numeral to simplest form.

- | | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|----|------------------|----------------|------------------|------------------|
| 1. | $2\frac{3}{9}$ | $2\frac{1}{3}$ | $3\frac{4}{8}$ | $2\frac{15}{18}$ |
| 2. | $1\frac{6}{16}$ | $1\frac{3}{8}$ | $5\frac{4}{12}$ | $7\frac{2}{6}$ |
| 3. | $7\frac{15}{20}$ | $7\frac{3}{4}$ | $6\frac{12}{15}$ | $8\frac{9}{24}$ |
| 4. | $4\frac{9}{18}$ | $4\frac{1}{2}$ | $3\frac{3}{6}$ | $1\frac{8}{10}$ |
| 5. | $3\frac{8}{12}$ | $3\frac{2}{3}$ | $4\frac{5}{3}$ | $9\frac{6}{9}$ |
| 6. | $1\frac{9}{5}$ | $2\frac{4}{5}$ | $7\frac{4}{32}$ | $4\frac{7}{28}$ |

Subtraction of Fractional Numbers



Use after page 203.

Copy. Find each difference. Change each difference to simplest form, if needed.

- | a | b |
|---|---|
| 1. $\frac{7}{9} - \frac{2}{9} = a$ $\frac{5}{9}$ | $\frac{9}{10} - \frac{5}{10} = b$ $\frac{2}{5}$ |
| 2. $\frac{8}{12} - \frac{5}{12} = c$ $\frac{1}{4}$ | $\frac{8}{8} - \frac{3}{8} = d$ $\frac{5}{8}$ |
| 3. $\frac{13}{18} - \frac{7}{18} = e$ $\frac{1}{3}$ | $\frac{13}{12} - \frac{6}{12} = f$ $\frac{7}{12}$ |
| 4. $\frac{11}{15} - \frac{6}{15} = g$ $\frac{1}{3}$ | $\frac{11}{21} - \frac{8}{21} = h$ $\frac{1}{7}$ |

Copy. Find each difference in simplest form.

- | a | b | c | d |
|---|---|---|--|
| 5. $\frac{5}{8} - \frac{3}{8} = \frac{1}{4}$ | $\frac{7}{9} - \frac{1}{9} = \frac{2}{3}$ | $\frac{7}{10} - \frac{2}{10} = \frac{1}{2}$ | $\frac{13}{15} - \frac{3}{15} = \frac{2}{3}$ |
| 6. $\frac{16}{20} - \frac{9}{20} = \frac{7}{20}$ | $\frac{16}{16} - \frac{10}{16} = \frac{3}{8}$ | $\frac{9}{18} - \frac{3}{18} = \frac{1}{3}$ | $\frac{20}{24} - \frac{13}{24} = \frac{7}{24}$ |
| 7. $\frac{21}{25} - \frac{8}{25} = \frac{13}{25}$ | $\frac{18}{28} - \frac{9}{28} = \frac{9}{28}$ | $\frac{17}{32} - \frac{8}{32} = \frac{9}{32}$ | $\frac{18}{24} - \frac{6}{24} = \frac{1}{2}$ |
| 8. $\frac{19}{25} - \frac{9}{25} = \frac{2}{5}$ | $\frac{9}{16} - \frac{2}{16} = \frac{7}{16}$ | $\frac{12}{20} - \frac{7}{20} = \frac{1}{4}$ | $\frac{15}{32} - \frac{9}{32} = \frac{3}{16}$ |

9. $\frac{24}{30} - \frac{17}{30} = \frac{7}{30}$ $\frac{21}{32} - \frac{14}{32} = \frac{7}{32}$ $\frac{16}{36} - \frac{9}{36} = \frac{7}{36}$ $\frac{20}{40} - \frac{19}{40} = \frac{1}{40}$
10. $\frac{14}{16} - \frac{9}{16} = \frac{5}{16}$ $\frac{13}{18} - \frac{8}{18} = \frac{5}{18}$ $\frac{11}{21} - \frac{9}{21} = \frac{2}{21}$ $\frac{17}{24} - \frac{6}{24} = \frac{11}{24}$

The least common denominator of $\frac{3}{8}$ and $\frac{5}{6}$ is the least common multiple of their denominators.

$$8 \times 6 = 48$$

$$48 \div 2 = 24$$

Use after page 205.

Find the least common denominator of the fractional numbers in each pair of numbers below.

- | a | b |
|---|-------------------------------------|
| 1. $\frac{2}{6}$ and $\frac{1}{2}$ 6 | $\frac{2}{3}$ and $\frac{1}{4}$ 12 |
| 2. $\frac{1}{2}$ and $\frac{3}{8}$ 8 | $\frac{5}{9}$ and $\frac{2}{3}$ 9 |
| 3. $\frac{2}{5}$ and $\frac{3}{6}$ 30 | $\frac{2}{4}$ and $\frac{3}{8}$ 8 |
| 4. $\frac{3}{5}$ and $\frac{7}{10}$ 10 | $\frac{2}{6}$ and $\frac{5}{8}$ 24 |
| 5. $\frac{3}{8}$ and $\frac{2}{5}$ 40 | $\frac{4}{6}$ and $\frac{6}{24}$ 24 |
| 6. $\frac{1}{4}$ and $\frac{10}{16}$ 16 | $\frac{3}{4}$ and $\frac{1}{12}$ 12 |
| 7. $\frac{2}{3}$ and $\frac{13}{15}$ 15 | $\frac{8}{9}$ and $\frac{5}{6}$ 18 |
| 8. $\frac{6}{8}$ and $\frac{3}{16}$ 16 | $\frac{1}{4}$ and $\frac{2}{5}$ 20 |
| 9. $\frac{1}{3}$ and $\frac{5}{12}$ 12 | $\frac{2}{5}$ and $\frac{3}{8}$ 40 |
| 10. $\frac{1}{5}$ and $\frac{3}{8}$ 40 | $\frac{2}{7}$ and $\frac{4}{21}$ 21 |

Addition and Subtraction of Fractional Numbers

To find the sum of $\frac{3}{8}$ and $\frac{2}{3}$, rename both addends as fractions with the same denominator.

$$\begin{array}{r} \frac{3}{8} \\ + \frac{2}{3} \\ \hline \end{array} \longrightarrow \begin{array}{r} \frac{9}{24} \\ + \frac{16}{24} \\ \hline \frac{9+16}{24} = \frac{25}{24} \\ = 1\frac{1}{24} \end{array}$$

Use after page 207.

Copy. Find each sum as a fraction or a mixed numeral in simplest form.

- | a | b | c |
|--|--|---|
| 1. $\frac{2}{6} + \frac{1}{3} = \frac{2}{3}$ | $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$ | $\frac{1}{2} + \frac{5}{8} = 1\frac{1}{8}$ |
| 2. $\frac{3}{5} + \frac{2}{6} = \frac{14}{15}$ | $\frac{4}{9} + \frac{2}{3} = 1\frac{1}{9}$ | $\frac{3}{7} + \frac{3}{8} = \frac{45}{56}$ |
| 3. $\frac{1}{3} + \frac{5}{8} = \frac{23}{24}$ | $\frac{4}{7} + \frac{7}{21} = \frac{19}{21}$ | $\frac{3}{5} + \frac{2}{4} = 1\frac{1}{10}$ |
| 4. $\frac{1}{9} + \frac{5}{6} = \frac{17}{18}$ | $\frac{7}{9} + \frac{3}{18} = \frac{17}{18}$ | $\frac{3}{4} + \frac{5}{8} = 1\frac{3}{8}$ |
| 5. $\frac{2}{5} + \frac{1}{10} = \frac{1}{2}$ | $\frac{1}{2} + \frac{7}{8} = 1\frac{3}{8}$ | $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ |

Copy. Find each sum as a fraction or a mixed numeral in simplest form.

- | a | b | c | d |
|--|--|--|---|
| 6. $\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$ | $\frac{7}{8} + \frac{1}{2} = 1\frac{3}{8}$ | $\frac{2}{6} + \frac{5}{24} = \frac{13}{24}$ | $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ |
| 7. $\frac{2}{3} + \frac{2}{9} = \frac{8}{9}$ | $\frac{1}{4} + \frac{7}{16} = \frac{11}{16}$ | $\frac{5}{6} + \frac{1}{3} = 1\frac{1}{6}$ | $\frac{1}{3} + \frac{7}{8} = 1\frac{5}{24}$ |
| 8. $\frac{3}{8} + \frac{3}{5} = \frac{39}{40}$ | $\frac{3}{5} + \frac{7}{10} = 1\frac{3}{10}$ | $\frac{1}{6} + \frac{3}{8} = \frac{13}{24}$ | $\frac{3}{9} + \frac{4}{5} = 1\frac{2}{15}$ |

An easy way to find the simplest numeral for $\frac{2}{3} - \frac{2}{4}$ is to rename $\frac{2}{3}$ and $\frac{2}{4}$ as fractions with the same denominator.

$$\begin{array}{r} \frac{2}{3} \\ - \frac{2}{4} \\ \hline \end{array} \longrightarrow \begin{array}{r} \frac{8}{12} \\ - \frac{6}{12} \\ \hline \frac{2}{12} = \frac{1}{6} \end{array}$$

Use after page 209.

Copy. Find each difference in simplest form.

- | a | b | c |
|---|--|--|
| 1. $\frac{2}{3} - \frac{1}{6} = \frac{1}{2}$ | $\frac{9}{16} - \frac{1}{4} = \frac{5}{16}$ | $\frac{5}{8} - \frac{1}{2} = \frac{1}{8}$ |
| 2. $\frac{7}{8} - \frac{2}{4} = \frac{3}{8}$ | $\frac{2}{3} - \frac{3}{9} = \frac{1}{3}$ | $\frac{4}{5} - \frac{3}{10} = \frac{1}{2}$ |
| 3. $\frac{11}{16} - \frac{5}{8} = \frac{1}{16}$ | $\frac{7}{8} - \frac{1}{2} = \frac{3}{8}$ | $\frac{14}{15} - \frac{2}{3} = \frac{4}{15}$ |
| 4. $\frac{7}{9} - \frac{5}{18} = \frac{1}{2}$ | $\frac{9}{12} - \frac{3}{4} = 0$ | $\frac{5}{6} - \frac{1}{3} = \frac{1}{2}$ |
| 5. $\frac{6}{7} - \frac{2}{21} = \frac{16}{21}$ | $\frac{4}{6} - \frac{3}{24} = \frac{13}{24}$ | $\frac{8}{9} - \frac{2}{5} = \frac{22}{45}$ |

Copy. Find each difference in simplest form.

- | a | b | c | d |
|---|--|--|---|
| 6. $\frac{11}{12} - \frac{2}{3} = \frac{1}{4}$ | $\frac{4}{6} - \frac{2}{3} = 0$ | $\frac{7}{10} - \frac{1}{5} = \frac{1}{2}$ | $\frac{9}{12} - \frac{1}{4} = \frac{1}{2}$ |
| 7. $\frac{23}{24} - \frac{4}{6} = \frac{7}{24}$ | $\frac{25}{28} - \frac{5}{7} = \frac{5}{28}$ | $\frac{5}{8} - \frac{2}{6} = \frac{7}{24}$ | $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$ |
| 8. $\frac{6}{7} - \frac{3}{5} = \frac{9}{35}$ | $\frac{7}{8} - \frac{1}{2} = \frac{3}{8}$ | $\frac{9}{10} - \frac{1}{3} = \frac{17}{30}$ | $\frac{9}{11} - \frac{2}{3} = \frac{5}{33}$ |

Fractional Numbers

An easy way to find the simplest numeral for $2\frac{3}{4} + \frac{2}{3}$ is shown below.

$$\begin{array}{r} 2\frac{3}{4} \\ + \frac{2}{3} \\ \hline \end{array} \longrightarrow \begin{array}{r} 2\frac{9}{12} \\ + \frac{8}{12} \\ \hline 2\frac{17}{12} = 3\frac{5}{12} \end{array}$$

Use after page 211.

Copy. Find each sum in simplest form.

- | a | b |
|--------------------------------------|------------------|
| 1. $3\frac{1}{2} + \frac{7}{12} = a$ | $4\frac{1}{12}$ |
| 2. $2\frac{3}{4} + \frac{1}{2} = c$ | $3\frac{1}{4}$ |
| 3. $2\frac{1}{3} + \frac{5}{6} = e$ | $3\frac{1}{6}$ |
| 4. $7\frac{5}{8} + \frac{1}{3} = g$ | $7\frac{23}{24}$ |
| 5. $1\frac{7}{12} + \frac{2}{3} = i$ | $2\frac{1}{4}$ |

Copy. Find each sum in simplest form.

- | a | b | c | d |
|--------------------------------------|----------------------------------|-----------------------------------|------------------|
| 6. $7\frac{5}{8} + \frac{1}{4} = a$ | $7\frac{7}{8}$ | $6\frac{1}{2} + \frac{3}{8} = b$ | $6\frac{7}{8}$ |
| 7. $6\frac{3}{4} + \frac{4}{8} = c$ | $7\frac{1}{3} + \frac{5}{6} = d$ | $3\frac{1}{2} + \frac{7}{10} = e$ | $4\frac{1}{5}$ |
| 8. $2\frac{4}{5} + \frac{5}{10} = f$ | $4\frac{3}{8} + \frac{3}{4} = g$ | $2\frac{5}{14} + \frac{3}{7} = h$ | $2\frac{11}{14}$ |
| 9. $3\frac{7}{8} + \frac{3}{4} = i$ | $2\frac{5}{9} + \frac{2}{3} = j$ | $3\frac{5}{8} + \frac{1}{6} = k$ | $3\frac{19}{24}$ |

You can find the simplest numeral for $1 - \frac{3}{12}$ by replacing 1 by $\frac{12}{12}$ and subtracting as shown below.

$$\begin{array}{r} 1 - \frac{3}{12} = \frac{12}{12} - \frac{3}{12} \\ = \frac{12-3}{12} \\ = \frac{9}{12} \\ = \frac{3}{4} \end{array} \quad \begin{array}{r} 1 \\ - \frac{3}{12} \\ \hline \end{array} \longrightarrow \begin{array}{r} \frac{12}{12} \\ - \frac{3}{12} \\ \hline \frac{9}{12} = \frac{3}{4} \end{array}$$

Use after page 213.

Copy. Find each difference in simplest form.

- | a | b |
|---------------------------|----------------|
| 1. $1 - \frac{3}{5} = a$ | $\frac{2}{5}$ |
| 2. $1 - \frac{8}{9} = c$ | $\frac{1}{9}$ |
| 3. $1 - \frac{5}{10} = e$ | $\frac{1}{2}$ |
| 4. $1 - \frac{8}{15} = g$ | $\frac{7}{15}$ |

Copy. Find each difference in simplest form.

- | a | b | c | d |
|----------------------------|-----------------|-------------------------|----------------|
| 5. $1 - \frac{3}{15} = a$ | $\frac{4}{5}$ | $1 - \frac{12}{15} = b$ | $\frac{1}{5}$ |
| 6. $1 - \frac{7}{20} = c$ | $\frac{13}{20}$ | $1 - \frac{13}{20} = d$ | $\frac{7}{20}$ |
| 7. $1 - \frac{15}{24} = e$ | $\frac{3}{8}$ | $1 - \frac{9}{24} = f$ | $\frac{5}{8}$ |
| 8. $1 - \frac{9}{12} = g$ | $\frac{1}{4}$ | $1 - \frac{6}{15} = h$ | $\frac{3}{5}$ |

Lines, Line Segments, Rays

A line segment has two endpoints. A line has no endpoints.



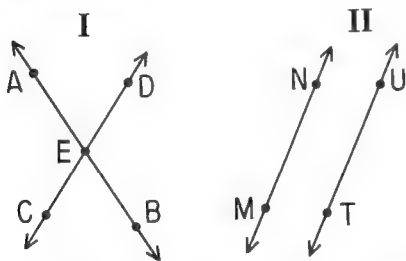
Line segment RS



Line XY

Use after page 221.

Use the figures below to answer the following.



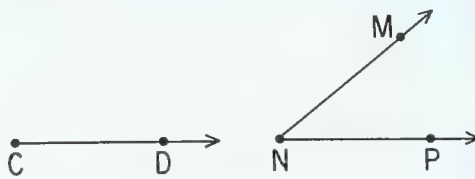
1. In I, name the points that are labeled. **points A, E, B, C, D**

2. In I, how many points do lines AB and CD have in common? **1** What is that point? **E** Are lines AB and CD intersecting or parallel lines? **intersecting**

3. In II, what line segment contains points M and N and all the points between them? **line segment MN**

4. The two lines in II are in the same plane and do not intersect. What would you call them? **parallel lines**

A ray has only one endpoint. Two rays with a common endpoint form an angle.

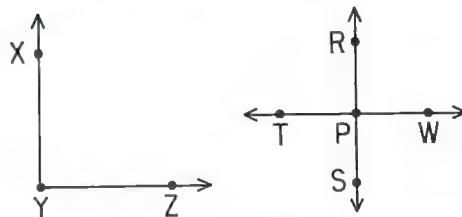


Ray CD

Angle MNP

Use after page 225.

Study the figures below.



1. Give two names for the angle at the left. **$\angle XYZ$ and $\angle ZYX$**

2. What are the two sides of $\angle XYZ$? **ray YX and ray YZ**

3. What is the vertex of $\angle XYZ$? **point Y**

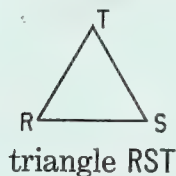
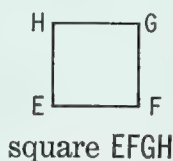
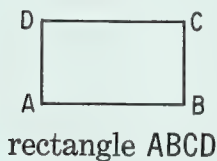
4. In the figure on the right, lines RS and TW are perpendicular lines. What kind of angles are formed at their intersection? **right angles**

5. In the figure on the right, name six different line segments. Name four different rays. Name the four angles formed by lines RS and TW.

line segments RP, PS, RS, TP, PW, TW
rays PR, PS, PT, PW, SR, RS, WT, TW
angles TPS, SPW, WPR, RPT

Simple Closed Figures

A simple closed figure has only 1 interior and 1 exterior.



Use after page 235.

Use the figures shown above to answer the following.

1. Name the sides of rectangle ABCD. **sides AB, BC, CD, DA**

2. Give two names for each angle of rectangle ABCD. **See below.**

3. What side is opposite side AB? **DC**
Opposite side AD? **BC** Do you think opposite sides of a rectangle have the same length? **Yes**
What do you call two line segments that have the same length? **congruent line segments**

4. Name the sides of square EFGH. **sides EF, FG, GH, HE**

5. Do the sides of a square have the same length? **Yes**

6. Give two names for each angle of square EFGH. **$\angle HEF$ and $\angle FEH$;
 $\angle EFG$ and $\angle GFE$; $\angle FGH$ and $\angle HGF$;
 $\angle GHE$ and $\angle EHG$**

7. What kind of angles does a square have? **right angles**

8. Is every square a rectangle? **Yes**

9. Is every rectangle a square? **No**

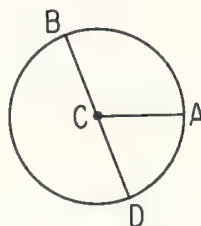
10. Name the sides of triangle RST. **sides RS, ST, TR**

11. How many angles are there in a triangle? **3** A rectangle? **4** A square? **4**

12. Give two names for each angle of triangle RST. **$\angle RST$ and $\angle TSR$;
 $\angle STR$ and $\angle RTS$; $\angle TRS$ and $\angle SRT$**

13. What do you call a triangle that has one right angle? **a right triangle**

Use the figure below to answer the following.



14. What point is the center of circle C? **point C**

15. Is line segment CA called a diameter or a radius? **a radius**

16. Is line segment BD called a diameter or a radius? **a diameter**

17. How many times longer is a diameter than a radius? **two**

Use after page 235. 1. **$\angle DAB$ and $\angle BAD$; $\angle ABC$ and $\angle CBA$; $\angle BCD$ and $\angle DCB$;
 $\angle CDA$ and $\angle ADC$**

Decimals

A fractional number that has a denominator of 10 or 100 can be expressed as a decimal.

$$\frac{7}{10} = .7$$

$$\frac{8}{100} = .08$$

Use after page 240.

Copy. Change the fractions or mixed numerals to decimals and change the decimals to fractions or mixed numerals.

a	b	c	d	e
1. $\frac{3}{10}$	$1\frac{3}{10}$	$6\frac{29}{100}$.09	1.9
$\frac{9}{10}$	$5\frac{4}{100}$	$3\frac{3}{100}$.9	7.10
$\frac{4}{100}$	$7\frac{2}{10}$	$4\frac{1}{10}$.24	3.04
$\frac{20}{100}$	$5\frac{50}{100}$	$7\frac{8}{10}$.40	4.75

Some fractions that do not have a denominator of 10 or 100 are changed to decimals as shown below.

$$\frac{3}{5} = \frac{3}{5} \times 1 = \frac{3}{5} \times \frac{2}{2} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10} = .6$$

You can also change $\frac{3}{5}$ to a decimal which is read as hundredths.

$$\frac{3}{5} = \frac{3}{5} \times 1 = \frac{3}{5} \times \frac{20}{20} = \frac{3 \times 20}{5 \times 20} = \frac{60}{100} = .60$$

You can change a decimal to a fraction in simplest form as shown below.

$$.08 = \frac{8}{100} = \frac{2 \times 4}{25 \times 4} = \frac{2}{25} \times \frac{4}{4} = \frac{2}{25} \times 1 = \frac{2}{25}$$

Use after page 241.

Change each fraction below to a decimal read as hundredths.

a	b	c	d	e
1. $\frac{4}{5}$	$\frac{3}{4}$	$\frac{4}{25}$	$\frac{5}{25}$	$\frac{1}{5}$
2. $\frac{2}{4}$	$\frac{1}{2}$	$\frac{25}{50}$	$\frac{7}{50}$	$\frac{6}{25}$

Change each of the following to a fraction in simplest form.

a	b	c	d	e
3. $.3$	$.6$	$.7$	$.40$	$.35$
4. $.9$	$.07$	$.05$	$.10$	$.85$

You can add 1.82 and .74 as shown below.

0	Ts	Hs	
1	8	2	1.82
+	7	4	+.74
2	5	6	2.56

Add the Hs. _____
 Add the Ts. _____
 Add the Ones. _____

Use after page 243.

Copy. Find each sum.

a	b	c	d
1. $.7$	1.8	6.21	14.29
$+.2$	$+.4$	$+.38$	$+3.70$
$.9$	2.2	6.59	17.99
2. $.14$	6.3	3.47	.82
$+.29$	$+.28$	$+.08$	$+1.79$
$.43$	9.1	3.55	2.61
$.17$	2.14	8.74	17.31
$+.33$	$+.19$	$+.92$	$+6.09$
$.50$	2.33	9.66	23.40

Decimals and Units of Length

You can subtract .28 from 1.60 as shown below.

0 Ts Hs	
5 10	
1.60	1.60
- .28	- .28
1.32	1.32

Subtract the Hs.

Subtract the Ts.

Subtract the Ones.

Use after page 245.

Copy. Find each difference.

	a	b	c	d
1.	$\begin{array}{r} .7 \\ -.2 \\ \hline .5 \end{array}$	$\begin{array}{r} .13 \\ -.05 \\ \hline .08 \end{array}$	$\begin{array}{r} .8 \\ -.3 \\ \hline .5 \end{array}$	$\begin{array}{r} .74 \\ -.15 \\ \hline .59 \end{array}$
2.	$\begin{array}{r} .43 \\ -.28 \\ \hline .15 \end{array}$	$\begin{array}{r} .76 \\ -.25 \\ \hline .51 \end{array}$	$\begin{array}{r} .90 \\ -.28 \\ \hline .62 \end{array}$	$\begin{array}{r} .31 \\ -.15 \\ \hline .16 \end{array}$
3.	$\begin{array}{r} 1.22 \\ -.17 \\ \hline 1.05 \end{array}$	$\begin{array}{r} 1.8 \\ -.9 \\ \hline .9 \end{array}$	$\begin{array}{r} 7.05 \\ -.92 \\ \hline 6.13 \end{array}$	$\begin{array}{r} 12.4 \\ -.8.9 \\ \hline 3.5 \end{array}$

Copy. Find each sum or difference.

	a	b	c	d
4.	$\begin{array}{r} .82 \\ +.14 \\ \hline .96 \end{array}$	$\begin{array}{r} .28 \\ -.19 \\ \hline .09 \end{array}$	$\begin{array}{r} 1.31 \\ +.09 \\ \hline 1.40 \end{array}$	$\begin{array}{r} 1.71 \\ -.82 \\ \hline .89 \end{array}$
5.	$\begin{array}{r} 1.49 \\ +.31 \\ \hline 1.80 \end{array}$	$\begin{array}{r} .73 \\ -.39 \\ \hline .34 \end{array}$	$\begin{array}{r} 2.40 \\ +.99 \\ \hline 3.39 \end{array}$	$\begin{array}{r} 2.00 \\ -1.59 \\ \hline .41 \end{array}$
6.	$\begin{array}{r} 5.00 \\ +.65 \\ \hline 5.65 \end{array}$	$\begin{array}{r} .75 \\ -.39 \\ \hline .36 \end{array}$	$\begin{array}{r} 1.85 \\ +.92 \\ \hline 2.77 \end{array}$	$\begin{array}{r} 1.00 \\ -.51 \\ \hline .49 \end{array}$
7.	$\begin{array}{r} 2.98 \\ +1.49 \\ \hline 4.47 \end{array}$	$\begin{array}{r} .50 \\ -.29 \\ \hline .21 \end{array}$	$\begin{array}{r} 3.49 \\ +.98 \\ \hline 4.47 \end{array}$	$\begin{array}{r} 3.00 \\ -2.08 \\ \hline .92 \end{array}$

To change a number of feet to a number of inches, multiply the number of feet by 12. To change a number of inches to a number of feet, divide the number of inches by 12.

Since 1 ft. = 12 in.,

4 ft. = (4×12) in. or 48 in.

Since 12 in. = 1 ft.,

36 in. = $(36 \div 12)$ ft. or 3 ft.

Use after page 253.

Copy. Find the numeral that would make each sentence true.

	a	b
1.	7 ft. = $\frac{84}{\text{in.}}$	48 in. = $\frac{4}{\text{ft.}}$
2.	4 ft. = $\frac{48}{\text{in.}}$	18 ft. = $\frac{6}{\text{yd.}}$
3.	6 yd. = $\frac{18}{\text{ft.}}$	72 in. = $\frac{6}{\text{ft.}}$
4.	4 yd. = $\frac{144}{\text{in.}}$	108 in. = $\frac{3}{\text{yd.}}$
5.	3 yd. = $\frac{108}{\text{in.}}$	24 ft. = $\frac{8}{\text{yd.}}$
6.	24 in. = $\frac{2}{\text{ft.}}$	80 cm. = $\frac{8}{\text{dm.}}$
7.	5 dm. = $\frac{50}{\text{cm.}}$	50 dm. = $\frac{5}{\text{m.}}$
8.	20 m. = $\frac{200}{\text{dm.}}$	500 cm. = $\frac{5}{\text{m.}}$
9.	8 m. = $\frac{800}{\text{cm.}}$	30 dm. = $\frac{3}{\text{m.}}$
10.	50 m. = $\frac{500}{\text{dm.}}$	30 cm. = $\frac{3}{\text{dm.}}$
11.	600 cm. = $\frac{6}{\text{m.}}$	400 cm. = $\frac{40}{\text{dm.}}$

Money, Weight, Capacity

Three ways to express an amount of money are shown below.

405¢ \$4.05 4.05 dollars

Use after page 256.

Give two other ways to express each of the following. **Representative answers for 1a, 1b, and 1c only.**

- | | a | b | c |
|----|--------------|---------------------|--------------------------------|
| 1. | 19¢
\$.19 | 209¢
.19 dollars | 2.09 dollars
225¢
\$2.25 |
| 2. | 50¢ | \$1.05 | .27 dollars |
| 3. | 4¢ | \$5.10 | 3.50 dollars |
| 4. | 75¢ | \$7.50 | .08 dollars |
| 5. | 20¢ | \$8.32 | .98 dollars |
| 6. | 82¢ | \$6.49 | 7.44 dollars |

Ounce, pound, and ton are units of weight.

Use after page 257.

Copy. Replace each — with a numeral that makes each sentence true.

- | | a | b |
|----|-----------------------------------|--|
| 1. | 5 lb. = <u>80</u> oz. | 48 oz. = <u>3</u> lb. |
| 2. | 2 T. = <u>4000</u> lb. | 6000 lb. = <u>3</u> T. |
| 3. | $\frac{3}{4}$ lb. = <u>12</u> oz. | 8 oz. = <u>$\frac{1}{2}$</u> lb. |
| 4. | 6 T. = <u>12,000</u> lb. | 500 lb. = <u>$\frac{1}{4}$</u> T. |
| 5. | 6 lb. = <u>96</u> oz. | 64 oz. = <u>4</u> lb. |

- | | | |
|----|----------------------------------|---|
| 6. | $\frac{1}{2}$ lb. = <u>8</u> oz. | 10,000 lb. = <u>5</u> T. |
| 7. | 8 lb. = <u>128</u> oz. | 80 oz. = <u>5</u> lb. |
| 8. | 10 T. = <u>20,000</u> lb. | 1000 lb. = <u>$\frac{1}{2}$</u> T. |

Fluid ounce, cup, pint, quart, and gallon are units of liquid measure.

Pint, quart, peck, and bushel are units of dry measure.

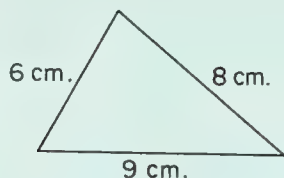
Use after page 258.

Copy. Replace each — with a numeral that makes each sentence true.

- | | a | b |
|-----|---------------------------------------|-----------------------|
| 1. | 4 qt. = <u>8</u> pt. | 6 pt. = <u>3</u> qt. |
| 2. | 8 c. = <u>4</u> pt. | 16 qt. = <u>2</u> pk. |
| 3. | 8 pt. = <u>4</u> qt. | 8 pk. = <u>2</u> bu. |
| 4. | $\frac{1}{2}$ pt. = <u>8</u> fl. oz. | 3 bu. = <u>12</u> pk. |
| 5. | 7 gal. = <u>28</u> qt. | 32 qt. = <u>4</u> pk. |
| 6. | 12 qt. = <u>3</u> gal. | 10 pt. = <u>5</u> qt. |
| 7. | 4 pt. = <u>64</u> fl. oz. | 24 qt. = <u>3</u> pk. |
| 8. | 16 c. = <u>8</u> pt. | 16 pk. = <u>4</u> bu. |
| 9. | 5 gal. = <u>20</u> qt. | 5 bu. = <u>20</u> pk. |
| 10. | $\frac{3}{4}$ pt. = <u>12</u> fl. oz. | 20 pk. = <u>5</u> bu. |
| 11. | 12 pt. = <u>6</u> qt. | 6 bu. = <u>24</u> pk. |

Metric Geometry

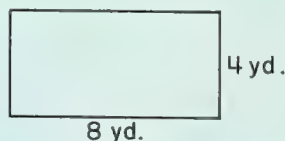
To find the perimeter measure of a figure, add the measures of all its sides.



$$p = 6 + 8 + 9 \\ = 23$$

Perimeter is 23 centimeters.

To find the area measure of a rectangle, multiply the measure of its length by the measure of its width.



$$A = 8 \times 4 \text{ or } 32$$

Area is 32 square yards.

Use after page 265.

Find the perimeter of each figure.

- | | <i>a</i> | <i>b</i> |
|----|---|--|
| 1. | <p style="text-align: center;">5 m.
20 m.</p> | <p style="text-align: center;">6 in.
22 in.</p> |
| 2. | <p style="text-align: center;">9 ft.
4 ft.
8 ft.
30 ft.</p> | <p style="text-align: center;">14 yd.
10 yd.
14 yd.
48 yd.</p> |

Solve each of the following.

3. A rug is in the shape of a square with each side 10 feet long. What is the perimeter of this rug? **40 ft.**

4. What is the perimeter of a rectangle 18 inches long and 12 inches wide? **60 in.**

Use after page 271.

Find the area of each rectangle below.

- | | <i>a</i> | <i>b</i> |
|----|---|---|
| 1. | <p style="text-align: center;">18 cm.
9 cm.
162 sq. cm.</p> | <p style="text-align: center;">12 ft.
144 sq. ft.</p> |
| 2. | <p style="text-align: center;">8 yd.
6 yd.
48 sq. yd.</p> | <p style="text-align: center;">9 in.
81 sq. in.</p> |

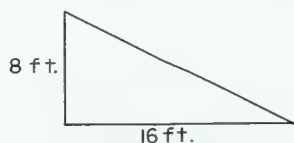
Solve each of the following.

3. A patio is in the shape of a rectangle 15 feet long and 18 feet wide. What is its area? **270 sq. ft.**

4. A backyard is in the shape of a square with each side 25 yards long. What is its area? **625 sq. yds.**

Metric Geometry

To find the area measure of a right triangle, find one half of the product of the measures of the two legs.



$$A = \frac{1}{2} \times (8 \times 16) = \frac{1}{2} \times 128 = 64$$

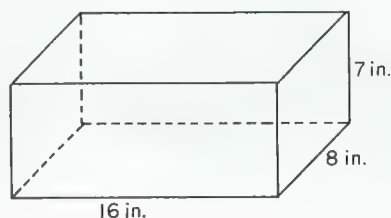
Area is 64 square feet.

Use after page 273.

Find the area of each right triangle below.

- | | <i>a</i> | <i>b</i> |
|----|-------------------|-------------------|
| 1. | <p>9 sq. cm.</p> | <p>55 sq. ft.</p> |
| 2. | <p>36 sq. m.</p> | <p>12 sq. yd.</p> |
| 3. | <p>40 sq. yd.</p> | <p>56 sq. m.</p> |
| 4. | <p>16 sq. in.</p> | <p>25 sq. ft.</p> |

To find the volume measure of a rectangular solid, find the product of the measures of its length, its width, and its height.



$$V = (16 \times 8) \times 7 = 128 \times 7 = 896$$

Volume is 896 cu. in.

Use after page 279.

Find the volume of each rectangular solid whose length, width, and height are listed below.

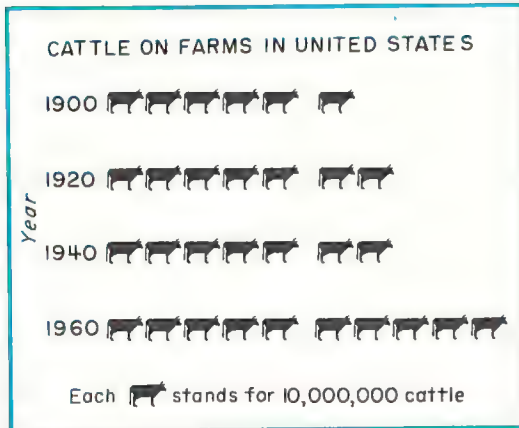
	Length	Width	Height	Volume
1.	15 in.	7 in.	4 in.	420 —cu. in.
2.	22 cm.	10 cm.	14 cm.	3080 —cu. cm.
3.	18 yd.	7 yd.	3 yd.	378 —cu. yd.
4.	8 ft.	4 ft.	6 ft.	192 —cu. ft.
5.	18 in.	9 in.	6 in.	972 —cu. in.
6.	4 ft.	3 ft.	5 ft.	60 —cu. ft.
7.	50 m.	25 m.	8 m.	10,000 —cu. m.
8.	75 ft.	28 ft.	10 ft.	21,000 —cu. ft.

Graphs

A picture graph is a drawing which displays information by using a picture or a symbol to stand for a certain number of things.

Use after page 287.

Use the following graph to answer the questions below.



1. What is its title? **Cattle on Farms in United States**
2. What is the graph about? **See below.**
3. What does each stand for? **10,000,000 cattle**
4. Copy and complete the following table for this graph.

Number of Cattle on Farms	
1900	60,000,000
1920	70,000,000
1940	70,000,000
1960	100,000,000

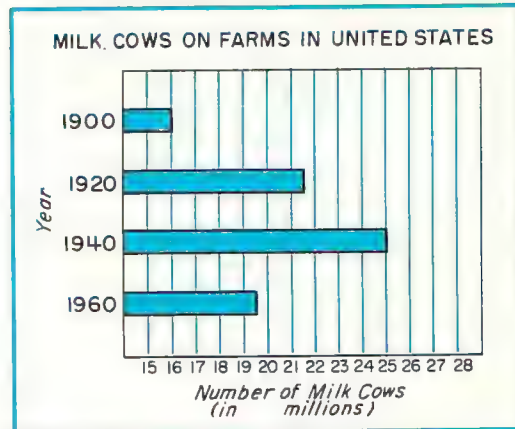
Use after page 287.

Use after page 289.

A bar graph is a drawing which displays information by using bars.

Use after page 289.

Use the following graph to answer the questions below.



1. What is its title? **Milk Cows on Farms in United States**
2. What is the graph about? **See below.**
3. In what year were there about 25,000,000 milk cows on farms in the United States? **1940**
4. Copy and complete the following table for this graph.

Number of Milk Cows on Farms	
1900	16,000,000
1920	21,500,000
1940	25,000,000
1960	19,500,000

2. the number of cattle on farms in United States by years **333**

2. the number of milk cows on farms in United States by years

Line Graphs

You can represent the pairs of numbers 1 and 2, 2 and 8, and so on, in the following table as ordered pairs of numbers.

A	1	2	3	4	5
B	2	8	2	8	10

{(1,2), (2,8), (3,2), (4,8), (5,10)}

Use after page 291.

List the set of ordered pairs shown in each table below. Take the first number of each pair from the top row of the table. **See below.**

1.

X	1	2	3	4	5	6
Y	1	4	9	9	4	1

2.

M	1	2	3	4	5	6
N	1	4	1	4	1	4

Use the set of ordered pairs below to complete the table that follows.

3. {(1,4), (2,6), (3,9), (4,8), (5,3)}

A	1	2	3	4	5
B	4	6	9	8	3

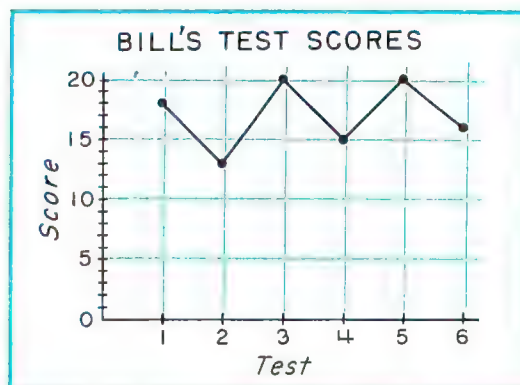
4. {(2,4), (3,6), (4,2), (5,1), (6,7)}

C	2	3	4	5	6
D	4	6	2	1	7

A line graph is a drawing which displays information by using points and line segments.

Use after page 295.

Use the graph below to answer the following questions.



1. What is shown on the horizontal number line? What is shown on the vertical number line? **numbers of the tests; test scores**
2. What is its title? **Bill's Test Scores**
3. What is the graph about? **the scores Bill made on 6 tests**
4. On which test or tests was the highest score made? What is the highest score that was made? **3 and 5; 20**
5. On which test was the lowest score made? What is that score? **test 2; 13**
6. On which test was a score of 18 made? Of 15 made? **test 1; test 4**
7. How many tests were taken by Bill? **6**

1. {(1,1), (2,4), (3,9), (4,9), (5,4), (6,1)}
2. {(1,1), (2,4), (3,1), (4,4), (5,1), (6,4)}

Diagnostic Self-Tests

The tests on pages 335–338 will help you and your teacher to see how much you have learned in mathematics. Do each test and check your work before you give your paper to your teacher.

Self-Test 1—Numeration

Write the decimal numeral for each of the following.

- eight tens plus four ones **84**
- $1000+400+50+7$ **1457**
- two million eight **2,000,008**
- $(3 \times 10,000) + (0 \times 1000) + (8 \times 100) + (1 \times 10) + (5 \times 1)$ **30,815**

Write the expanded notation for each number named below.

<i>a</i>	<i>b</i>	<i>c</i>
5. 28	315	6402
20+8	300+10+5	6000+400+2

Round off each number named below to the nearest 10, the nearest 100, and the nearest 1000.

<i>a</i>	<i>b</i>	<i>c</i>
9460	6750	10,960
6. 9458	6753	10,955
9500	6800	11,000
9000	7000	11,000

Use commas to separate each numeral below into periods. Then name each number by using words.

See below.

<i>a</i>	<i>b</i>
7. 1007600	2700006450

Self-Test 2—Addition

1. Which number is the identity number of addition? **zero**

Solve each open sentence below.

<i>a</i>	<i>b</i>
2. $7+9=\square$	$8+2+7=\square$
3. $\square=8+4$	$4+9+1=\square$
4. $\square-6=5$	$7+3+9=\square$
5. $\square-3=9$	$8+6+4=\square$
6. $8+5=\square$	$6+7+3=\square$

Copy. Find each sum.

<i>a</i>	<i>b</i>	<i>c</i>
7. $\begin{array}{r} 43 \\ +36 \\ \hline 79 \end{array}$	$\begin{array}{r} 125 \\ +234 \\ \hline 359 \end{array}$	$\begin{array}{r} 146 \\ +346 \\ \hline 492 \end{array}$
8. $\begin{array}{r} 241 \\ +39 \\ \hline 280 \end{array}$	$\begin{array}{r} 4173 \\ +1264 \\ \hline 5437 \end{array}$	$\begin{array}{r} 7464 \\ +287 \\ \hline 7751 \end{array}$
9. $\begin{array}{r} 61731 \\ +4248 \\ \hline 65979 \end{array}$	$\begin{array}{r} 44263 \\ +18198 \\ \hline 62461 \end{array}$	$\begin{array}{r} 18726 \\ +41295 \\ \hline 60021 \end{array}$
10. $\begin{array}{r} 321 \\ 486 \\ +349 \\ \hline 1156 \end{array}$	$\begin{array}{r} 4291 \\ 1803 \\ +2024 \\ \hline 8118 \end{array}$	$\begin{array}{r} 64310 \\ 10698 \\ +14013 \\ \hline 89021 \end{array}$

Self-Test 1

7a. one million seven thousand six hundred

7b. two billion seven hundred million six thousand four hundred fifty

Self-Test 3—Subtraction

Solve each open sentence below.

- | a | b |
|-------------------------|----------------------|
| 1. $15 - 9 = \boxed{6}$ | $\boxed{8} + 4 = 12$ |
| 2. $13 - \boxed{7} = 6$ | $6 + \boxed{5} = 11$ |

Copy. Replace each \odot with the correct symbol ($=$, $<$, or $>$).

- | a | b | c |
|---------------------|------------------|-------------|
| 3. $6 \odot 7 - 1$ | $10 - 3 \odot 6$ | $5 \odot 7$ |
| 4. $7 \odot 12 - 4$ | $13 - 5 \odot 8$ | $8 \odot 4$ |

Copy. Find each difference.

- | a | b | c |
|--|---|--|
| 5. $\begin{array}{r} 85 \\ -31 \\ \hline 54 \end{array}$ | $\begin{array}{r} 572 \\ -32 \\ \hline 540 \end{array}$ | $\begin{array}{r} 1784 \\ -1204 \\ \hline 580 \end{array}$ |
| 6. $\begin{array}{r} 764 \\ -37 \\ \hline 727 \end{array}$ | $\begin{array}{r} 294 \\ -168 \\ \hline 126 \end{array}$ | $\begin{array}{r} 2914 \\ -1307 \\ \hline 1607 \end{array}$ |
| 7. $\begin{array}{r} 524 \\ -81 \\ \hline 443 \end{array}$ | $\begin{array}{r} 486 \\ -192 \\ \hline 294 \end{array}$ | $\begin{array}{r} 6711 \\ -4140 \\ \hline 2571 \end{array}$ |
| 8. $\begin{array}{r} 9402 \\ -7137 \\ \hline 2265 \end{array}$ | $\begin{array}{r} 3682 \\ -1590 \\ \hline 2092 \end{array}$ | $\begin{array}{r} 64072 \\ -15978 \\ \hline 48094 \end{array}$ |

Self-Test 4—Multiplication

Solve each open sentence below.

- | a | b |
|------------------------------|------------------------------------|
| 1. $8 \times 6 = \boxed{48}$ | $4 \times 3 \times 2 = \boxed{24}$ |
| 2. $9 \times 7 = \boxed{63}$ | $\boxed{42} \div 7 = 6$ |

Answer the following.

3. Write $6+6+6+6$ as a multiplication numeral. 4×6

4. Write two division sentences for $7 \times 6 = 42$. $42 \div 6 = 7$, $42 \div 7 = 6$

Copy. Find each product.

- | a | b | c |
|--|--|---|
| 5. $\begin{array}{r} 21 \\ \times 4 \\ \hline 84 \end{array}$ | $\begin{array}{r} 432 \\ \times 5 \\ \hline 2160 \end{array}$ | $\begin{array}{r} 492 \\ \times 6 \\ \hline 2952 \end{array}$ |
| 6. $\begin{array}{r} 707 \\ \times 6 \\ \hline 4242 \end{array}$ | $\begin{array}{r} 2121 \\ \times 4 \\ \hline 8484 \end{array}$ | $\begin{array}{r} 9487 \\ \times 8 \\ \hline 75896 \end{array}$ |
| 7. $\begin{array}{r} 12 \\ \times 14 \\ \hline 168 \end{array}$ | $\begin{array}{r} 47 \\ \times 36 \\ \hline 1692 \end{array}$ | $\begin{array}{r} 134 \\ \times 29 \\ \hline 3886 \end{array}$ |
| 8. $\begin{array}{r} 725 \\ \times 90 \\ \hline 65250 \end{array}$ | $\begin{array}{r} 671 \\ \times 132 \\ \hline 88572 \end{array}$ | $\begin{array}{r} 483 \\ \times 264 \\ \hline 127512 \end{array}$ |

Self-Test 5—Division

Solve each open sentence below.

- | a | b |
|------------------------------|---------------------------|
| 1. $48 \div 6 = \boxed{8}$ | $480 \div 6 = \boxed{80}$ |
| 2. $8 \times \boxed{9} = 72$ | $\boxed{7} \times 5 = 35$ |
| 3. $72 \div 6 = \boxed{12}$ | $45 \div \boxed{5} = 9$ |
| 4. $91 \div 7 = \boxed{13}$ | $63 \div \boxed{9} = 7$ |

Answer the following.

5. Write a closed multiplication sentence for $24 \div 6 = 4$. $4 \times 6 = 24$

6. Change $\frac{80}{100}$ to a fraction in simplest form. $\frac{4}{5}$

Copy. Find each sum or difference in simplest form.

	<i>a</i>	<i>b</i>	<i>c</i>
	$\begin{array}{r} 5 \\ 9 \overline{)45} \\ \underline{24} \end{array}$	$\begin{array}{r} 13 \\ 7 \overline{)91} \\ \underline{189} \end{array}$	$\begin{array}{r} 16r1 \\ 4 \overline{)65} \\ \underline{84r1} \end{array}$
6.	$\begin{array}{r} 6 \overline{)144} \\ \underline{233} \end{array}$	$\begin{array}{r} 5 \overline{)945} \\ \underline{692} \end{array}$	$\begin{array}{r} 3 \overline{)253} \\ \underline{582r3} \end{array}$
7.	$\begin{array}{r} 4 \overline{)932} \\ \underline{1839} \end{array}$	$\begin{array}{r} 6 \overline{)4152} \\ \underline{1143r3} \end{array}$	$\begin{array}{r} 9 \overline{)5241} \\ \underline{4} \end{array}$
8.	$\begin{array}{r} 4 \overline{)7356} \\ \underline{6r3} \end{array}$	$\begin{array}{r} 7 \overline{)8004} \\ \underline{16} \end{array}$	$\begin{array}{r} 19 \overline{)76} \\ \underline{18r2} \end{array}$
9.	$\begin{array}{r} 16 \overline{)99} \\ \underline{81} \end{array}$	$\begin{array}{r} 24 \overline{)384} \\ \underline{52} \end{array}$	$\begin{array}{r} 22 \overline{)398} \\ \underline{51r2} \end{array}$
10.	$\begin{array}{r} 37 \overline{)2997} \end{array}$	$\begin{array}{r} 38 \overline{)1976} \end{array}$	$\begin{array}{r} 45 \overline{)2297} \end{array}$

	a	b	c	d
7.	$\begin{array}{r} \frac{3}{8} \\ +\frac{4}{8} \\ \hline \end{array}$	$\begin{array}{r} \frac{8}{12} \\ +\frac{5}{12} \\ \hline \end{array}$	$\begin{array}{r} \frac{12}{15} \\ -\frac{8}{15} \\ \hline \end{array}$	$\begin{array}{r} \frac{14}{16} \\ -\frac{8}{16} \\ \hline \end{array}$
	$\frac{7}{8}$	$\frac{1}{12}$	$\frac{4}{15}$	$\frac{3}{8}$
8.	$\begin{array}{r} \frac{5}{9} \\ +\frac{2}{3} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{5} \\ +\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{4} \\ -\frac{1}{8} \\ \hline \end{array}$	$\begin{array}{r} \frac{1}{2} \\ -\frac{1}{3} \\ \hline \end{array}$
	$\frac{1}{9}$	$\frac{7}{20}$	$\frac{5}{8}$	$\frac{1}{6}$
9.	$\begin{array}{r} 2\frac{1}{2} \\ +\frac{3}{8} \\ \hline \end{array}$	$\begin{array}{r} 2\frac{3}{4} \\ +5\frac{5}{12} \\ \hline \end{array}$	$\begin{array}{r} 1 \\ -\frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} 1 \\ -\frac{5}{8} \\ \hline \end{array}$
	$2\frac{7}{8}$	$6\frac{1}{12}$	$\frac{1}{2}$	$3\frac{3}{8}$

Copy. Find each sum or difference.

Solve each open sentence below.

$$\begin{array}{l} \text{1.} \quad \frac{3}{9} + \frac{4}{9} = a \frac{7}{9} \quad \frac{3}{12} + \frac{4}{12} + \frac{2}{12} = c \frac{9}{12} \text{ or } \frac{3}{4} \quad \begin{array}{r} 10. \quad .9 \quad 1.48 \quad 2.94 \quad 8.62 \\ +.6 \quad +.26 \quad -1.31 \quad -2.39 \\ \hline 1.5 \quad 1.74 \quad 1.63 \quad 6.23 \end{array} \\ \text{2.} \quad \frac{7}{12} - \frac{2}{12} = b \frac{5}{12} \quad 2\frac{3}{4} + \frac{1}{4} = d 2\frac{4}{4} \text{ or } 3 \end{array} \quad \text{Self-Test 7—Geometry}$$

Express each product below as a repeated addition and as a mixed numeral in simplest form. See below for the repeated addition.

3. $5 \times \frac{3}{8} \quad 1\frac{7}{8} \times \frac{2}{9} \quad 1\frac{7}{9} \times \frac{1}{4} \quad 1\frac{1}{2} \times \frac{2}{6} \quad 2\frac{1}{3}$

Copy. Find each product in simplest form.

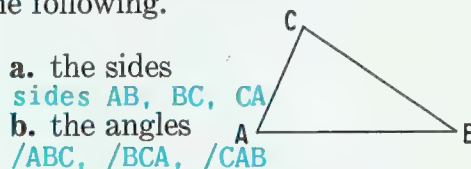
4. $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$ $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$ $\frac{2}{5} \times \frac{7}{8} = \frac{7}{20}$ $\frac{4}{5} \times \frac{5}{8} = \frac{1}{2}$

Answer the following.

5. Change $\frac{2}{5}$ to a decimal that is read as hundredths. **.40**

Answer the following.

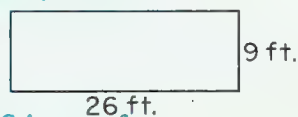
1. Using the figure below, name the following.



2. In the figure above, are the sides lines, line segments, or rays?
line segments

3. Find the perimeter and the area of the rectangle below.

perimeter: 70 ft.



area: 234 sq. ft.

(Turn the page.)

337

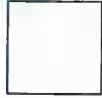
Self-Test 6

3a. $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$ 3b. $\frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9}$
3c. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ 3d. $\frac{2}{6} + \frac{2}{6} + \frac{2}{6} + \frac{2}{6} + \frac{2}{6} + \frac{2}{6} + \frac{2}{6}$

Self-Test 7 (continued)

4. Find the perimeter and the area of the square below.

perimeter: 16 m.



area: 16 sq. m.

5. Find the area of a right triangle whose legs are 14 meters and 7 meters long. 49 sq. m.

6. Find the volume of a box that is 25 inches long, 16 inches wide, and 10 inches high. 4000 cu. in.

Self-Test 8—Measurement

Answer the following.

1. Use a ruler to find the length of the line segment below to the nearest $\frac{1}{2}$ inch. To the nearest centimeter. $1\frac{1}{2}$ in.; 4 cm.



2. Which is longer: 1 foot or 10 inches? How much longer? 1 foot; 2 inches

3. Which is longer: 1 yard or 40 inches? How much longer? 40 inches; 4 inches

Replace each $\underline{\hspace{1cm}}$ with a numeral to make each sentence true.

- | a | b |
|---------------------------------|------------------------------|
| 4. 4 ft. = $\frac{48}{100}$ in. | 18 ft. = $\frac{6}{100}$ yd. |
| 5. 6 ft. = $\frac{72}{144}$ in. | 12 ft. = $\frac{4}{5}$ yd. |
| 6. 4 yd. = $\frac{144}{60}$ in. | 60 in. = $\frac{5}{60}$ ft. |

- | | |
|-----------------------------------|-------------------------------|
| 7. 20 dm. = $\frac{200}{100}$ cm. | 200 cm. = $\frac{2}{100}$ m. |
| 8. 3 lb. = $\frac{48}{16}$ oz. | 1000 lb. = $\frac{1}{2}$ T. |
| 9. 2 pt. = $\frac{4}{100}$ c. | 10 c. = $\frac{5}{100}$ pt. |
| 10. 12 qt. = $\frac{3}{4}$ gal. | 12 qt. = $\frac{24}{100}$ pt. |
| 11. 20 pk. = $\frac{5}{100}$ bu. | 3 bu. = $\frac{12}{100}$ pk. |

Self-Test 9—Problem Solving

Write an open sentence for each problem. Solve the open sentence. Answer the problem.

1. Tom weighed 106 pounds last year. This year he gained 12 pounds. How much does he weigh now?
 $106+12=\square$; 118 pounds

2. Bill and Harry together drove 485 miles. Bill drove 237 miles. How many miles did Harry drive?
 $485-237=\square$; 248 miles

3. A rope was 425 feet long. It was cut into pieces each 25 feet long. How many pieces of rope were there?
 $425\div 25=\square$; 17 pieces

4. Mr. Bond drove 448 miles. In doing so, he used 28 gallons of gasoline. How many miles did he travel on each gallon of gasoline?
 $448\div 28=\square$; 16 miles

5. Sally had 36 tickets to sell. She was able to sell $\frac{2}{3}$ of them. How many of the tickets was she able to sell?
 $\frac{2}{3}\times 36=\square$; 24 tickets

6. Jill bought $2\frac{1}{4}$ yards of ribbon. She had $\frac{1}{2}$ yard of ribbon. How much ribbon does she now have?
 $2\frac{1}{4}+\frac{1}{2}=\square$; $2\frac{3}{4}$ yards

HANDBOOK

Many of your questions about mathematics can be answered by using this handbook. An example is given for each idea or computation.

The handbook is easy to use. Its twelve sections are arranged in alphabetical order. The parts of each section are listed in the order they appear throughout the book. The numerals in () tell the pages in the book that give more information on that topic.

Addition

Terms and Concepts

1. Unnamed addend An unnamed addend is named by a placeholder. (12)

$$\begin{array}{rcl} \square + 4 = 13 & & 7 + a = 15 \\ \text{└────────── unnamed addend ─────────┘} \end{array}$$

2. Addends The numbers being added are called addends. (12, 33)

$$\begin{array}{r} 6 + 4 = 10 \\ \swarrow \searrow \\ \text{addends} \end{array}$$

3. Sum The result in addition is called the sum. (33)

$$\begin{array}{r} 8 + 13 = 21 \\ \text{└──────── sum ─────────┘} \end{array}$$

4. Addition numeral An addition numeral names a sum. (33)

$$\begin{array}{c} 5 + 9 \\ \text{addition numeral} \end{array}$$

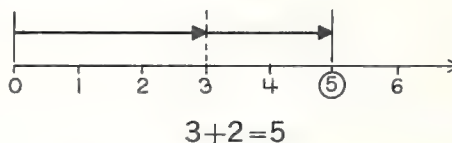
5. Commutative property of addition The order of two addends does not change the sum. (34)

$$4 + 5 = 5 + 4 \qquad 18 + 22 = 22 + 18$$

6. Identity number of addition Zero is called the identity number of addition. (35)

$$9 + 0 = 9 = 0 + 9$$

7. Addition on a number line Addition can be shown on a number line as follows. (37)



8. Addition and subtraction as inverse operations Addition and subtraction undo each other. (38)

$$\text{If } 3 + 4 = 7, \text{ then } 7 - 4 = 3.$$

9. Associative property of addition The way we group addends does not change the sum. (40)

$$\begin{aligned}(4+6)+5 &= 4+(6+5) \\ 10+5 &= 4+11 \\ 15 &= 15\end{aligned}$$

How to Add

1. Adding two numbers The examples below show how to add two numbers.

a.

$\begin{array}{r} 15 \\ +32 \\ \hline 47 \end{array}$	$\begin{array}{r l} & 0 \\ 1 & 5 \\ +3 & 2 \\ \hline 4 & 7 \end{array}$	(44)

b.

$$\begin{array}{r} 147 \\ +532 \\ \hline 679 \end{array}$$

Add the ones. Add the tens. Add the hundreds. (44)

c.

$\begin{array}{r} 39 \\ +24 \\ \hline 63 \end{array}$	$\begin{array}{r} 1 \\ 39 \\ +24 \\ \hline 63 \end{array}$
---	--

Add the ones. If the sum is greater than 9, name it in expanded notation. Add the tens. (48)

d.

$\begin{array}{r} 347 \\ +526 \\ \hline 873 \end{array}$	$\begin{array}{r} 1 \\ 347 \\ +526 \\ \hline 873 \end{array}$
--	---

Add the ones. Rename this sum if it is greater than 9. Add the tens. Add the hundreds. (48)

e.

$\begin{array}{r} 342 \\ +295 \\ \hline 637 \end{array}$	$\begin{array}{r} 1 \\ 342 \\ +295 \\ \hline 637 \end{array}$
--	---

Add the ones. Add the tens. Rename this sum if it is greater than 90. Add the hundreds. (52)

f.

$$\begin{array}{r} 1250 \\ +4139 \\ \hline 5389 \end{array}$$

Add the ones. Add the tens. Add the hundreds. Add the thousands. (96)

g.

$$\begin{array}{r} 43276 \\ +41403 \\ \hline 84679 \end{array}$$

Add the ones. Add the tens. Add the hundreds. Add the thousands. Add the ten thousands. (96)

h.

$\begin{array}{r} 24685 \\ +16776 \\ \hline 41461 \end{array}$	$\begin{array}{r} 1111 \\ 24685 \\ +16776 \\ \hline 41461 \end{array}$
--	--

Add the ones, the tens, the hundreds, and so on. Rename a sum when necessary. (100)

2. Adding three numbers The example below shows how to add three numbers.

$\begin{array}{r} 2362 \\ 1342 \\ +3469 \\ \hline 7173 \end{array}$	$\begin{array}{r} 111 \\ 2362 \\ 1342 \\ +3469 \\ \hline 7173 \end{array}$
---	--

Add the ones, the tens, the hundreds, and so on. Rename a sum when necessary. (100)

Decimals

Terms and Concepts

1. Decimals Fractional numbers can be renamed as decimals. (239)

$$\frac{9}{10} = .9$$

$$\frac{9}{100} = .09$$

2. Reading decimals Decimals are read as shown below. (239)

.6 $\left\{ \begin{array}{l} \text{six tenths} \\ \text{or} \\ \text{point six} \end{array} \right.$

.06 $\left\{ \begin{array}{l} \text{six hundredths} \\ \text{or} \\ \text{point zero six} \end{array} \right.$

How to Use Decimals

1. Common fractional equivalents You can change $\frac{3}{5}$ to a decimal numeral. (241)

$$\frac{3}{5} = \frac{3}{5} \times 1 = \frac{3}{5} \times \frac{20}{20} = \frac{3 \times 20}{5 \times 20} = \frac{60}{100} = .60$$

You can change a decimal to a fraction in simplest form. (241)

$$.50 = \frac{50}{100} = \frac{1 \times 50}{2 \times 50} = \frac{1}{2} \times \frac{50}{50} = \frac{1}{2} \times 1 = \frac{1}{2}$$

2. Decimals in addition You can add numbers expressed as decimals like you add whole numbers. Add the hundredths. Add the tenths. Add the ones. Rename a sum when necessary. (242, 243)

$$\begin{array}{r} 1.42 \\ +.78 \\ \hline 2.20 \end{array} \qquad \begin{array}{r} \overset{1}{1}.4 \overset{2}{2} \\ +.7 \overset{8}{8} \\ \hline 2.2 \overset{0}{0} \end{array}$$

3. Decimals in subtraction You can subtract .87 from 2.24 as shown below. Rename the minuend so that you can subtract in every place-value position. Subtract the hundredths. Subtract the tenths. Subtract the ones. (244)

$$\begin{array}{r} 2.24 \\ - .87 \\ \hline 1.37 \end{array} \qquad \begin{array}{r} \overset{11}{2} \overset{14}{2} 4 \\ - .8 \overset{7}{7} \\ \hline 1.3 \overset{7}{7} \end{array}$$

Distributive Property

Terms and Concepts

1. Multiplication distributes over addition You can distribute multiplication over addition if one of the factors is named as a sum. (110)

$$\begin{aligned} 5 \times 14 &= 5 \times (10 + 4) \\ &= (5 \times 10) + (5 \times 4) \\ &= 50 + 20 \\ &= 70 \end{aligned}$$

$$\begin{array}{r} 10 + 4 \\ \times 5 \\ \hline 50 + 20 = 70 \end{array}$$

2. Division distributes over addition You can distribute division over addition if the dividend is named as a sum. (122)

$$\begin{aligned} 84 \div 6 &= (60 + 24) \div 6 \\ &= (60 \div 6) + (24 \div 6) \\ &= 10 + 4 \\ &= 14 \end{aligned}$$

$$\begin{array}{r} 10 + 4 \\ 6 \overline{) 84} \qquad 6 \overline{) 60 + 24} \end{array}$$

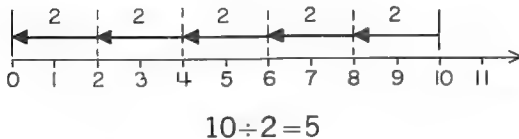
Division

Terms and Concepts

1. Quotient When one number is divided by another the result is called the quotient. (64)

$$32 \div 4 = 8 \quad \begin{array}{r} \text{quotient} \\ 4 \overline{) 32} \end{array}$$

2. Repeated subtraction Division can be thought of as repeated subtraction. (65)



3. Zero in division Division by zero is meaningless. We never divide by zero. (65)

4. Multiplication and division as inverse operations Multiplication and division undo each other. (66)

$$\text{If } 6 \times 5 = 30, \text{ then } 6 = 30 \div 5.$$

5. Divisor The number by which we divide is called the divisor. (72)

$$28 \div 7 = 4 \quad \begin{array}{r} \text{divisor} \\ 7 \overline{) 28} \end{array}$$

6. Dividend The number being divided is called the dividend. (72)

$$48 \div 8 = 6 \quad \begin{array}{r} 8 \overline{) 48} \\ \text{dividend} \end{array}$$

7. One in division If any number, except zero, is divided by itself, the quotient is one. (74)

$$8 \div 8 = 1 \quad 24 \div 24 = 1$$

8. Naming quotients A quotient may be expressed as follows. (120)

Twelve divided by three is equal to four.

$$12 \div 3 = 4 \quad \begin{array}{r} 4 \\ 3 \overline{) 12} \end{array} \quad \frac{12}{3} = 4$$

9. Checking division results To check a division result, multiply the quotient by the divisor. This product should be the dividend. (125)

$$\text{If } 35 \div 5 = 7, \text{ then } 7 \times 5 = 35.$$

10. Remainder The number that remains undivided is called the remainder. (130)

$$\begin{array}{r} 3 \\ 5 \overline{) 17} \\ \underline{15} \\ 2 \text{ --- remainder} \end{array}$$

How to Divide

1. Find an unnamed factor To find an unnamed factor, express the multiplication as a division and find the quotient. (76)

$$\begin{array}{ll} \square \times 8 = 24 & \text{so } \square = 24 \div 8 \\ 8 \times \square = 24 & \text{and } \square \times 8 = 24 \\ & \text{so } \square = 24 \div 8 \end{array}$$

2. Dividing tens and hundreds

Knowing the quotients in a division table helps you find quotients involving tens and hundreds. (82)

If $24 \div 4 = 6$, then $240 \div 4 = 60$.

3. Finding a quotient You can find a quotient as follows.

a.
$$\begin{array}{r} 14 \\ 3 \overline{) 42} \text{ --- } \left\{ \begin{array}{l} 3 \times 10 < 42 \\ 3 \times 20 > 42 \end{array} \right. \\ \underline{30} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

Think of 42 as $30 + 12$. Distribute division over the addition. (125)

b.
$$\begin{array}{r} 123 \\ 6 \overline{) 738} \text{ --- } \left\{ \begin{array}{l} 6 \times 100 < 738 \\ 6 \times 200 > 738 \end{array} \right. \\ \underline{600} \\ 138 \\ \underline{120} \\ 18 \\ \underline{18} \\ 0 \end{array}$$

Subtract multiples of the divisor from the dividend. (128)

c.
$$\begin{array}{r} 1336 \\ 7 \overline{) 9352} \text{ --- } \left\{ \begin{array}{l} 7 \times 1000 < 9352 \\ 7 \times 2000 > 9352 \end{array} \right. \\ \underline{7000} \\ 2352 \\ \underline{2100} \\ 252 \\ \underline{210} \\ 42 \\ \underline{42} \\ 0 \end{array}$$

Subtract multiples of the divisor from the dividend. (129)

d.
$$\begin{array}{r} 5 \\ 13 \overline{) 65} \text{ --- } 1T \times \text{ --- } < 6T \\ \underline{65} \\ 0 \end{array}$$

Round off the divisor and the dividend to the nearest ten and use $1T \times \text{ --- } < 6T$ to find the quotient. (146)

e.
$$\begin{array}{r} 12 \\ 18 \overline{) 216} \text{ --- } 2T \times \text{ --- } T < 216 \\ \underline{180} \\ 36 \\ \underline{36} \\ 0 \end{array}$$

Use sentences like $2T \times \text{ --- } T < 216$ to determine the tens digit of the quotient numeral. (148)

f.
$$\begin{array}{r} 39 \\ 41 \overline{) 1599} \text{ --- } \left\{ \begin{array}{l} 4T \times \text{ --- } H < 2Th \\ 4T \times \text{ --- } T < 16H \\ 4T \times \text{ --- } T > 16H \end{array} \right. \\ \underline{1230} \\ 369 \\ \underline{369} \\ 0 \end{array}$$

When dividing, you are subtracting multiples of the divisor from the dividend. Since $4T \times 4T = 16H$ and $16H > 1599$ we know that the quotient is less than 40. (150)

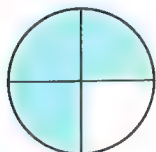
4. Find a remainder To find a remainder, divide until the result of subtracting is less than the divisor. (130)

$$\begin{array}{r} 42 \\ 42 \overline{) 1766} \\ \underline{1680} \\ 86 \\ \underline{84} \\ 2 \text{ --- remainder --- } 0 \end{array} \quad \begin{array}{r} 54 \\ 35 \overline{) 1890} \\ \underline{1750} \\ 140 \\ \underline{140} \\ 0 \end{array}$$

Fractional Numbers

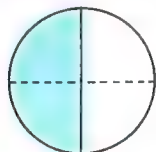
Terms and Concepts

1. Meaning of a fraction $\frac{3}{4}$ means 3 out of 4 parts of the same size. (161-163)



$\frac{3}{4}$ of the interior is colored.

2. Equivalent fractions Fractions that name the same number are called equivalent fractions. (166)



$$\frac{1}{2} = \frac{2}{4}$$

3. Kinds of fractions Fractions can name numbers less than one, greater than one, or one. (168)

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{3}{8}$ —less than one

$\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{8}{3}$ —greater than one

$\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}$ —one

4. Addition is commutative For all numbers that can be named by fractions, addition is commutative. (172)

$$\frac{3}{8} + \frac{2}{8} = \frac{2}{8} + \frac{3}{8}$$

5. Addition is associative For all numbers that can be named by fractions, addition is associative. (174)

$$\left(\frac{3}{9} + \frac{4}{9}\right) + \frac{2}{9} = \frac{3}{9} + \left(\frac{4}{9} + \frac{2}{9}\right)$$

6. Multiplication is commutative For all numbers that can be named by fractions, multiplication is commutative. (186)

$$\frac{2}{5} \times \frac{3}{7} = \frac{3}{7} \times \frac{2}{5}$$

7. Identity number of multiplication A fractional number is not changed when it is multiplied by the number one. You call the number one the identity number of multiplication. (187)

$$\frac{3}{4} \times 1 = \frac{3}{4}$$

$$1 \times \frac{2}{3} = \frac{2}{3}$$

8. Mixed numerals A mixed numeral names the sum of a whole number and a fractional number. (192)

$$4 + \frac{3}{5} = 4\frac{3}{5}$$

How to Use Fractions

1. Adding fractional numbers The sum of $\frac{3}{9}$ and $\frac{4}{9}$ is found as shown below. (170)

$$\frac{3}{9} + \frac{4}{9} = \frac{3+4}{9} = \frac{7}{9}$$

2. Fractional part of a number The product of 5 and $\frac{2}{3}$ is found as shown below. (183)

$$5 \times \frac{2}{3} = \frac{5 \times 2}{3} = \frac{10}{3}$$

3. Multiplication of fractional numbers The product of $\frac{2}{3}$ and $\frac{3}{4}$ is found as shown below. (184)

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12}$$

4. Renaming a fractional number

The identity number of multiplication helps you find many names for every fractional number. (188)

$$\begin{array}{l} \frac{2}{3} = \frac{2}{3} \times 1 \\ = \frac{2}{3} \times \frac{2}{2} \\ = \frac{2 \times 2}{3 \times 2} \\ = \frac{4}{6} \end{array} \quad \begin{array}{l} \frac{2}{3} = \frac{2}{3} \times 1 \\ = \frac{2}{3} \times \frac{3}{3} \\ = \frac{2 \times 3}{3 \times 3} \\ = \frac{6}{9} \end{array}$$

5. Fractions in simplest form

You can change a fraction to its simplest form as shown below. (190)

$$\begin{array}{l} \frac{5}{10} = \frac{5 \times 1}{5 \times 2} \\ = \frac{5}{5} \times \frac{1}{2} \\ = 1 \times \frac{1}{2} \\ = \frac{1}{2} \end{array} \quad \begin{array}{l} \frac{4}{8} = \frac{1 \times 4}{2 \times 4} \\ = \frac{1}{2} \times \frac{4}{4} \\ = \frac{1}{2} \times 1 \\ = \frac{1}{2} \end{array}$$

6. Mixed numerals and fractions

You change $3\frac{1}{4}$ to a fraction as shown in A. You change $\frac{13}{4}$ to a mixed numeral as shown in B. (194)

A	B
$3\frac{1}{4} = 3 + \frac{1}{4}$	$\frac{13}{4} = \frac{12+1}{4}$
$= \frac{12}{4} + \frac{1}{4}$	$= \frac{12}{4} + \frac{1}{4}$
$= \frac{12+1}{4}$	$= 3 + \frac{1}{4}$
$= \frac{13}{4}$	$= 3\frac{1}{4}$

7. Mixed numerals in simplest form A mixed numeral is in simplest form if the fraction is in simplest form and it names a number less than one. (196)

$$\begin{aligned} 2\frac{3}{6} &= 2 + \frac{3}{6} \\ &= 2 + (\frac{3 \times 1}{3 \times 2}) \\ &= 2 + (\frac{3}{3} \times \frac{1}{2}) \\ &= 2 + (1 \times \frac{1}{2}) \\ &= 2 + \frac{1}{2} \text{ or } 2\frac{1}{2} \end{aligned}$$

8. Subtracting fractional numbers

To subtract a fractional number like $\frac{2}{9}$ from $\frac{7}{9}$, use the following method. (202)

$$\frac{7}{9} - \frac{2}{9} = \frac{7-2}{9} = \frac{5}{9}$$

9. Adding fractional numbers having different denominators Rename both numbers with a common denominator. Then add. (206)

$$\begin{array}{r} \frac{2}{3} \\ + \frac{3}{5} \\ \hline \end{array} \longrightarrow \begin{array}{r} \frac{10}{15} \\ + \frac{9}{15} \\ \hline \frac{19}{15} = 1\frac{4}{15} \end{array}$$

10. Subtracting fractional numbers having different denominators Rename both numbers with a common denominator. Then subtract. (208)

$$\begin{array}{r} \frac{4}{5} \\ - \frac{2}{3} \\ \hline \end{array} \longrightarrow \begin{array}{r} \frac{12}{15} \\ - \frac{10}{15} \\ \hline \frac{2}{15} \end{array}$$

11. Adding fractional numbers Rename both numbers with a common denominator. Then add. (210)

$$\begin{array}{r} 3\frac{1}{4} \\ + \frac{2}{3} \\ \hline \end{array} \longrightarrow \begin{array}{r} 3\frac{3}{12} \\ + \frac{8}{12} \\ \hline 3\frac{11}{12} \end{array}$$

12. Subtracting a fractional number from the number one Rename the minuend as a fraction having the same denominator. Then subtract. (212)

$$\begin{array}{r} 1 \\ - \frac{3}{4} \\ \hline \end{array} \longrightarrow \begin{array}{r} \frac{4}{4} \\ - \frac{3}{4} \\ \hline \frac{1}{4} \end{array}$$

Geometry

Terms and Concepts

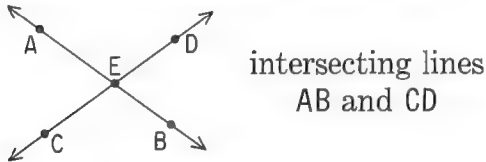
1. Line segment A set of points between and including two points is called a line segment. (217)



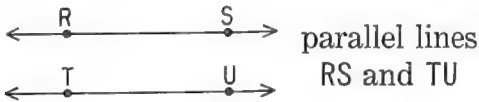
2. Line If you extend a line segment in both directions, you are thinking of a line. (218)



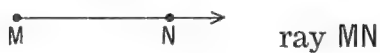
3. Intersecting lines Two lines that have only one point in common are called intersecting lines. (221)



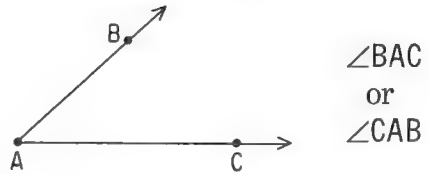
4. Parallel lines Two lines in the same plane that do not intersect are called parallel lines. (221)



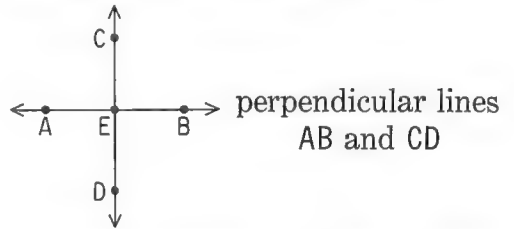
5. Ray A ray is part of a line which extends in only one direction from a point. (222)



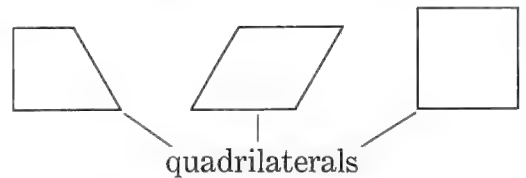
6. Angle A figure formed by two rays with a common endpoint is called an angle. (222)



7. Perpendicular lines When two lines intersect so that four right angles are formed, the lines are perpendicular to each other. (224)



8. Quadrilateral A simple closed figure formed by four line segments is called a quadrilateral. (228)



9. Rectangle A quadrilateral that has four right angles is called a rectangle. (228)

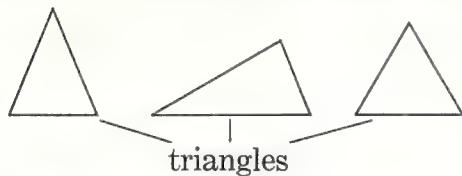


10. Square A rectangle that has four congruent line segments as its sides is called a square. (230)



square

11. Triangle A simple closed figure formed by three line segments as its sides is called a triangle. (232)

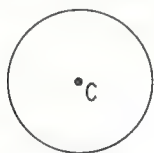


12. Right triangle A triangle that has one right angle is called a right triangle. (232)



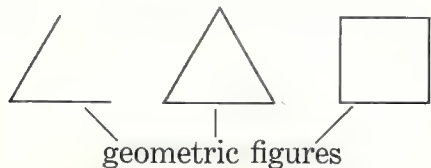
right triangle

13. Circle A circle is a set of points in a plane that are all the same distance from a point called the center of the circle. (234)



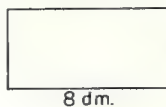
circle C

14. Geometry Geometry is the study of geometric figures. (263)



How to Find Perimeter, Area, and Volume

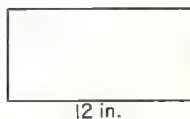
1. Perimeter Add the measures of the sides. (264)



$$p = 8 + 4 + 8 + 4 \\ = 24$$

Perimeter is 24 dm.

2. Area of a rectangle Multiply the measure of the length by the measure of the width. (268)



$$A = 12 \times 6 \\ = 72$$

Area is 72 sq. in.

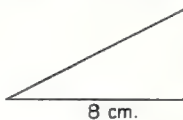
3. Area of a square Multiply the measure of one of its sides by itself. (270)



$$A = 4 \times 4 \\ = 16$$

Area is 16 sq. yd.

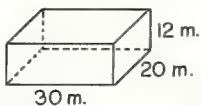
4. Area of a right triangle Find one half the product of the measures of the two legs. (272)



$$A = \frac{1}{2} \times (8 \times 4) \\ = 16$$

Area is 16 sq. cm.

5. Volume of a rectangular solid Multiply the measures of the length, width, and height. (278)



$$V = 30 \times 20 \times 12 \\ = 7200$$

Volume is 7200 cu. m.

Graphs

Terms and Concepts

1. Picture graph A picture graph is a drawing which displays information by using a picture or a symbol to stand for a certain number of things. (283)

2. Bar graph A bar graph is a drawing which displays information by using bars. (288)

3. Ordered pair Ordered pairs can be expressed as shown. (290)

(1,1), (2,4), (3,9), (4,16)

4. Line graph A line graph is a drawing which displays information by using points and line segments. (294)

How to Make a Picture Graph

Making picture graphs To make a picture graph use the following steps. (286)

a. Choose a symbol that is easy to draw.

b. Let each symbol stand for a convenient number of things.

c. Decide what each row of symbols stands for.

d. Draw the correct number of symbols neatly in each row.

e. Write a title above the graph. Write a statement telling what each symbol stands for.

Measurement

Terms and Concepts

1. Measure and unit of measure The number of units is the measure. The kind of unit is the unit of measure. (246)

12 inches
measure ——— unit of measure

2. Measurement A measurement names both a measure and a unit. (246)

10 feet 148 cubic yards
——— measurements ———

How to Rename a Length

1. Yards to feet You can change a number of yards to a number of feet as shown below. (250)

Since $1 \text{ yd.} = 3 \text{ ft.}$,

$4 \text{ yd.} = (4 \times 3) \text{ ft. or } 12 \text{ ft.}$

2. Feet to yards You can change a number of feet to a number of yards as shown below. (250)

Since $3 \text{ ft.} = 1 \text{ yd.}$,

$15 \text{ ft.} = (15 \div 3) \text{ yd. or } 5 \text{ yd.}$

Multiplication

Terms and Concepts

1. Factors Numbers that are multiplied are called factors. (62)

$$\begin{array}{c} 4 \times 7 = 28 \\ \swarrow \searrow \\ \text{factors} \end{array}$$

2. Product When two numbers are multiplied, the result is called the product. (62)

$$\begin{array}{c} 5 \times 9 = 45 \\ | \\ \text{product} \end{array}$$

3. Commutative property of multiplication Changing the order of two factors does not change the product. (68)

$$5 \times 4 = 4 \times 5 \qquad 8 \times 19 = 19 \times 8$$

4. Zero as a factor The product of zero and any number is zero. (69)

$$0 \times 4 = 0 \qquad 0 \times 21 = 0$$

5. Identity number of multiplication One is called the identity number of multiplication. (69)

$$1 \times 7 = 7 \qquad 25 \times 1 = 25$$

6. Unnamed factor An unnamed factor is not named by a numeral, only by a placeholder like \square , \triangle , a , or n . (76)

$$\begin{array}{c} \square \times 7 = 56 \qquad 9 \times a = 54 \\ \text{unnamed factors} \end{array}$$

7. Inverse operations Multiplication and division undo each other. (76)

$$\text{If } 28 \div 7 = 4, \text{ then } 4 \times 7 = 28.$$

8. Associative property of multiplication When multiplying 3 numbers, it does not matter which way you associate or group the factors. (78)

$$\begin{array}{l} (3 \times 4) \times 6 = 3 \times (4 \times 6) \\ 12 \times 6 = 3 \times 24 \\ 72 = 72 \end{array}$$

How to Multiply

1. Multiples of 10 and 100 Knowing the multiples of 1, of 10, and of 100 helps you find products like the one shown below. (80)

$$\begin{array}{l} 4 \times 50 = 4 \times (5 \times 10) \\ = (4 \times 5) \times 10 \\ = 20 \times 10 \\ = 200 \end{array}$$

2. Multiplying two numbers The examples below show how to multiply two numbers.

$$\begin{array}{r} \text{a.} \quad \begin{array}{r} 46 \\ \times 9 \\ \hline 54 \end{array} \quad \begin{array}{r} 40 + 6 \\ \times 9 \\ \hline 54 \end{array} \end{array}$$

$$\begin{array}{r} 360 \text{ --- } 9 \times 6 \text{ --- } 54 \\ 414 \text{ --- } 9 \times 40 \text{ --- } 360 \\ 414 \end{array}$$

Rename one factor as a sum. Distribute multiplication over addition. (112)

$$\begin{array}{r}
 \text{b. } 324 \\
 \times 4 \\
 \hline
 16 \text{ --- } 4 \times 4 \\
 80 \text{ --- } 4 \times 20 \\
 1200 \text{ --- } 4 \times 300 \\
 \hline
 1296
 \end{array}$$

Name 324 in expanded notation and distribute multiplication over addition. (114)

$$\begin{array}{r}
 \text{c. } 3421 \\
 \times 5 \\
 \hline
 5 \text{ --- } 5 \times 1 \\
 100 \text{ --- } 5 \times 20 \\
 2000 \text{ --- } 5 \times 400 \\
 15000 \text{ --- } 5 \times 3000 \\
 \hline
 17105
 \end{array}$$

Rename the multiplicand in expanded notation and distribute multiplication over addition. (116)

$$\begin{array}{r}
 \text{d. } 28 \\
 \times 13 \\
 \hline
 24 \text{ --- } 3 \times 8 \\
 60 \text{ --- } 3 \times 20 \\
 80 \text{ --- } 10 \times 8 \\
 200 \text{ --- } 10 \times 20 \\
 \hline
 364
 \end{array}
 \qquad
 \begin{array}{r}
 28 \\
 \times 13 \\
 \hline
 84 \\
 280 \\
 \hline
 364
 \end{array}$$

Distribute multiplication over addition, but write the results in shortened form as shown above. (138)

$$\begin{array}{r}
 \text{e. } 531 \\
 \times 415 \\
 \hline
 2655 \text{ --- } 5 \times 531 \\
 5310 \text{ --- } 10 \times 531 \\
 212400 \text{ --- } 400 \times 531 \\
 \hline
 220365
 \end{array}$$

Distribute multiplication over addition, but write the results in shortened form as shown above. (142)

Problem Solving

Terms and Concepts

1. Open sentence A sentence which is neither true nor false is called an open sentence. (13)

$$\square + 4 = 12 \qquad 6 \times a = 24$$

2. Closed sentence A sentence which is either true or false is called a closed sentence. (17)

$$24 \div 4 = 6 \qquad 13 - 5 = 7$$

3. Solution A solution makes an open sentence true. (18)

The solution of $4 + \square = 7$ is 3.

How to Solve Problems

Steps to use in problem solving
The procedure given below can be very helpful in solving story problems. (152)

- a. Read the problem carefully.
- b. Draw a diagram, if needed.
- c. Decide on which operation to use.
- d. Write an open sentence.
- e. Solve the open sentence.
- f. Answer the problem.

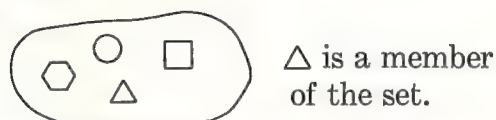
Sets

Terms and Concepts

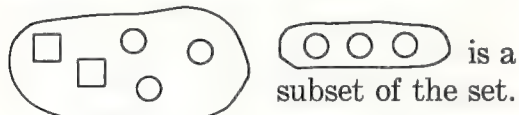
1. Set A set is any collection of objects. (5)



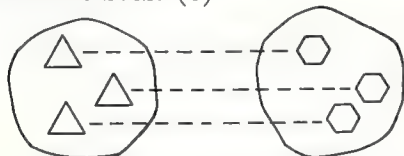
2. Member of a set Each object in a set is called a member of that set. (5)



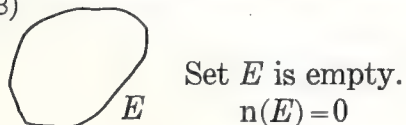
3. Subset A set contained within another set is called a subset of the set which contains it. (5)



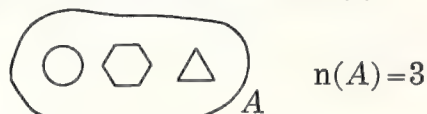
4. Equivalent sets Sets which can be matched one-to-one are called equivalent sets. (6)



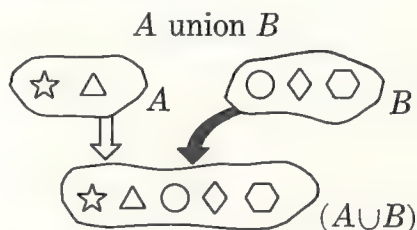
5. Empty set The set that contains no members is called the empty set. (8)



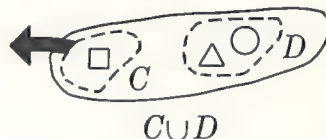
6. Number of a set The number of objects in a set is called the number of the set. We use $n(A)$ to mean the number of set A . (8)



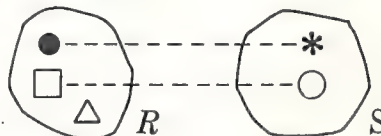
7. Union of sets When two sets are joined, the resulting set is called their union. (10)



8. Removing a subset You can remove a subset from any set that is not empty. (13)



9. Non-equivalent sets Two sets which cannot be matched one-to-one are called non-equivalent sets. (14)



$n(R) > n(S)$ 3 is greater than 2
 $n(S) < n(R)$ 2 is less than 3

10. Replacement set The set of numbers which may be named in place of the placeholder in an open sentence is called the replacement set. (18)

Open sentence: $\square + 3 = 7$

Replacement set: $\{3, 4, 5\}$

Using each member of the replacement set, you get these closed sentences.

$$3 + 3 = 7 \quad 4 + 3 = 7 \quad 5 + 3 = 7$$

11. Solution set The set of numbers in a replacement set which make the open sentence true is called the solution set. (19)

Open sentence: $14 - \square > 8$

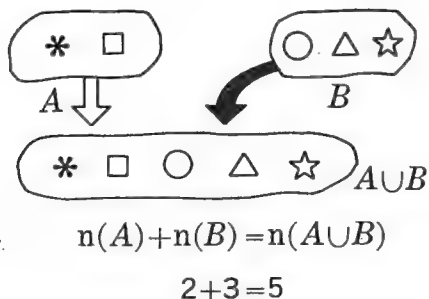
Replacement set: $\{4, 5, 6\}$

Using each member of the replacement set, you get these closed sentences.

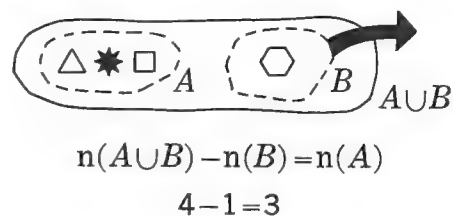
$$14 - 4 > 8 \quad 14 - 5 > 8 \quad 14 - 6 > 8$$

Solution set: $\{4, 5\}$

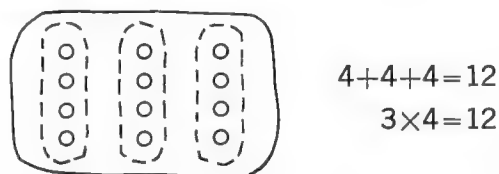
12. Union of sets and addition The union of sets may be used to explain the addition of numbers. (33)



13. Sets and subtraction Sets can be used to illustrate subtraction. (36)



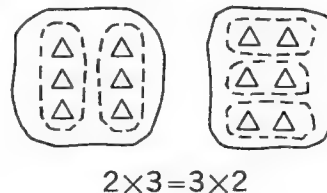
14. Sets and multiplication Sets can be used to illustrate that multiplication is a short way of finding a sum when all addends are the same. (59)



15. Sets and division Sets can be used to illustrate that division undoes multiplication. (63)



16. Sets and the commutative property of multiplication Sets can be used to illustrate that changing the order of two factors does not change the product. (68)



Subtraction

Terms and Concepts

1. Subtraction Subtraction is an operation on two numbers resulting in a third number called the difference. (36)

$$8-2=6$$

2. Subtraction numeral We read $8-5$ as *eight minus five*. We call $8-5$ a subtraction numeral. (36)

Subtraction numerals

$14-9$ — fourteen minus nine

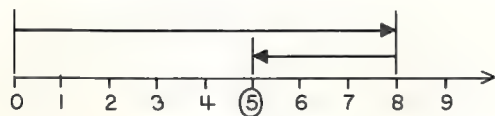
$6-2$ — six minus two

$12-4$ — twelve minus four

3. Difference When one number is subtracted from another, the result is called the difference. (36)

$$\begin{array}{r} 13-5=8 \\ \quad | \\ \text{difference} \end{array}$$

4. Subtraction on a number line Subtraction can be shown as follows. (37)



$$8-3=5$$

5. Subtraction and addition as inverse operations Subtraction and addition undo each other. (38)

If $3+4=7$, then $7-4=3$.

If $4+3=7$, then $7-3=4$.

and

If $7-4=3$, then $3+4=7$.

If $7-3=4$, then $4+3=7$.

6. Checking subtraction results Add the difference and the number subtracted. This sum should be the number subtracted from. (38)

Subtract	$\left\{ \begin{array}{r} 48 \\ -25 \\ \hline 23 \end{array} \right.$	Same
Check	$\left\{ \begin{array}{r} 23 \\ +25 \\ \hline 48 \end{array} \right.$	

7. Subtracting a number from itself When any number is subtracted from itself, the difference is zero. (39)

$$6-6=0$$

$$36-36=0$$

8. Subtracting zero Subtracting zero does not change a number. (39)

$$4-0=4$$

$$25-0=25$$

9. Repeated subtraction Division can be thought of as repeated subtraction. When dividing you are subtracting multiples of the divisor from the dividend. (65)

$$12 \div 3 = 4$$

$$\begin{array}{r} 12 \\ -3 \\ \hline 9 \end{array} \rightarrow \begin{array}{r} 9 \\ -3 \\ \hline 6 \end{array} \rightarrow \begin{array}{r} 6 \\ -3 \\ \hline 3 \end{array} \rightarrow \begin{array}{r} 3 \\ -3 \\ \hline 0 \end{array}$$

How to Subtract

1. Subtracting one number from another number You can subtract one number from another number as shown in the examples below.

$$\begin{array}{r} \text{a.} \quad \begin{array}{r} 65 \\ -34 \\ \hline 31 \end{array} \end{array}$$

Think of naming the minuend and the subtrahend in expanded notation. Use the idea of place value and subtract the ones. Subtract the tens. (46)

$$\begin{array}{r} \text{b.} \quad \begin{array}{r} 859 \\ -436 \\ \hline 423 \end{array} \end{array}$$

Check to see if you can subtract in every place-value position. Subtract the ones. Subtract the tens. Subtract the hundreds. (46)

$$\begin{array}{r} \text{c.} \quad \begin{array}{r} 492 \\ -238 \\ \hline 254 \end{array} \end{array}$$

Rename the minuend so you can subtract the ones. Subtract the ones, the tens, and the hundreds. (50)

$$\begin{array}{r} \text{d.} \quad \begin{array}{r} 624 \\ -391 \\ \hline 233 \end{array} \end{array}$$

Rename the minuend so you can subtract the tens. Subtract the ones, the tens, and the hundreds. (54)

$$\begin{array}{r} \text{e.} \quad \begin{array}{r} 8473 \\ -5132 \\ \hline 3341 \end{array} \end{array}$$

Th	H	T	O
8	4	7	3
-5	1	3	2
3	3	4	1

You need not write the expanded notation. Instead, use the idea of place value when subtracting the ones, the tens, the hundreds, and the thousands. (98)

$$\begin{array}{r} \text{f.} \quad \begin{array}{r} 7438 \\ -2654 \\ \hline 4784 \end{array} \end{array}$$

Th	H	T	O
7	4	3	8
-2	6	5	4
4	7	8	4

Check to see if you can subtract in every place-value position. Rename the minuend so you can subtract in every place-value position. Then subtract the ones, the tens, the hundreds, and the thousands. (98)

$$\begin{array}{r} \text{g.} \quad \begin{array}{r} 24987 \\ -11456 \\ \hline 13531 \end{array} \end{array}$$

TTh	Th	H	T	O
2	4	9	8	7
-1	1	4	5	6
1	3	5	3	1

Check to see if you can subtract in every place-value position. Then subtract the ones, the tens, the hundreds, and so on. (99)

$$\begin{array}{r} \text{h.} \quad \begin{array}{r} 85431 \\ -28868 \\ \hline 56563 \end{array} \end{array}$$

TTh	Th	H	T	O
8	5	4	3	1
-2	8	8	6	8
5	6	5	6	3

If you cannot subtract in every place-value position, rename the minuend so that you can subtract in every place-value position. Then subtract the ones, the tens, the hundreds, and so on. (102)

Tables of Measures

Use the tables of measures below for quick reference or for review. The = sign is used to mean *is equivalent to*.

Length (English)

12 inches (in.) = 1 foot (ft.)
3 feet (ft.) = 1 yard (yd.)
36 inches (in.) = 1 yard (yd.)
5280 feet (ft.) = 1 mile (mi.)
1760 yards (yd.) = 1 mile (mi.)

Length (Metric)

10 millimeters (mm.) = 1 centimeter (cm.)
10 centimeters (cm.) = 1 decimeter (dm.)
10 decimeters (dm.) = 1 meter (m.)
100 centimeters (cm.) = 1 meter (m.)
1000 meters (m.) = 1 kilometer (km.)

Weight

16 ounces (oz.) = 1 pound (lb.)
2000 pounds (lb.) = 1 ton (T.)

Liquid

16 fluid ounces (fl. oz.) = 1 pint (pt.)
2 cups (c.) = 1 pint (pt.)
2 pints (pt.) = 1 quart (qt.)
4 quarts (qt.) = 1 gallon (gal.)

Time

60 seconds (sec.) = 1 minute (min.)
60 minutes (min.) = 1 hour (hr.)
24 hours (hr.) = 1 day
7 days = 1 week (wk.)
12 months = 1 year (yr.)

Dry

2 pints (pt.) = 1 quart (qt.)
8 quarts (qt.) = 1 peck (pk.)
4 pecks (pk.) = 1 bushel (bu.)

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